Minimal Little Higgs Model and Dark Matter

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Outline

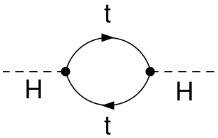
1. Motivation
   - Little Hierarchy Problem
   - Little Higgs Model
   - One Example
2. The Model
   - Bosonic Sector
   - Fermionic Sector: Z_2 broken
   - Fermionic Sector: Z_2 unbroken
3. Dark Matter
   - Dark Matter
   - Relic Abundance
   - Direct Detection

Explore New Physics beyond the Standard Model

- The Electroweak Symmetry Breaking:
  - Higgs Mechanism: described in the standard model and predicts the Higgs Boson.
  - Dynamical Symmetry Breaking: Technicolor, Topcolor or Walking Technicolor model.
- The Dark Matter:
  - What is the particle content of it? Scalars, Fermions or Gauge Bosons?
  - How does it couple to ordinary particles?
- Other Puzzles:
  - Hierarchies of fermion masses and their mixings?
  - The large mixings in the lepton sector versus the small mixings in the quark sector?
  - ......

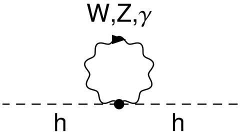
Radiative Corrections to the Higgs Boson Mass

- The mass of the Higgs field is not stable against radiative corrections:



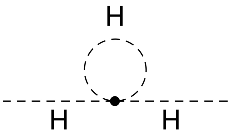
$$-\frac{3\lambda_t^2}{8\pi^2}\Lambda^2$$

$\Lambda = 10$ TeV: $\sim$ (2 TeV)^2



$$\frac{3(3g^2+g_y^2)}{64\pi^2}\Lambda^2$$

$\sim$ (700 GeV)^2



$$\frac{\lambda^2}{16\pi^2}\Lambda^2$$

$\sim$ (500 GeV)^2

- LEP paradox [Barbieri and Strumia, 2000]:
  - The mass of Higgs boson is less than 250 GeV.
  - The cutoff Λ of relevant higher-dimensional operators must be greater than 5-10 GeV.

Little Higgs Model [Arkani-Hamed, Cohen and Georgi, 2001]

- Identify the Higgs doublet as a pseudo-Nambu-Goldstone boson (PNGB) of a spontaneously broken global symmetry.
- Collective Symmetry Breaking: two or more couplings are needed to explicitly break the global symmetry.
- Consequence: only logarithmically divergent potentials of the Higgs doublet are generated at one-loop level. The weak scale can be protected up to 5-10 TeV.

One Example

- The VEV of a triplet spontaneously breaks $U(3)$ to $U(2)$:

$$U(3) \xrightarrow{\langle\phi\rangle=(0,0,f)^T} U(2)\,.$$

- 5 Goldstone Bosons: one doublet and one singlet of $U(2)$:

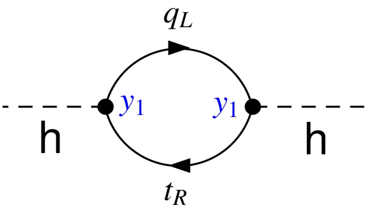
$$\phi \;=\; exp\left[\frac{i}{f}\begin{pmatrix} & h \\ h^\dagger & \end{pmatrix}\right]\begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} = \begin{pmatrix} h \\ f - \frac{h^2}{2\,f} \end{pmatrix} + \cdots$$

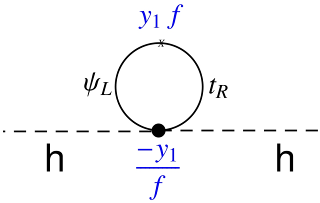
- Using Yukawa couplings to explicitly break the global symmetry:

$$\begin{aligned} & y_1 \bar{Q}_L\, \phi\, t_R \,+\, y_2\, f\, \bar{\psi}_L \psi_R \,+\, h.c. \\ = \;& y_1 \bar{q}_L\, h\, t_R \,-\, \frac{y_1}{2\,f} \bar{\psi}_L\, h^2\, t_R \,+\, y_1\, f\, \bar{\psi}_L\, t_R \,+\, y_2\, f\, \bar{\psi}_L \psi_R \,+\, h.c. \end{aligned}$$

with $\bar{Q}_L \equiv (\bar{q}_L, \bar{\psi}_L)$, a triplet of $U(3)$.

- The cutoff-squared terms are cancelled:



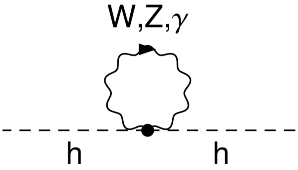


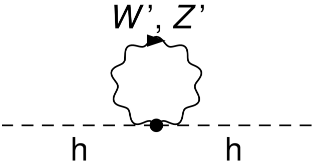
- One-Loop Effective Potential [Coleman and Weinberg, 1973]

$$m_f^2 = m_f\, m_f^\dagger = \begin{pmatrix} y_1^2\, f^2 \sin^2 \frac{h}{f} & y_1^2\, f^2 \sin \frac{h}{f} \cos \frac{h}{f} \\ y_1^2\, f^2 \sin \frac{h}{f} \cos \frac{h}{f} & y_1^2\, f^2 \cos^2 \frac{h}{f} + y_2^2\, f^2 \end{pmatrix}$$

$$V_{CW} = -\frac{3}{16\pi^2}\Lambda^2\, \mathrm{Tr}[m_f^2] + \frac{3}{16\pi^2}\mathrm{Tr}[m_f^4 \log(\frac{\Lambda^2}{m_f^2} + \frac{3}{2})]$$

- Cancellations in the gauge sector:





- Gauge symmetries in various little Higgs models [$SU(3)_c$ is not included]:
  - The minimal moose model: $SU(3) \times SU(2) \times U(1)$.
  - The littlest Higgs model: $[SU(2) \times U(1)]^2$.
  - The simplest little Higgs model: $SU(3) \times U(1)$.
- Predict Z', W'; t' and partners of other light quarks; extra scalars including triplets and singlets.

- How about the most minimal extension of the standard model gauge group: $SU(2) \times U(1) \times U(1)$?

- What is the symmetry for this cancellation?

Linear Realization

- The field content under the gauge symmetry:

	$SU(2)_w$	$U(1)_1$	$U(1)_2$
H	2	1/2	1/2
S	1	5/3	-5/3

.

- The kinetic terms of scalars:

$$|(\partial_\mu + ig\, t^a\, W^a_\mu + i\frac{g'}{2\sqrt{2}}(B_{1\mu} + B_{2\mu}))H|^2 + |(\partial_\mu + i\frac{5g'}{3\sqrt{2}}(B_{1\mu} - B_{2\mu}))S|^2$$

- A Z_2 interchanging symmetry: $g_1 = g_2 = \sqrt{2}g'$

g' is the gauge coupling of $U(1)_Y$; g is the gauge coupling of $SU(2)_w$.

- The Λ^2 contributions to scalar masses from gauge bosons are:

$$V_g = \frac{3\,\Lambda^2}{64\pi^2}\left[(3g^2 + g'^2)HH^\dagger + \frac{100}{9}g'^2 SS^\dagger\right] + \cdots,$$

$$\approx \frac{25g'^2\Lambda^2}{48\pi^2}\left[HH^\dagger + SS^\dagger\right] + \cdots . \Rightarrow \text{Approximate } U(3) \text{ global symmetry}$$

$\sin^2\theta_w = g'^2/(g^2 + g'^2) \approx 0.23$

Nonlinear Realization

- Write H and S together as a triplet of $U(3)$: $\phi = (H, S)^T$
- $\langle \phi \rangle = (0, 0, f)^T$ from underlying dynamics

global symmetry:	$U(3) \to U(2)$
gauge symmetry:	$SU(2)_w \times U(1)_1 \times U(1)_2 \to SU(2)_w \times U(1)_Y$

- Below the cutoff $\Lambda \approx 4\pi f$, the EFT contains 9-4=5 GB's.
  - One is eaten by the massive neutral gauge boson: $B' \equiv (B_1 - B_2)/\sqrt{2}$
  - The other 4 become PNGB's and identified as the Higgs doublet: h

$$\phi^T = f\left(\frac{ih}{\langle h \rangle} \sin \frac{\langle h \rangle}{f}, \cos \frac{\langle h \rangle}{f}\right) = \left(ih, f - \frac{\langle h \rangle^2}{2f}\right) + \cdots .$$

- The field dependent masses of gauge bosons are:

$$M_W^2(h) = c_w^2\, M_Z^2(h) = \frac{1}{2} g^2 f^2 \sin^2 \frac{\langle h \rangle}{f} \qquad M_{B'}^2(h) = \frac{50}{9} g'^2 f^2 \cos^2 \frac{\langle h \rangle}{f}$$

- Calculate the one-loop effective potential

$$V_{CW} = \frac{3}{32\pi^2} \Lambda^2\, \mathrm{Tr}[M_g^2] - \frac{3}{64\pi^2} \mathrm{Tr}[M_g^4 \log(\frac{\Lambda^2}{M_g^2} + \frac{3}{2})]$$

- The Higgs mass contributions from the gauge sector:

$$m_h^2|_g = \frac{3g'^2\Lambda^2}{32\pi^2}(\frac{27-118s_w^2}{9s_w^2}) + \frac{3M_{B'}^4}{32\pi^2 f^2}(\log \frac{\Lambda^2}{M_{B'}^2} + 1)$$

$$M_{B'} = 5\sqrt{2} g' f/3 \approx 0.8f$$

- For s_w^2 around 0.23, the Λ^2 term is even smaller than $\log\Lambda$ term.

$$m_h^2|_g \approx -(87\ GeV)^2 + (116\ GeV)^2\,,$$

for $f = 800$ Gev, $\Lambda = 10$ TeV, $s_w^2 = 0.23$.

Z_2 Broken Model

- The field content under the gauge symmetry:

	$SU(3)_c$	$SU(2)_w$	$U(1)_1$	$U(1)_2$
H	1	2	1/2	1/2
S	1	1	5/3	-5/3
q_L	3	2	1/6	1/6
t_R	3	1	2/3	2/3
b_R	3	1	-1/3	-1/3
ψ_L	3	1	7/3	-1
ψ_R	3	1	7/3	-1

Only a colored vector-like quark added; gauge anomalies are still cancelled.

- The Yukawa couplings in the top sector are:

$$\mathcal{L}_t = y_1(\bar{q}_L, \bar{\psi}_L)\,\phi\, t_R + y_2\, f\, \bar{\psi}_L\, \psi_R = y_1(\bar{q}_L\, \tilde{H} + \bar{\psi}_L\, S)\, t_R + y_2\, f\, \bar{\psi}_L\, \psi_R + h.c.$$

Z_2 symmetry is manifestly broken

Z_2 Broken Model

- Higgs boson masses from the top sector:

$$m_h^2|_t = -\frac{3}{8\pi^2} y_t^2\, m_{t'}^2 (\log \frac{\Lambda^2}{m_{t'}^2} + 1)$$

No Λ^2 contribution: collective breaking mechanism protects it.

- Spontaneously electroweak symmetry breaking:

$$m_h^2 = m_h^2|_g + m_h^2|_t < 0$$

Minimizing the full potential, we get a light Higgs boson below 200 GeV.

- Spectrum: $m_t = y_t \langle h \rangle \quad y_t = \frac{y_1 y_2}{\sqrt{y_1^2 + y_2^2}} \quad m_{t'} = \sqrt{y_1^2 + y_2^2} f$

$$t_{L,m} \approx t_L \qquad t_{R,m} \approx (y_2\, t_R - y_1\, \psi_R)/\sqrt{y_1^2 + y_2^2}$$

$$t'_L \approx \psi_L \qquad t'_R \approx (y_1\, t_R + y_2\, \psi_R)/\sqrt{y_1^2 + y_2^2}$$

- Large mixing between the right-handed parts of t and t' quarks.
- Both Z and B' couple to t_R and t'_R with order one couplings.

Electroweak Precision Test

- At tree level, only the experimentally unmeasured top quark couplings to W and Z bosons are changed.
- At one-loop level, the strongest constraint comes from the T parameter:

$$\alpha T = \frac{3y_t^2 y_1^2 m_t^2}{16\pi^2 y_2^2 m_{t'}^2}(\log\frac{m_{t'}^2}{m_t^2} - 1 + \frac{y_1^2}{2y_2^2})$$

[From PDG, $\alpha T < 1.2 \times 10^{-3}$ at 95% confidence level for $m_h < 300$ GeV.]

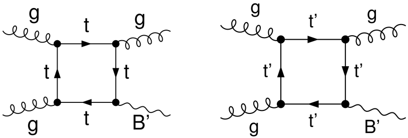
- For $y_1/y_2 < 3/4$, there is no bound on the symmetry breaking scale f. Hence, f can be as low as 400 GeV (to have the cutoff Λ above 5 TeV).

Signatures of the Z_2 Broken Model

- Two new parameters: y_2 and f (y_1 is determined by y_2 and y_t).
- Predicts two new particles: B' and t'.

$$M_{B'} = 5\sqrt{2}g'f/3 \approx 0.8f \qquad m_{t'} = \sqrt{y_1^2 + y_2^2}f \geq 2f$$

- For $f \geq 400$ GeV, $M_{B'} \geq 300$ GeV. This possible light neutral gauge boson only couples to top quarks (nonuniversal).
- B' can mainly be produced through loop diagrams at Hadron Colliders like:



- B' decays to two top quarks. Mainly look for $t\bar{t} + 1$ jet.

Z_2 Unbroken Model

- To have a cold dark matter candidate, we need to keep this Z_2 to be unbroken to have stable particles. [Low and Cheng, 2003]
- Introduce two more vector-like quarks:

	$SU(3)_c$	$SU(2)_w$	$U(1)_1$	$U(1)_2$
H	1	2	1/2	1/2
S	1	1	5/3	-5/3
${q_1}_L$	3	2	1/6	1/6
t_R	3	1	2/3	2/3
b_R	3	1	-1/3	-1/3
${\psi_1}_L$	3	1	7/3	-1
${\psi_1}_R$	3	1	7/3	-1
${\psi_2}_L$	3	1	-1	7/3
${\psi_2}_R$	3	1	-1	7/3
${q_2}_L$	3	2	1/6	1/6
q'_R	3	2	1/6	1/6

Gauge anomalies are cancelled.

Z_2 invariant

$$
\begin{aligned}
\mathcal{L}_t &= \frac{y_1}{\sqrt{2}}\,(\bar{q}_{1_L}\,\tilde{H} + \bar{\psi}_{1_L}\,S)\,t_R + y_2\,f\,\bar{\psi}_{1_L}\,\psi_{1_R} \\
&+ \frac{y_1}{\sqrt{2}}\,(\bar{q}_{2_L}\,\tilde{H} + \bar{\psi}_{2_L}\,S^\dagger)\,t_R + y_2\,f\,\bar{\psi}_{2_L}\,\psi_{2_R} \\
&+ \frac{y_3}{\sqrt{2}}\,f\,(\bar{q}_{1_L} - \bar{q}_{2_L})\,q'_R + h.c.
\end{aligned}
$$

Under the Z_2 transformation, we have

$$
\begin{aligned}
Z_2: \quad & q_{1_{L,R}} \leftrightarrow q_{2_{L,R}}, \quad \psi_{1_{L,R}} \leftrightarrow \psi_{2_{L,R}}, \quad q'_R \to -q'_R, \\
& B_1 \leftrightarrow B_2, \quad S \leftrightarrow S^\dagger
\end{aligned}
$$

and all other fields are invariant

Mass Spectrum

Z_2 is exact; all particles are Z_2 eigenstates.

- Z_2 even particles:

$$\mathrm{t}: \quad \mathrm{t}_{L,m} \approx t_L \qquad \mathrm{t}_{R,m} \approx \frac{y_2\, t_R - y_1\,(\psi_{1R}+\psi_{2R})/\sqrt{2}}{\sqrt{y_1^2+y_2^2}} \qquad m_t = \frac{y_1 y_2}{y_1^2+y_2^2}\langle h\rangle$$

$$\mathrm{t}'_+: \quad \mathrm{t}'_{+L} \approx \frac{\psi_{1L}+\psi_{2L}}{\sqrt{2}} \qquad \mathrm{t}'_{+R} \approx \frac{y_1\, t_R + y_2\,(\psi_{1R}+\psi_{2R})/\sqrt{2}}{\sqrt{y_1^2+y_2^2}} \qquad m_{t'_+} \approx \sqrt{y_1^2+y_2^2}\, f \geq 2\, f$$

The Λ^2 contribution to the Higgs mass from t is cancelled by t'_+.

All other standard model particles are also Z_2 even.

- Z_2 odd particles:

$$\mathrm{t}'_-: \quad \mathrm{t}'_{-L} \approx \frac{\psi_{1L}-\psi_{2L}}{\sqrt{2}} \qquad \mathrm{t}'_{-R} \approx \frac{\psi_{1R}-\psi_{2R}}{\sqrt{2}} \qquad m_{t'_-} = y_2\, f$$

$$\mathrm{q}'_-: \quad \mathrm{q}'_{-L} \approx \frac{q_{1L}-q_{2L}}{\sqrt{2}} \qquad \mathrm{q}'_{-R} \approx q'_R \qquad m_{q'_-} = y_3\, f$$

$$\mathrm{B}': \quad (\mathrm{B}_1 - B_2)/2 \qquad \mathrm{M}_{B'} \approx 0.8 f$$

- For $y_2, y_3 \geq 1$, B' is the lightest Z_2 odd particle and a potential dark matter candidate in this model.

Dark Matter

- From WMAP, the relic abundance of the dark matter is:

$$0.098 < \Omega_{dm} h^2 < 0.122\,(2\sigma)$$

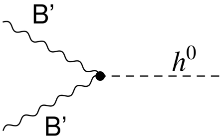
- In the non-relativistic limit, $\Omega_{dm} h^2$ is relating to sum of the quantities, $a(X) = v_r\,\sigma(B'B' \to X)$, as

$$\Omega_{dm}\,h^2 \approx \frac{1.04\times 10^9\,\text{GeV}^{-1}}{M_{pl}}\,\frac{x_F}{\sqrt{g^*}}\,\frac{1}{a_{tot}}$$

- Approximately, only need to calculate a_{tot} and require:

$$a_{tot} \approx 0.81 \pm 0.09\,pb$$

Couplings of B' to Higgs Boson



Minimal Little Higgs Model

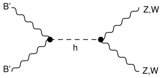
$\frac{50}{9}\, g'^2\, v$

Hypercharge-like Gauge Boson [LHT and UED]

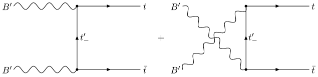
$\frac{1}{2}\, g'^2\, v$

Relic Abundance

The leading processes for $B'\ B'$ annihilation into SM particles:

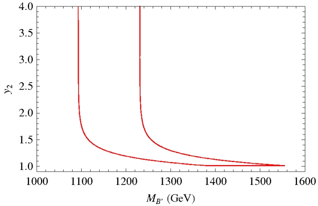


$$a(WW) = 2a(ZZ) = 2a(h^0 h^0) = \frac{2\pi\alpha^2}{3\cos^4\theta_W}\,\frac{5^4}{3^4}\,\frac{1}{M_{B'}^2}$$



$$a(\bar{t}t) = \frac{16\pi\alpha^2}{3\cos^4\theta_W}\,\frac{5^4}{3^4}\,\frac{y_t^4}{y_2^4}\,\frac{M_{B'}^2}{(M_{B'}^2 + m_{t'_-}^2)^2}$$

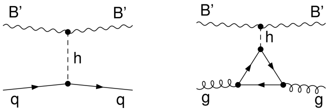
For $y_2 \gg 1$, $a(\bar{t}t)$ is negligible.



$[\ M_{B'} \approx 0.8\, f\ \ m_{t'_-} = y_2\, f\ ]$ $\quad 0.098 < \Omega_{dm}h^2 < 0.122\ (2\sigma) \Rightarrow a_{tot} \approx 0.81 \pm 0.09\ pb$

Direct Detection

- Measure the recoil energy in the elastic scattering of dark matter particles with nuclei.



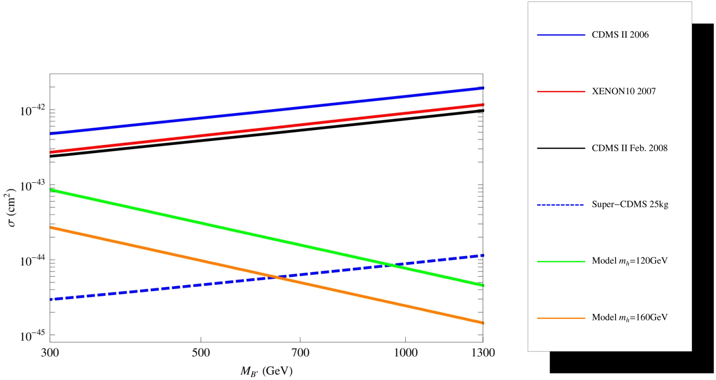
Only contribute to spin-independent cross section

- Using the matrix element of quarks and gluons in a nucleon state: [Ellis, Olive, Santoso, Spanos, 2005]

$$\sigma_{SI} \approx \frac{0.35^2\, g'^4}{16\pi\, M_{B'}^2} \frac{10^4}{3^4} \frac{m_p^4}{m_h^4} \approx 1.6 \times 10^{-44}\mathrm{cm}^2 \left(\frac{1\ \mathrm{TeV}}{M_{B'}}\right)^2 \left(\frac{100\ \mathrm{GeV}}{m_h}\right)^4$$

- Exchanging t'_{-} in the s- and t-channel can also contribute to spin-dependent cross section.

Direct Detection



Summary

- A very simple little Higgs model has been constructed based on the $SU(2)_w \times U(1)^2$ gauge symmetry.
- A Z_2 interchanging symmetry is introduced between these two $U(1)$'s.
- For Z_2 broken case: only B' and t' appears in the EFT. The mass of B' can be as light as 300 GeV.
- For Z_2 unbroken case:
  - B' is a stable particle and can serve as a dark matter candidate.
  - The direct detection of this B' dark matter is promising.
  - The $\sigma_{SI}(B'N)$ is two order of magnitude larger than a hypercharge-like neutral gauge boson dark matter candidate.