PRECISION DETERMINATION OF α_s FROM THRUST DISTRIBUTIONS AT LEP

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OVERVIEW

- Introduction
  - Event shape variables: thrust
  - α_s determinations and world average
  - NNLO fixed order calculation of thrust
- N^3LL resummed thrust distribution
  - Resummation by RG evolution in Soft Collinear Effective Theory (SCET)
  - Comparison with fixed order result
- Determination of α_s from thrust distributions at LEP I and LEP II

work in progress in collaboration with Matt Schwartz

EVENT-SHAPE VARIABLES

- Parameterize geometric properties of energy and momentum flow of an event in high energy collisions.
  - Collinear and infrared safe: can be evaluated in perturbation theory.
  - Used for QCD studies, measurements of α_s, to cut against backgrounds, ...
- The canonical event-shape variable is thrust.

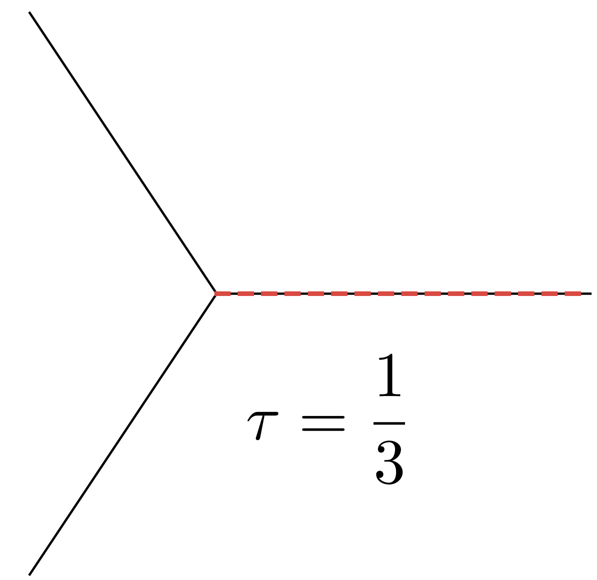

HERBIE HANCOCK
THRUST

THRUST T AND THRUST AXIS $\vec{n}$

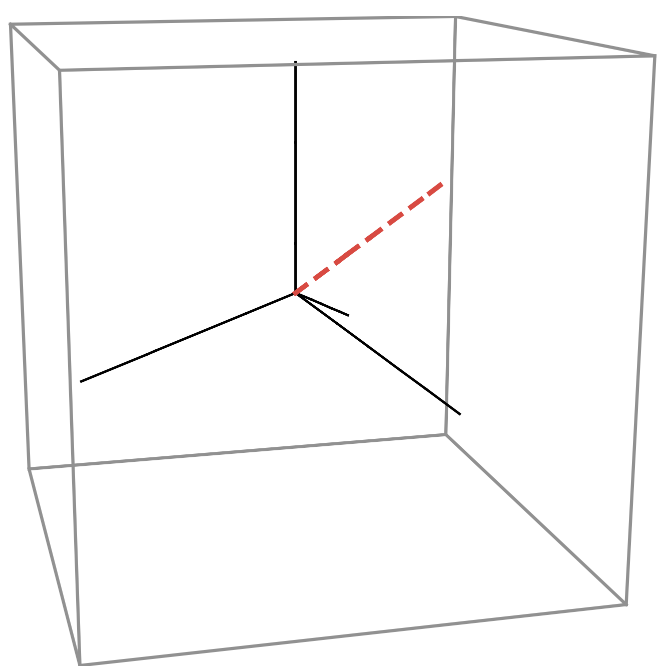
$\tau = 0$

$$T = \frac{1}{Q} \max_{\vec{n}} \sum_i |\vec{n} \cdot \vec{p_i}|$$

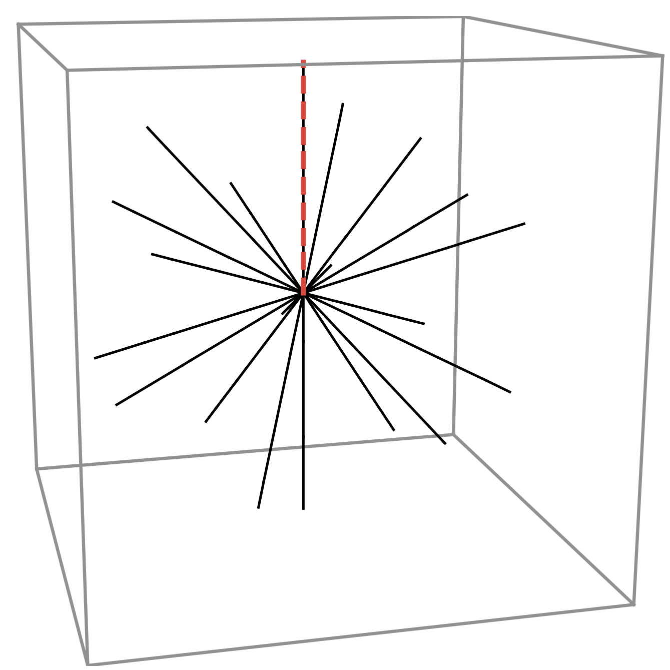
$$\tau = 1 - T$$



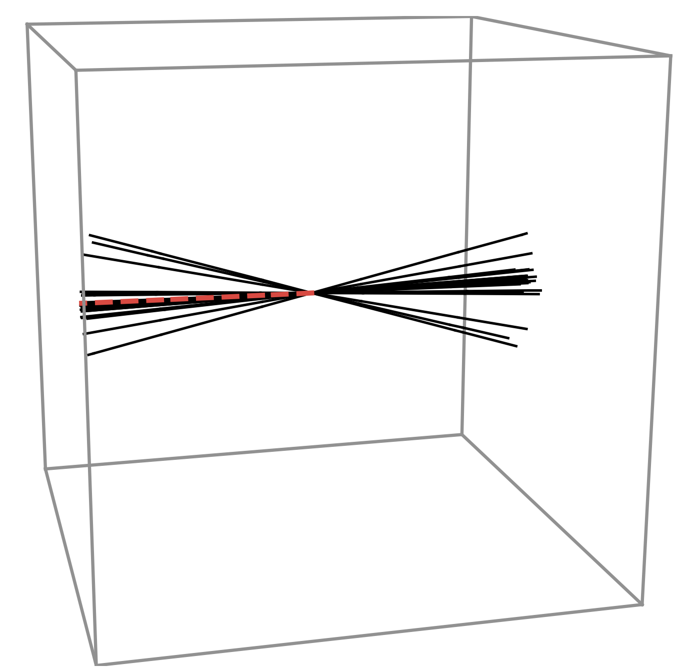
$\tau = \frac{1}{3}$



$\tau = 1 - \frac{1}{\sqrt{3}} = 0.42$

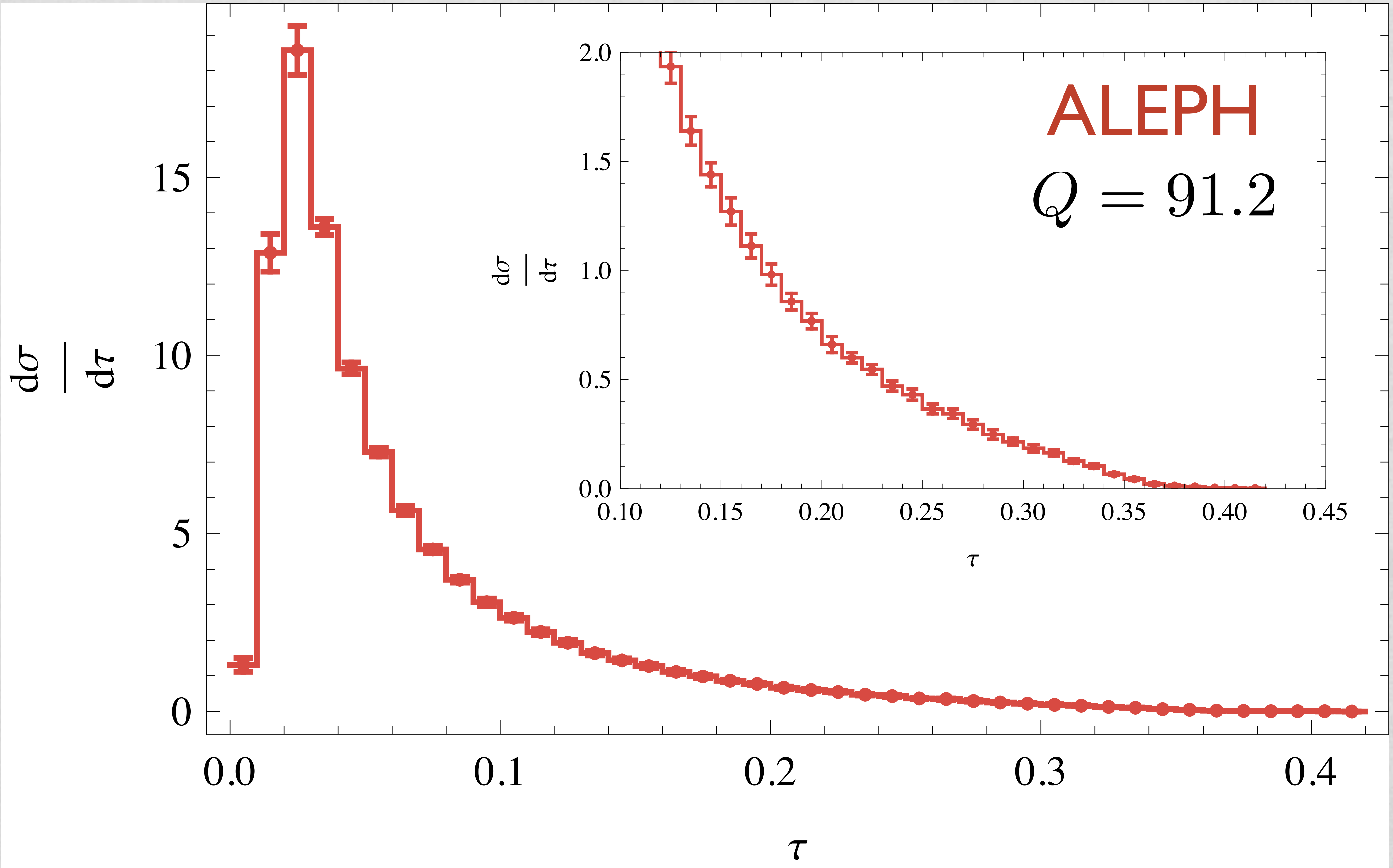


$\tau = 0.48$



$\tau = \frac{M_1^2 + M_2^2}{Q^2}$

MEASUREMENTS OF THRUST



- Will later use ALEPH and OPAL LEP I & II results

α_s DETERMINATIONS

S. Bethke '06

Process	Q [GeV]	$\alpha_s(Q)$	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s(M_{Z^0})$ exp.	$\Delta\alpha_s(M_{Z^0})$ theor.	Theory	refs.
DIS [pol. SF]	0.7 - 8		$0.113^{+0.010}_{-0.008}$	± 0.004	$^{+0.009}_{-0.006}$	NLO	[76]
DIS [Bj-SR]	1.58	$0.375^{+0.062}_{-0.081}$	$0.121^{+0.005}_{-0.009}$	–	–	NNLO	[77]
DIS [GLS-SR]	1.73	$0.280^{+0.070}_{-0.068}$	$0.112^{+0.009}_{-0.012}$	$^{+0.008}_{-0.010}$	0.005	NNLO	[78]
τ-decays	1.78	0.345 ± 0.010	0.1215 ± 0.0012	0.0004	0.0011	NNLO	[70]
DIS [ν; xF_3]	2.8 - 11		$0.119^{+0.007}_{-0.006}$	0.005	$^{+0.005}_{-0.003}$	NNLO	[79]
DIS [e/μ; F_2]	2 - 15		0.1166 ± 0.0022	0.0009	0.0020	NNLO	[80, 81]
DIS [e-p $\rightarrow$ jets]	6 - 100		0.1186 ± 0.0051	0.0011	0.0050	NLO	[67]
Υ decays	4.75	0.217 ± 0.021	0.118 ± 0.006	–	–	NNLO	[82]
$Q\overline{Q}$ states	7.5	0.1886 ± 0.0032	0.1170 ± 0.0012	0.0000	0.0012	LGT	[73]
e^+e^- [F_2^γ]	1.4 - 28		$0.1198^{+0.0044}_{-0.0054}$	0.0028	$^{+0.0034}_{-0.0046}$	NLO	[83]
e^+e^- [σ_{had}]	10.52	0.20 ± 0.06	$0.130^{+0.021}_{-0.029}$	$^{+0.021}_{-0.029}$	0.002	NNLO	[84]
e^+e^- [jets & shps]	14.0	$0.170^{+0.021}_{-0.017}$	$0.120^{+0.010}_{-0.008}$	0.002	$^{+0.009}_{-0.008}$	resum	[85]
e^+e^- [jets & shps]	22.0	$0.151^{+0.015}_{-0.013}$	$0.118^{+0.009}_{-0.008}$	0.003	$^{+0.009}_{-0.007}$	resum	[85]
e^+e^- [jets & shps]	35.0	$0.145^{+0.012}_{-0.007}$	$0.123^{+0.008}_{-0.006}$	0.002	$^{+0.008}_{-0.005}$	resum	[85]
e^+e^- [σ_{had}]	42.4	0.144 ± 0.029	0.126 ± 0.022	0.022	0.002	NNLO	[86, 32]
e^+e^- [jets & shps]	44.0	$0.139^{+0.011}_{-0.008}$	$0.123^{+0.008}_{-0.006}$	0.003	$^{+0.007}_{-0.005}$	resum	[85]
e^+e^- [jets & shps]	58.0	0.132 ± 0.008	0.123 ± 0.007	0.003	0.007	resum	[87]
$p\bar{p} \rightarrow b\bar{b}X$	20.0	$0.145^{+0.018}_{-0.019}$	0.113 ± 0.011	$^{+0.007}_{-0.006}$	$^{+0.008}_{-0.009}$	NLO	[88]
$p\bar{p}$, $pp \rightarrow \gamma X$	24.3	$0.135^{+0.012}_{-0.008}$	$0.110^{+0.008}_{-0.005}$	0.004	$^{+0.007}_{-0.003}$	NLO	[89]
$\sigma(p\bar{p} \rightarrow$ jets)	40 - 250		0.118 ± 0.012	$^{+0.008}_{-0.010}$	$^{+0.009}_{-0.008}$	NLO	[90]
e^+e^- $\Gamma(Z \rightarrow$ had)	91.2	$0.1226^{+0.0058}_{-0.0038}$	$0.1226^{+0.0058}_{-0.0038}$	± 0.0038	$^{+0.0043}_{-0.0005}$	NNLO	[91]
e^+e^- 4-jet rate	91.2	0.1176 ± 0.0022	0.1176 ± 0.0022	0.0010	0.0020	NLO	[92]
e^+e^- [jets & shps]	91.2	0.121 ± 0.006	0.121 ± 0.006	0.001	0.006	resum	[32]
e^+e^- [jets & shps]	133	0.113 ± 0.008	0.120 ± 0.007	0.003	0.006	resum	[32]
e^+e^- [jets & shps]	161	0.109 ± 0.007	0.118 ± 0.008	0.005	0.006	resum	[32]
e^+e^- [jets & shps]	172	0.104 ± 0.007	0.114 ± 0.008	0.005	0.006	resum	[32]
e^+e^- [jets & shps]	183	0.109 ± 0.005	0.121 ± 0.006	0.002	0.005	resum	[32]
e^+e^- [jets & shps]	189	0.109 ± 0.004	0.121 ± 0.005	0.001	0.005	resum	[32]
e^+e^- [jets & shps]	195	0.109 ± 0.005	0.122 ± 0.006	0.001	0.006	resum	[81]
e^+e^- [jets & shps]	201	0.110 ± 0.005	0.124 ± 0.006	0.002	0.006	resum	[81]
e^+e^- [jets & shps]	206	0.110 ± 0.005	0.124 ± 0.006	0.001	0.006	resum	[81]

Event shapes

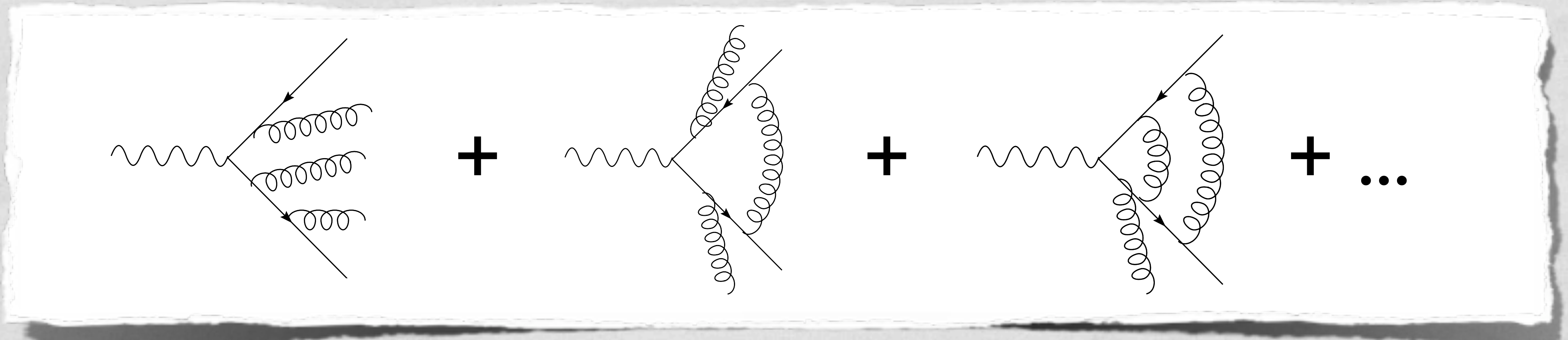
WORLD AVERAGE

S. Bethke '06

Process	Q [GeV]	$\alpha_s(M_{Z^0})$	excl. mean $\alpha_s(M_{Z^0})$	std. dev.
DIS [Bj-SR]	1.58	$0.121\ ^{+\ 0.005}_{-\ 0.009}$	0.1189 ± 0.0008	0.3
τ-decays	1.78	0.1215 ± 0.0012	0.1176 ± 0.0018	1.8
DIS [ν; xF_3]	2.8 - 11	$0.119\ ^{+\ 0.007}_{-\ 0.006}$	0.1189 ± 0.0008	0.0
DIS [e/μ; F_2]	2 - 15	0.1166 ± 0.0022	0.1192 ± 0.0008	1.1
DIS [e-p $\rightarrow$ jets]	6 - 100	0.1186 ± 0.0051	0.1190 ± 0.0008	0.1
Υ decays	4.75	0.118 ± 0.006	0.1190 ± 0.0008	0.2
$\mathrm{Q\overline{Q}}$ states	7.5	0.1170 ± 0.0012	0.1200 ± 0.0014	1.6
e^+e^- [$\Gamma(Z \rightarrow had)$	91.2	$0.1226^{+\ 0.0058}_{-\ 0.0038}$	0.1189 ± 0.0008	0.9
e^+e^- 4-jet rate	91.2	0.1176 ± 0.0022	0.1191 ± 0.0008	0.6
e^+e^- [jets & shps]	189	0.121 ± 0.005	0.1188 ± 0.0008	0.4

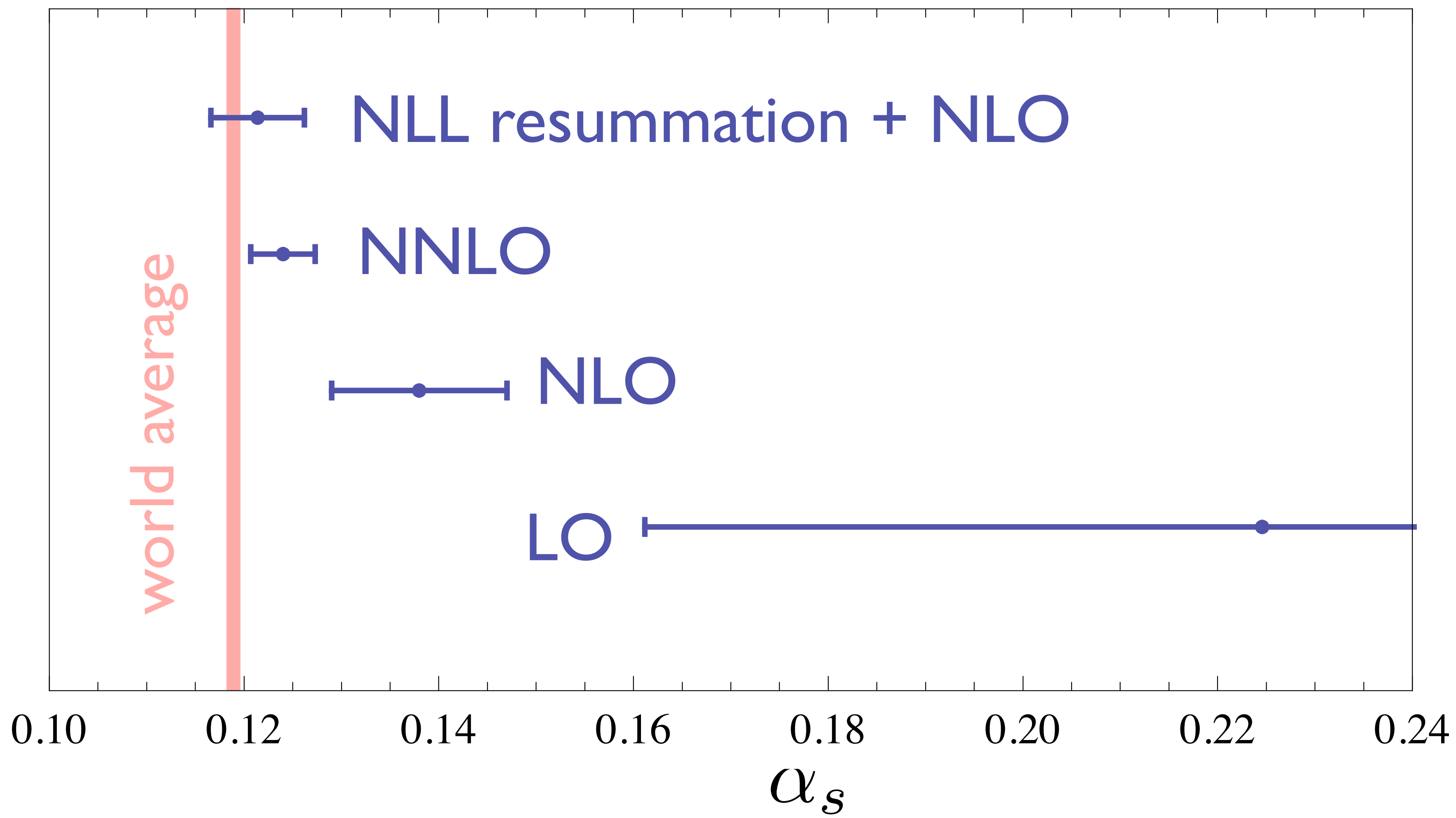
- Average $\alpha_s(M_{Z^0}) = 0.1189 \pm 0.0007$ dominated by τ-decays and LGT, which disagree by 2.7σ!

EVENT SHAPES AT NNLO



- After years of work, the NNLO calculation of $e^+e^- \rightarrow$ 3 jets has recently been completed.
  A. Gehrmann-De Ridder, T. Gehrmann, E.W.N. Glover, G. Heinrich '07
  - First time a subtraction scheme has been implemented at NNLO.
    - Real and virtual contributions are have collinear and soft divergences which cancel in the sum.
  - Implemented in fixed order event generator. First application: NNLO evaluation of event shapes.

α_s FROM EVENT SHAPES AT LEP I



- Perturbative uncertainty dominates. At NNLO

$$\alpha_s(M_Z^2) = 0.1240 \ \pm\ 0.0008\,(\text{stat}) \ \pm\ 0.0010\,(\text{exp}) \ \pm\ 0.0011\,(\text{had}) \ \pm\ 0.0029\,(\text{theo})$$

RESUMMATION

- All-order formalism for resummation of thrust distribution
- N^3LL resummation
- Comparison with fixed order

LOGARITHMICALLY ENHANCED CONTRIBUTIONS

- The LO thrust distribution has the form

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = \frac{2\alpha_s}{3\pi}\left[-\frac{3}{\tau}+6+9\tau+\frac{(6\tau^2-6\tau+4)}{(1-\tau)\tau}\ln\frac{1-2\tau}{\tau}\right]$$

$$= \frac{2\alpha_s}{3\pi}\left[\frac{-4\ln\tau-3}{\tau}+d_{\mathrm{regular}}(\tau)\right]$$

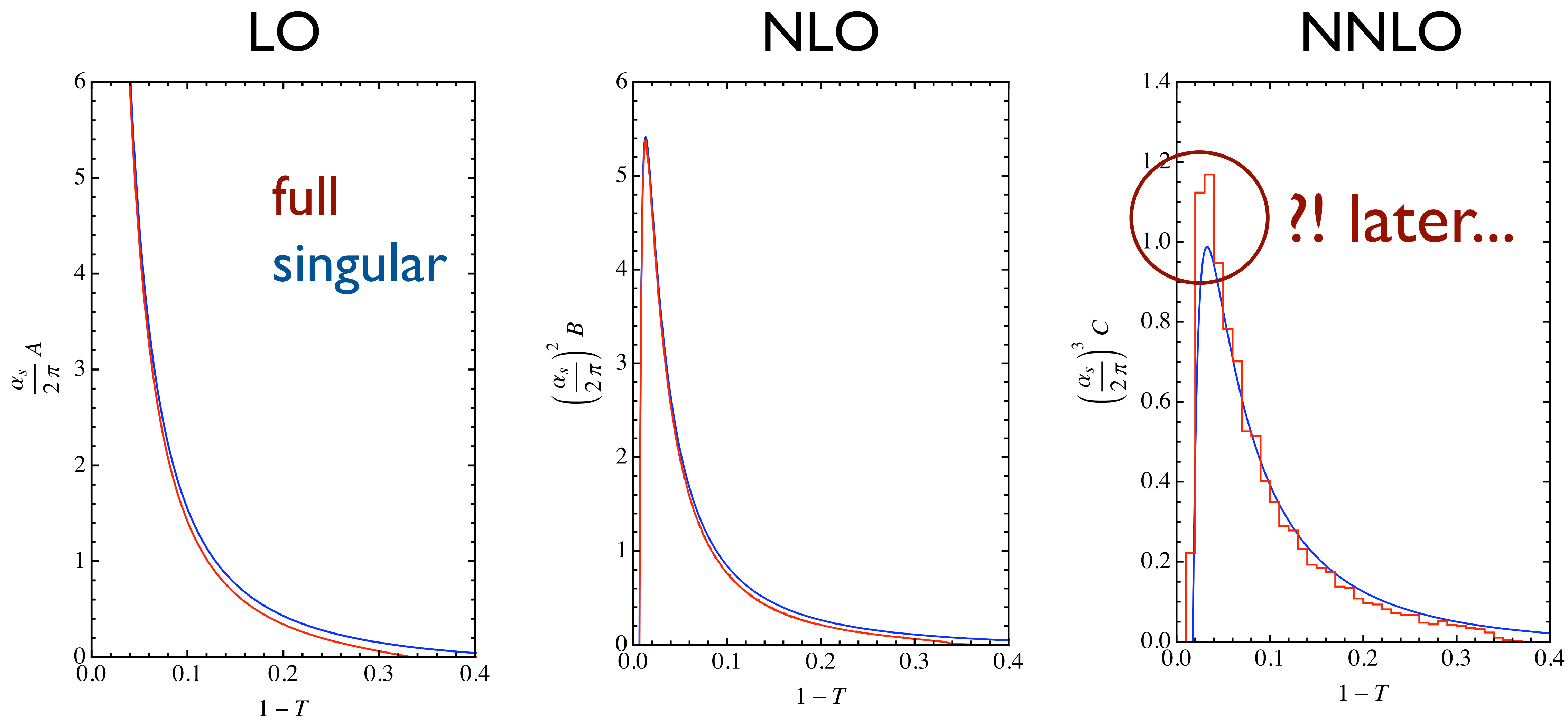
singular terms

- Integral over the end-point is

$$R(\tau) = \int_0^\tau d\tau' \frac{1}{\sigma_0}\frac{d\sigma}{d\tau'} = \frac{2\alpha_s}{3\pi}\left[-2\ln^2\tau - 3\ln\tau + \ldots\right]$$

Sudakov double logarithm

SINGULAR TERMS DOMINATE

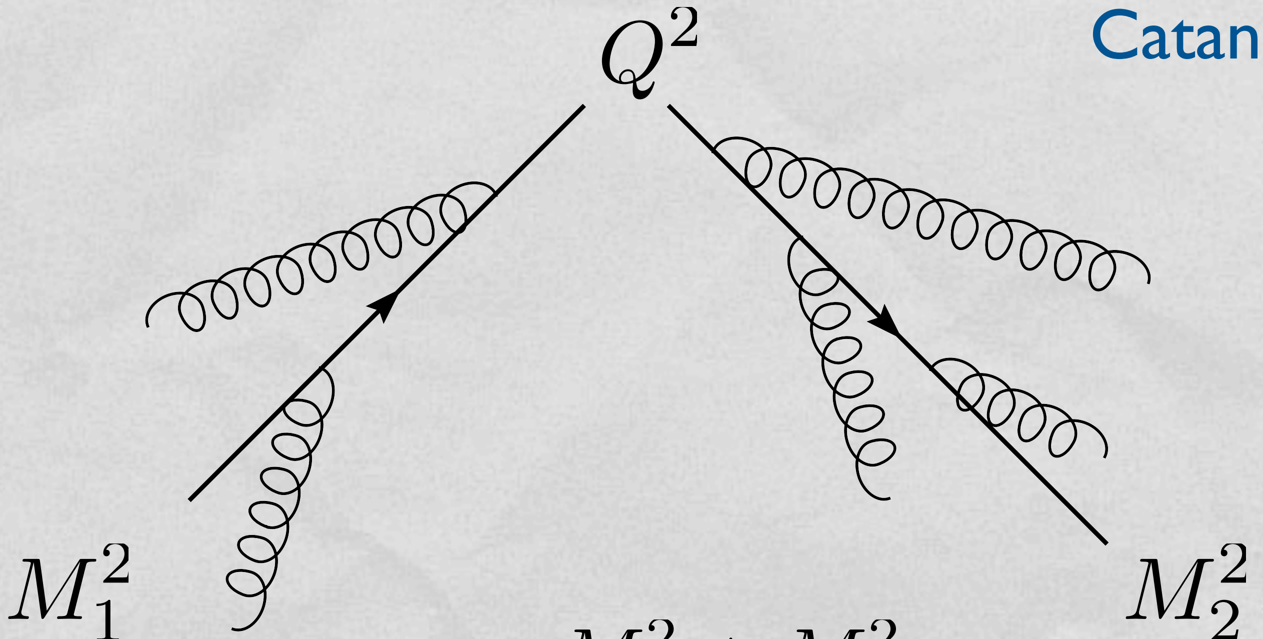


- Singular terms are predicted and later resummed to all orders with Soft-Collinear Effective Theory.
- Regular terms (difference of blue and red) are added back after resummation.

RESUMMATION: THE TRADITIONAL WAY

- Logarithmically enhanced contributions lead to slow convergence of perturbation theory
- The leading logarithms (LL) $\alpha_s^n \ln^{2n} \tau$ and next-to-leading log's (NLL) $\alpha_s^n \ln^{2n-1} \tau$ can be resummed using the "coherent branching algorithm"

Catani, Trentadue, Turnock, Webber '93



$$\tau = \frac{M_1^2 + M_2^2}{Q^2}$$

Note: NLL+NNLO calculation in progress by T. Gehrmann and G. Luisoni

EFFECTIVE THEORY RESUMMATION

- Using soft collinear effective theory, one can show that for $\tau \to 0$ the rate factorizes as

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = H(Q^2,\mu) \int dM_1^2 \int dM_2^2 \, J(M_1^2,\mu)\, J(M_2^2,\mu)\, S_T(\tau\, Q - \frac{M_1^2+M_2^2}{Q},\mu)$$

Fleming, Hoang, Mantry and Stewart ’07
Schwartz ’07
see also: Korchemsky ’98; Berger, Kucs, Sterman ‘03

- Three relevant scales:

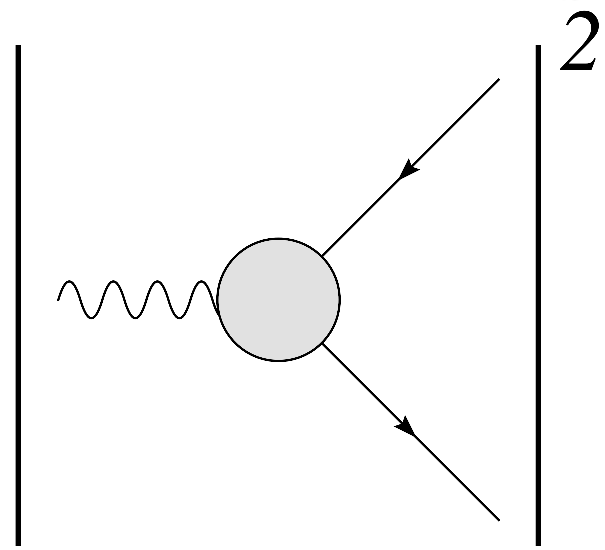
hard		jet		soft
Q^2	$\gg$	$M_1^2 \sim M_2^2 \sim \tau\, Q^2$	$\gg$	$\tau^2 Q^2$

(NO) DERIVATION

- Will refrain from using "incomprehensible SCET notation" A. Manohar to derive the theorem.
- However, will define hard, jet and soft functions in terms of matrix elements of QCD operators.
  - Same building blocks appear in many processes.
- Will discuss solution of RG equations for these functions.
  - Have elegant formalism to solve these equations using Laplace transformation. TB and Neubert '06
    - Used to perform resummations for B decays, DIS and DY.

HARD FUNCTION

- given by the on-shell form factor of a massless quark,

$H(Q^2)=$ 

  - known to two loops, logarithmic terms even to three loops. Moch, Vermaseren Vogt '05

- Same hard function appears in similar factorization theorems for DIS and DY in the end-point. TB, Neubert and Pecjak '06; TB and Neubert '07.

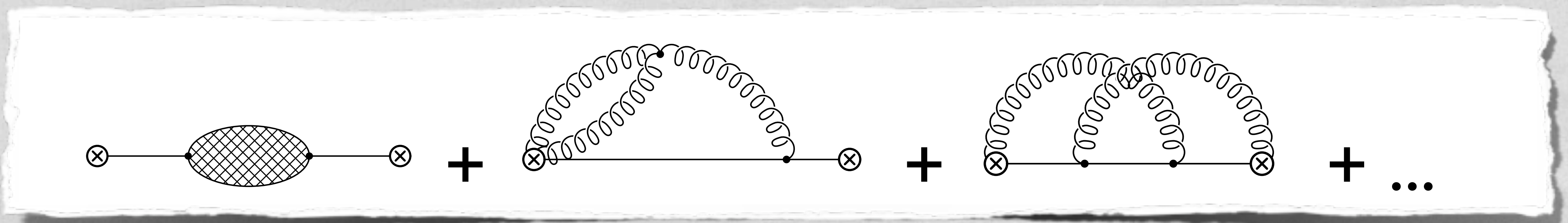
JET FUNCTION

- Imaginary part of propagator in light-cone gauge:

$$\langle 0 | \, W^\dagger(0)\psi(0)\,\bar{\psi}(x)W(x)\,|0\rangle$$

$$W(x) = \mathbf{P}\exp\left(ig\int_{-\infty}^{0} ds\,\bar{n}\cdot A(x+s\bar{n})\right)$$

- Known to two loops, anomalous dimension to three loops. TB and Neubert '06



  - Same jet function appears in B decays, DIS.

SOFT FUNCTION

- Soft function is given by Wilson lines along the directions of energetic particles $n^\mu = (1,0,0,1)$ and $\bar{n}^\mu = (1,0,0,-1)$

$$S_T(\omega) = \sum_X |\langle X| S_n^\dagger(0) S_{\bar{n}}(0) |0\rangle|^2 \delta(\omega - n\cdot p_{X_n} - \bar{n}\cdot p_{X_{\bar{n}}})$$

- Wilson lines $S_n(x) = \mathbf{P}\exp\left(ig\int_{-\infty}^{0} ds\, n\cdot A\ (x+sn)\right)$

- Kinematic constraint: $\omega = \dfrac{\delta M_1^2 + \delta M_2^2}{Q}$ is the change in jet-mass due to soft emissions

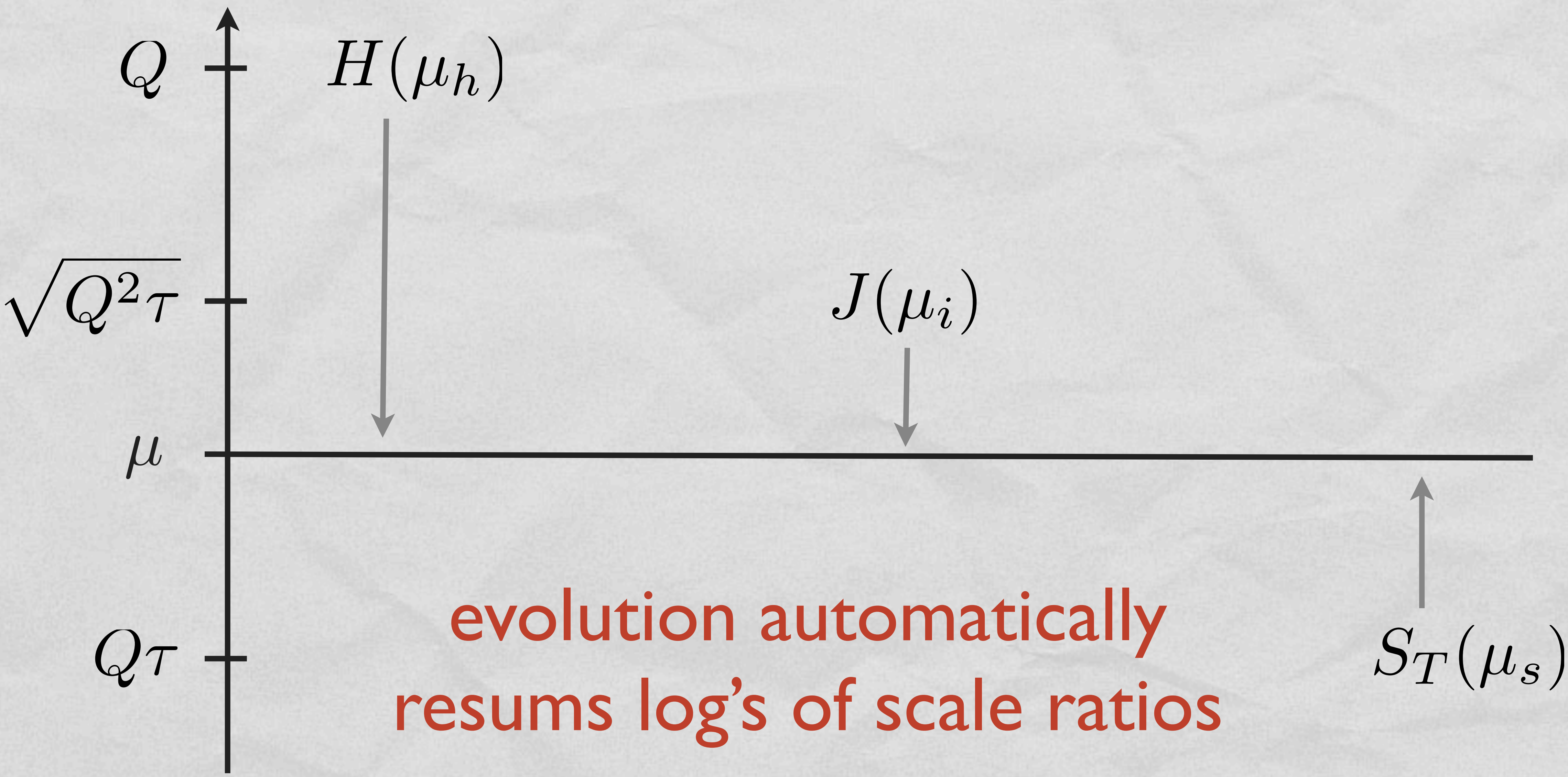
RESUMMATION

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = H(Q^2,\mu) \int dM_1^2 \int dM_2^2\, J(M_1^2,\mu)\, J(M_2^2,\mu)\, S_T(\tau\, Q - \frac{M_1^2 + M_2^2}{Q},\mu)$$

- The presence of the three separated scales leads to large perturbative logarithms.
  - Any choice of μ will produce large logarithms in either H, J or S.
- *H* and *J* are Wilson coefficients in SCET, *S* a matrix element,
  - fulfill renormalization group equation.

RESUMMATION BY RG EVOLUTION

- Evaluate each part at its characteristic scale, evolve to common scale:



LAPLACE TRANSFORM

- Factorization theorem and RG equations simplify after Laplace transform

$$\widetilde{t}(\nu) = \int_0^\infty d\tau\, e^{-s\tau}\, \frac{1}{\sigma_0}\frac{d\sigma}{d\tau}\,, \qquad s = \frac{1}{e^{\gamma_E}\nu}$$

- Factorization theorem

$$\widetilde{t}(\nu) = H(\ln\frac{Q^2}{\mu^2},\mu)\,\left[\widetilde{j}\Big(\ln\frac{\nu^2 Q^2}{\mu},\mu\Big)\right]^2 \widetilde{s}_T\Big(\ln\frac{\nu Q}{\mu},\mu\Big)$$

RG EQUATIONS

- RG's for hard and jet function TB and Neubert '06

$$\frac{d}{d\ln\mu} H(\ln\frac{Q^2}{\mu^2},\mu) = \left[2\Gamma_{\text{cusp}}(\alpha_s)\ln\frac{Q^2}{\mu^2} + 2\gamma^V(\alpha_s)\right] H(\ln\frac{Q^2}{\mu^2},\mu)$$

$$\frac{d}{d\ln\mu}\widetilde{j}\left(\ln\frac{\nu Q^2}{\mu^2},\mu\right) = -\left[2\Gamma_{\text{cusp}}(\alpha_s)\ln\frac{\nu Q^2}{\mu^2} + 2\gamma^J(\alpha_s)\right]\widetilde{j}\left(\ln\frac{\nu Q^2}{\mu^2},\mu\right)$$

  - Γ_{cusp} is anom. dim. of Wilson line with cusp.
- Since the rate does not depend on μ , this implies

$$\frac{d}{d\ln\mu}\widetilde{s}_T\left(\ln\frac{\nu Q}{\mu},\mu\right) = \left[2\Gamma_{\text{cusp}}(\alpha_s)\ln\frac{\nu^2 Q^2}{\mu^2} + 2\gamma^T(\alpha_s)\right]\widetilde{s}\left(\ln\frac{\nu Q}{\mu},\mu\right)$$

$\gamma^T = 2\gamma^J - \gamma^V$ $\rightarrow$ know soft anom. dim.'s to three loops!

SOLUTION TO RGE

$$\widetilde{s}\Big(\ln\frac{\nu Q}{\mu},\mu\Big) = \exp\left[4S(\mu_s,\mu) - 2a_{\gamma^T}(\mu_s,\mu)\right]\left(\frac{\nu Q}{\mu_s}\right)^{-4a_\Gamma(\mu_s,\mu)} \widetilde{s}\Big(\ln\frac{\nu Q}{\mu},\mu_s\Big)$$

$$S(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha\,\frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)}\int_{\alpha_s(\nu)}^{\alpha}\frac{d\alpha'}{\beta(\alpha')}\,, \qquad a_\Gamma(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha\,\frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)}$$

Sudakov double log's

single log's

- Equations for hard and jet functions have exactly the same structure and solution (with some obvious substitutions).

RESUMMED THRUST DISTRIBUTION

- Plug in solutions, do inverse Laplace transform

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = U(\mu_h,\mu_i,\mu_s)\left(\frac{Q^2}{\mu_h^2}\right)^{-2a_\Gamma(\mu_h,\mu_i)} H(Q^2,\mu_h)$$
$$\times\left[\widetilde{j}\left(\ln\frac{\mu_s Q}{\mu_i^2}+\partial_\eta,\mu_i\right)\right]^2 \widetilde{s}_T\left(\partial_\eta,\mu_s\right)\frac{1}{\tau}\left(\frac{\tau Q}{\mu_s}\right)^\eta \frac{e^{-\gamma_E\eta}}{\Gamma(\eta)}$$

- *U* is an evolution factor, $\eta = 4\,a_\Gamma(\mu_i,\mu_s)$
- For N3LL resummation, we need:
  - 4-loop Γ_{cusp} (use Pade approx. for 4-loop term),
  - 3-loop γ's,
  - 2-loop H, $\widetilde{j}$ and $\widetilde{s}$.

have everything except 2-loop soft function

TWO-LOOP SOFT FUNCTION

- Known 1-loop result und RG equation fixes all logarithmic terms $L = \ln \frac{\nu Q}{\mu}$ in the soft function:

$$\widetilde{s}_{\mathrm{T}}\Big(L,\mu\Big) = 1 + \frac{C_F\alpha_s}{4\pi}\left(-8L^2 - \pi^2\right) + C_F\left(\frac{\alpha_s}{4\pi}\right)^2\left[C_F S_F + C_A S_A + T_F n_f S_f\right]$$

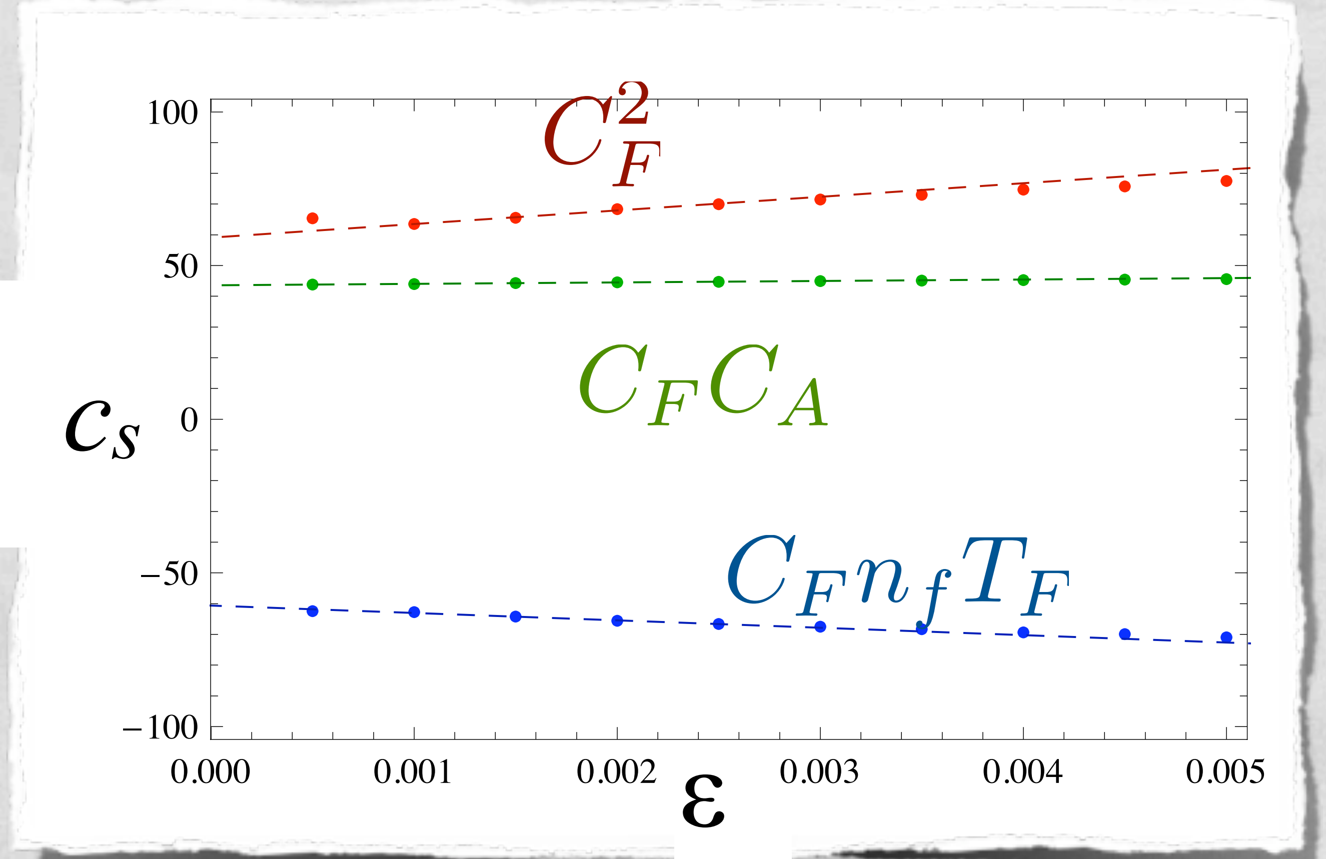
$$S_F = 32L^4 + 8\pi^2 L^2 + s_F\,,$$

$$S_A = \frac{176L^3}{9} + \left(-\frac{536}{9} + \frac{8\pi^2}{3}\right)L^2 + \left(\frac{1616}{27} + \frac{44\pi^2}{9} - 56\zeta(3)\right)L + s_A$$

$$S_f = -\frac{64L^3}{9} + \frac{160L^2}{9} + \left(-\frac{448}{27} - \frac{16\pi^2}{9}\right)L + s_f\,.$$

- We determine the constants s_F, s_A and s_f numerically.

CONSTANT TERMS IN S_T

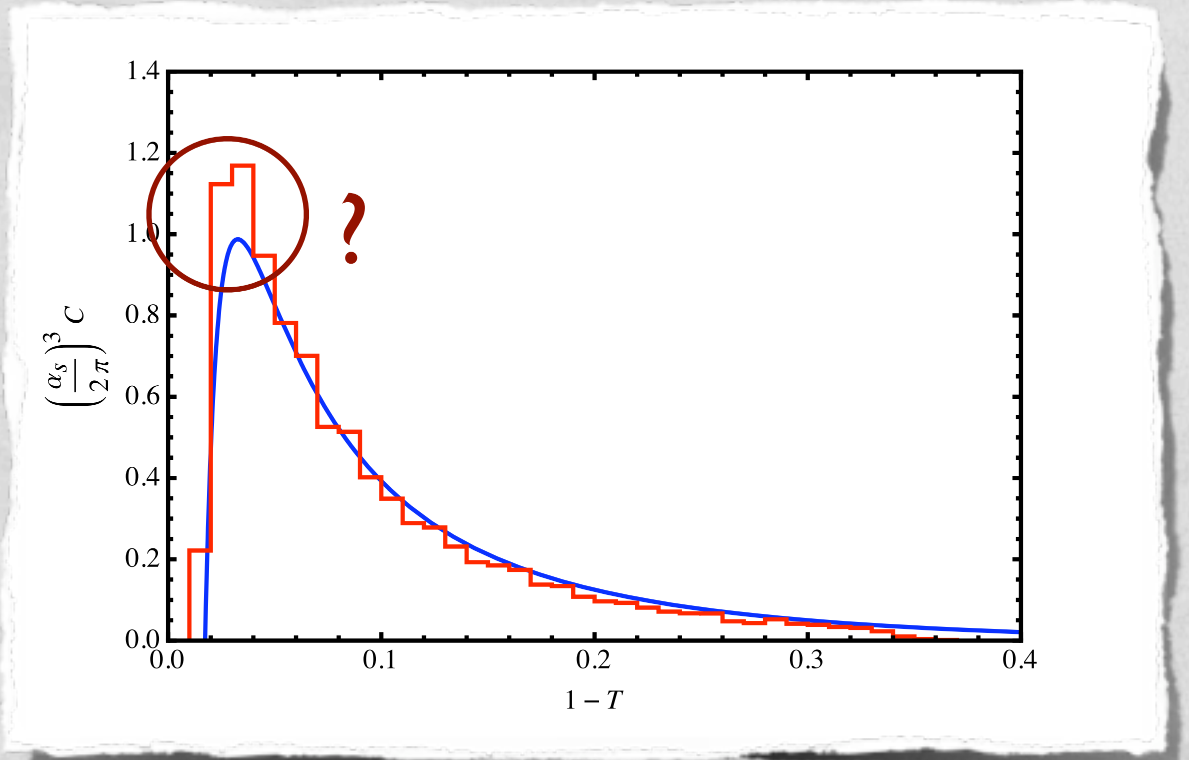


- are obtained from $\delta(\tau)$ terms at $O(\alpha_s^2)$.
- Use EVENT2 code by Catani and Seymour to numerically calculate

$$F(\epsilon) = \sigma_{\rm tot} - \int_{\epsilon>0} d\tau \left[\left(\frac{d\sigma}{d\tau}\right)_{\rm EVENT2} - \left(\frac{d\sigma}{d\tau}\right)_{\rm singular} \right]$$

NNLO SINGULAR TERMS

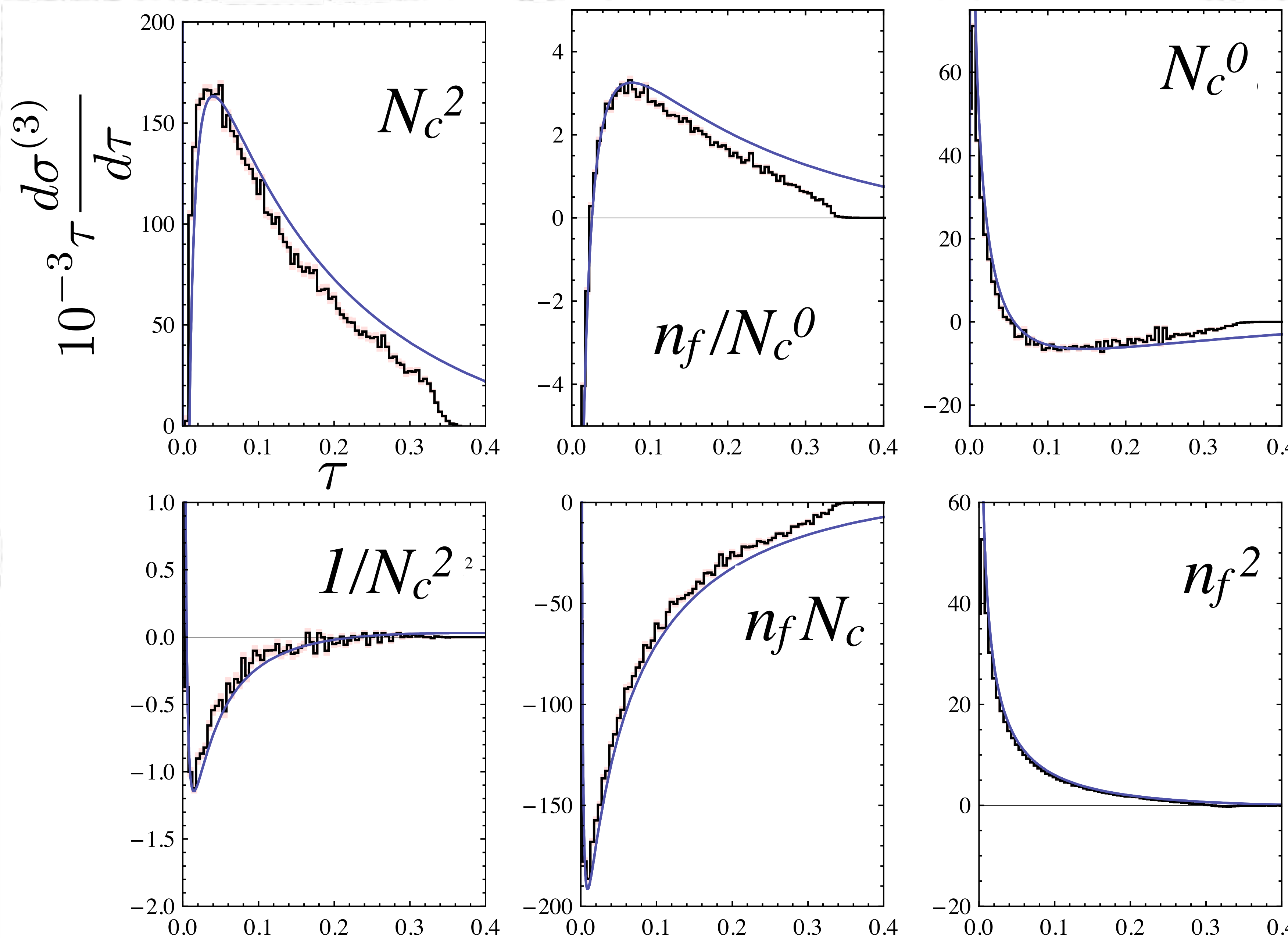
- With 2-loop soft function and 3-loop anomalous dimension we predict all singular terms at α_s^3.
- For small τ singular terms dominate full result: strong check of NNLO calculation of Gehrmann et al.



- nice agreement, except for the lowest few bins.

INDIVIDUAL COLOR STRUCTURES AT NNLO

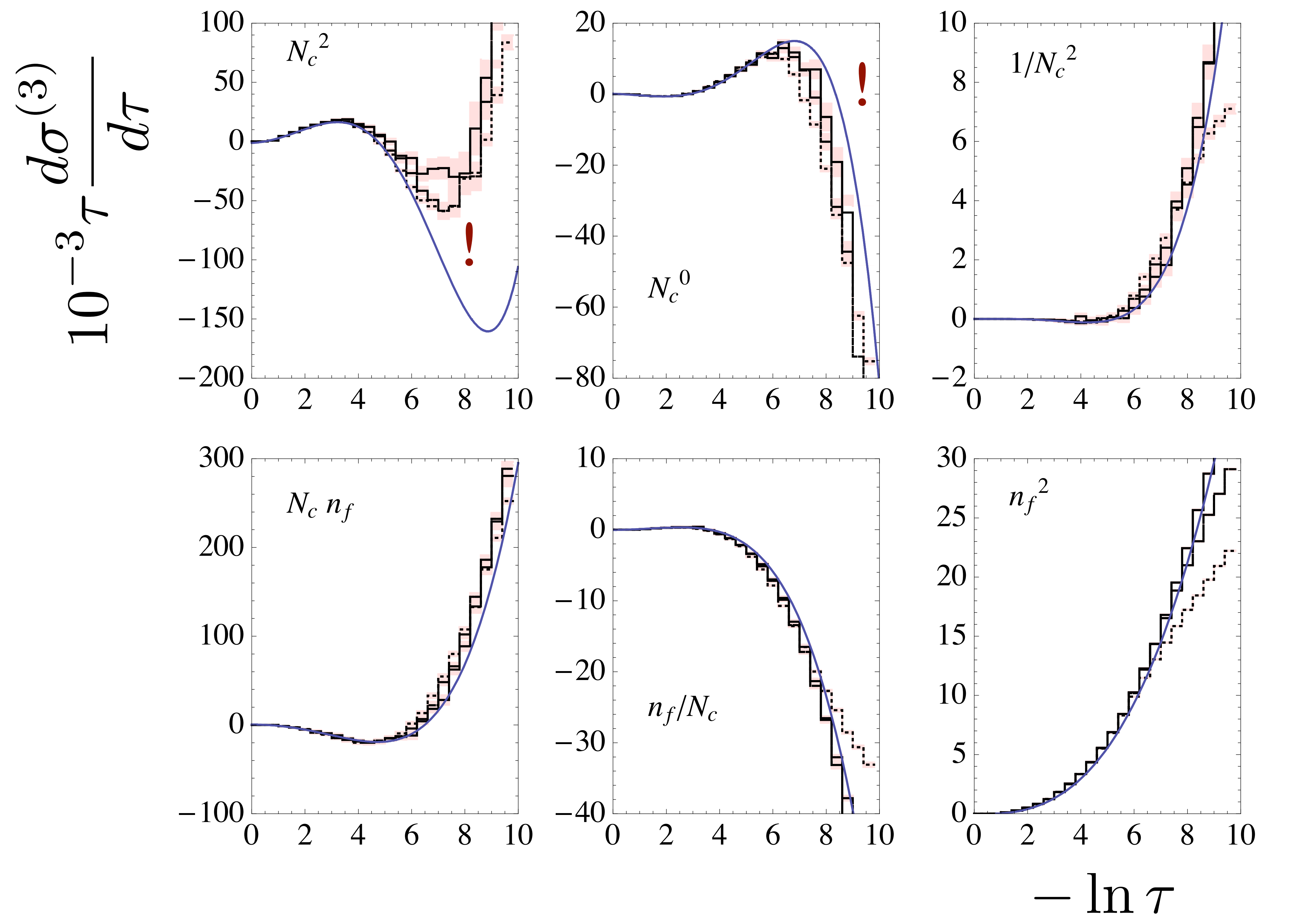
thanks to T. Gehrmann for providing the NNLO histograms!



black histograms: full NNLO (EVENT3 program)

blue lines: singular terms (SCET)

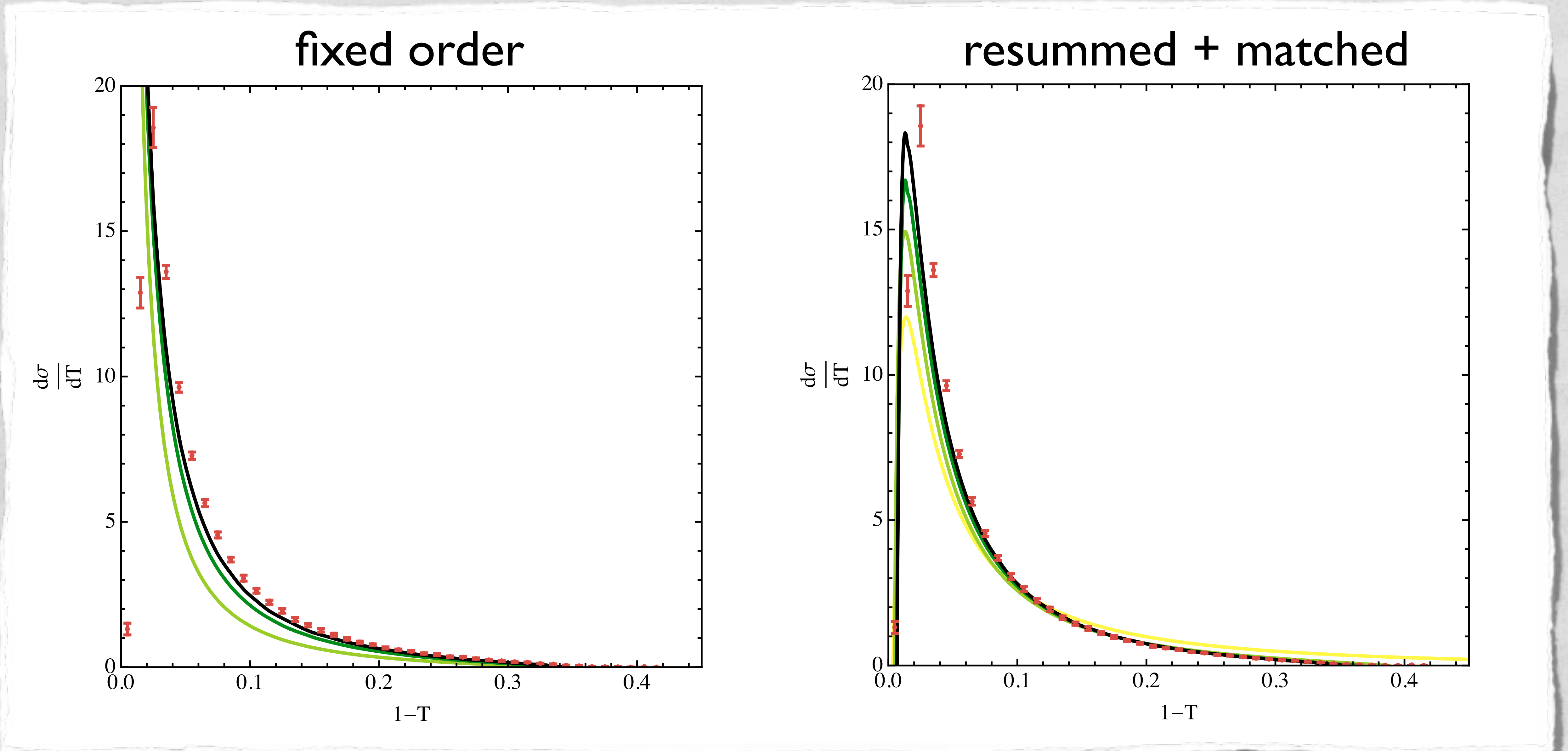
INDIVIDUAL COLOR STRUCTURES: SMALL τ



NUMERICS AT SMALL τ

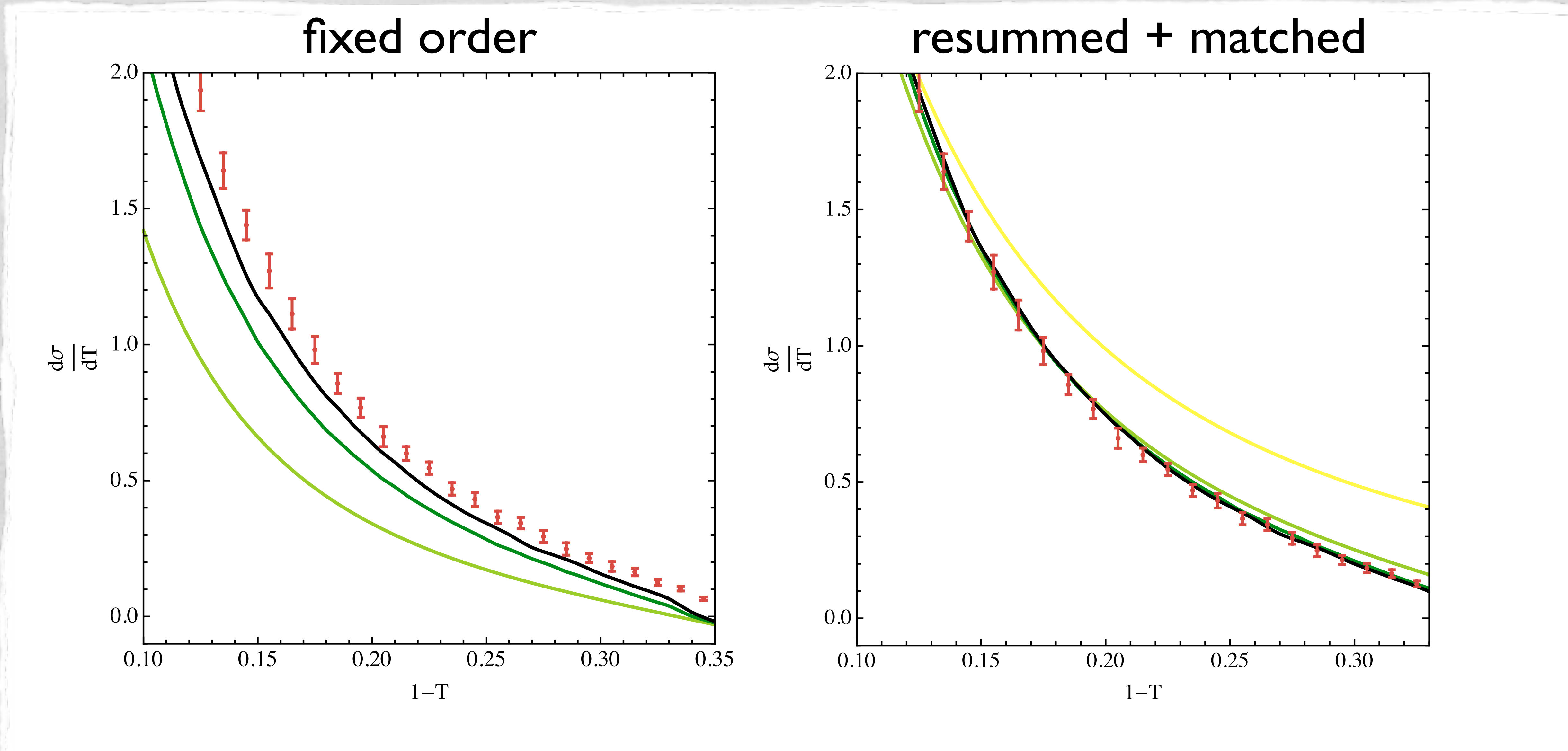
- Numerical evaluation at NNLO is quite involved.
  - Several months of computing time on cluster.
- Small τ especially nontrivial:
  - large numerical cancellation between amplitudes and subtraction terms,
  - negative weights.
- Gehrmann et al. confirm numerical problem with the two leading color structures.
  - Error estimates become unreliable at small τ.
- These numerical difficulties have no impact on α_s determination, since only region $\tau > 0.1$ is used.

RESUMMED VS. FIXED ORDER



- For PDG value $\alpha_s(M_Z)$=0.1176.

RESUMMED VS. FIXED ORDER



- For PDG value $\alpha_s(M_Z)$=0.1176
- This is the region relevant for α_s determination

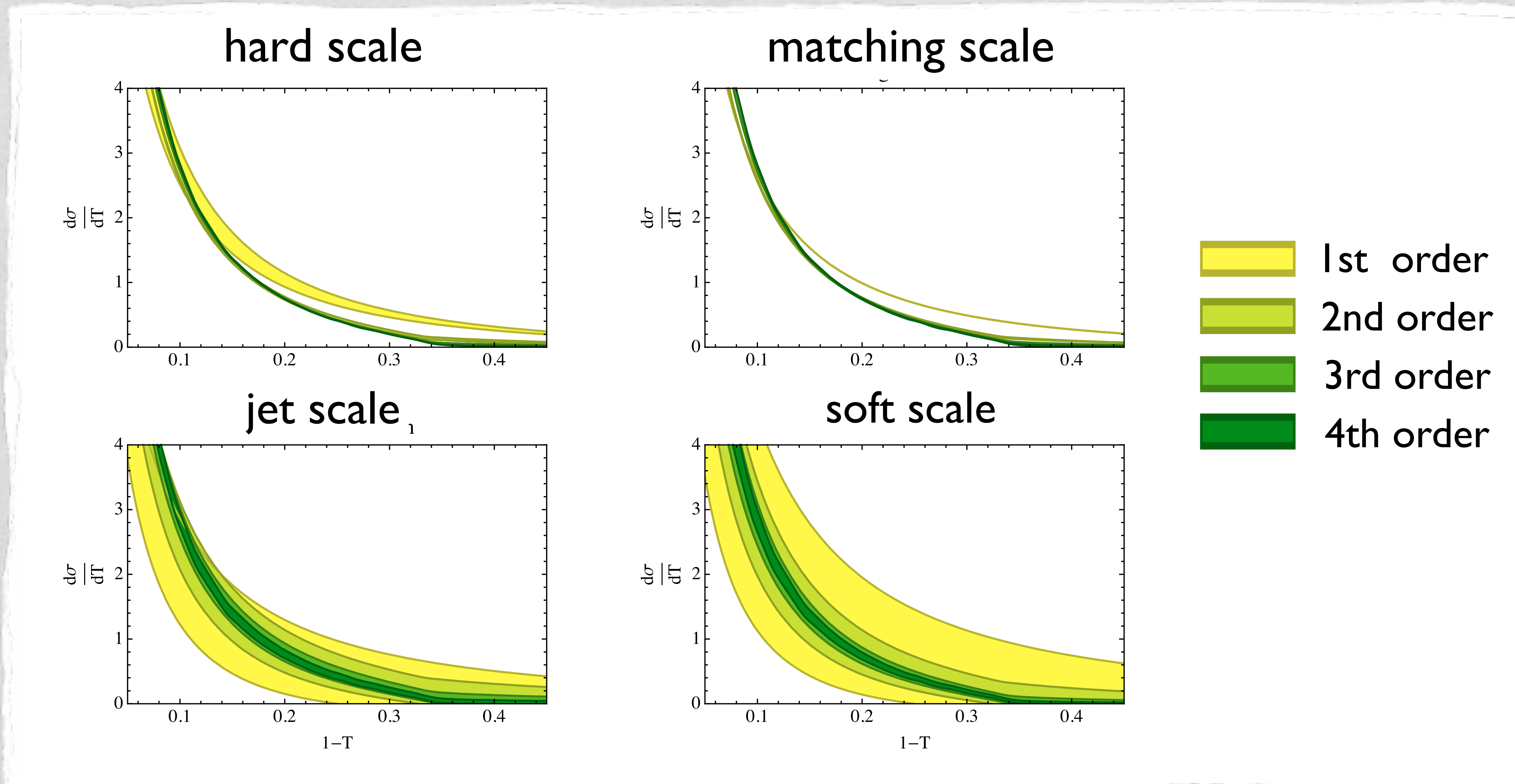
DETERMINATION OF α_s

- Scale variation, error band method
- Fit to ALEPH and OPAL LEP data

THEORETICAL UNCERTAINTY

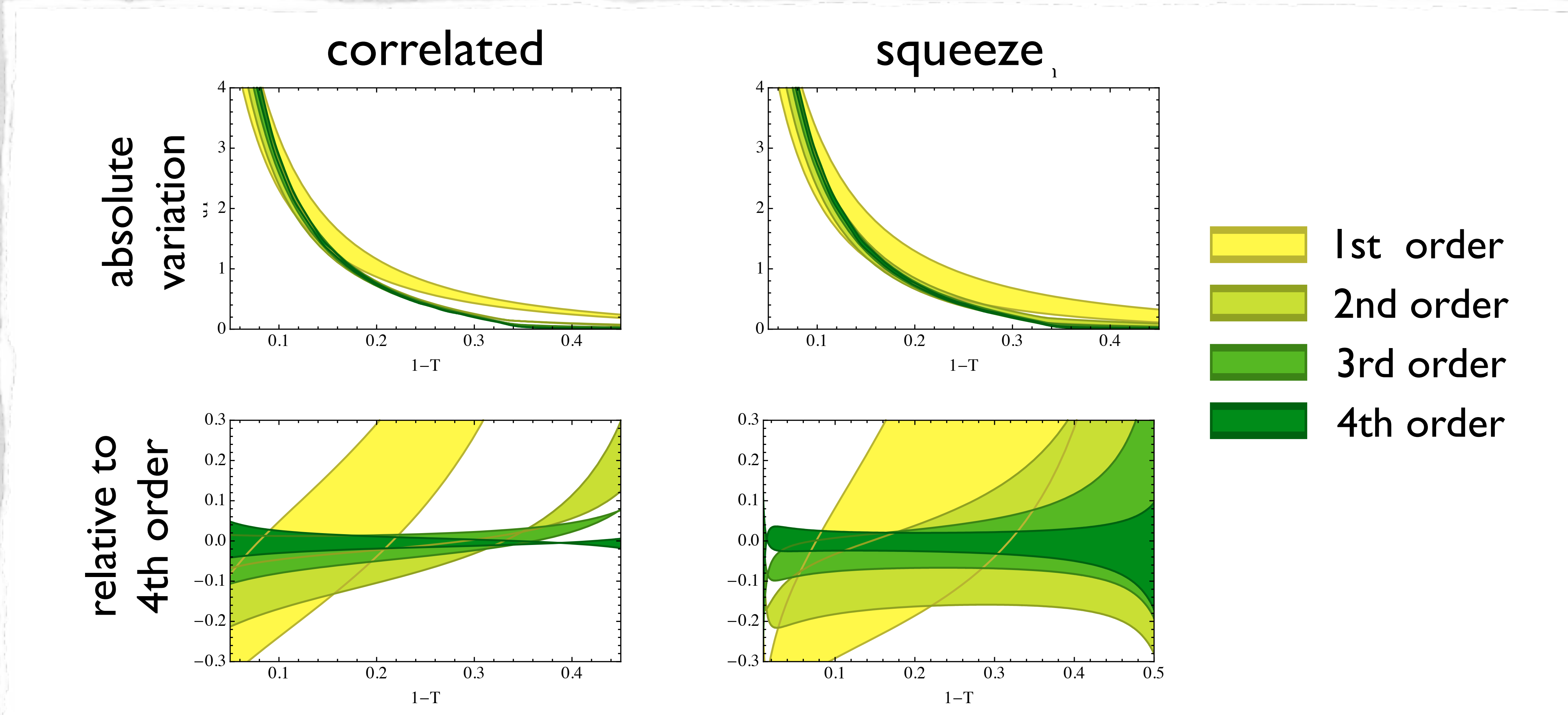
- We will assess the perturbative uncertainty in the standard way, by varying the renormalization (resp. matching) scales.
  - To the order of the calculation, the cross section is independent of these scales;
  - variation then is a measure of unknown higher order terms.
- We have four scales
  - $\mu_{\text{hard}}^2 \sim Q^2$: scale at which H is evaluated
  - $\mu_{\text{jet}}^2 \sim \tau\, Q^2$: scale at which J is evaluated
  - $\mu_{\text{soft}}^2 \sim \tau^2\, Q^2$: scale at which S_T is evaluated
  - μ_{match}^2 : scale of the regular terms

INDEPENDENT SCALE VARIATION



- Varying jet and soft scale independently by a factor 2 makes no sense at moderate τ (leads to $\mu_{soft} > \mu_{jet}$, etc.), overestimates the uncertainty.

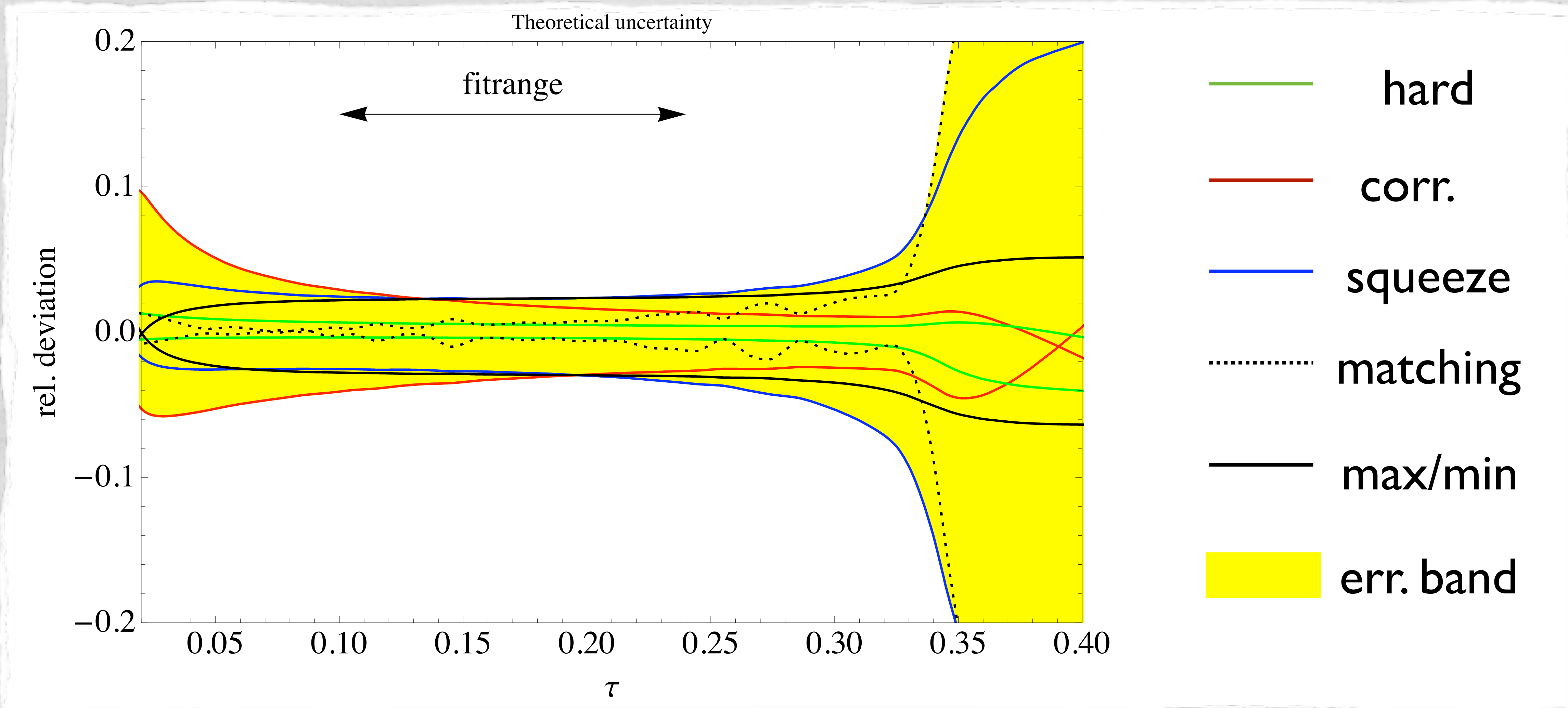
JET AND SOFT SCALE VARIATION



- Instead of independently varying the jet and soft scales, we vary as follows
  - correlated: $\mu_{jet} \to \alpha\, \mu_{jet},\ \ \mu_{soft} \to \alpha\, \mu_{soft}$ with $1/2 < \alpha < 2$
  - squeeze: $\mu_{jet} \to \sqrt{\alpha}\, \mu_{jet},\ \ \mu_{soft} \to \alpha\, \mu_{soft}$ with $1/\sqrt{2} < \alpha < \sqrt{2}$

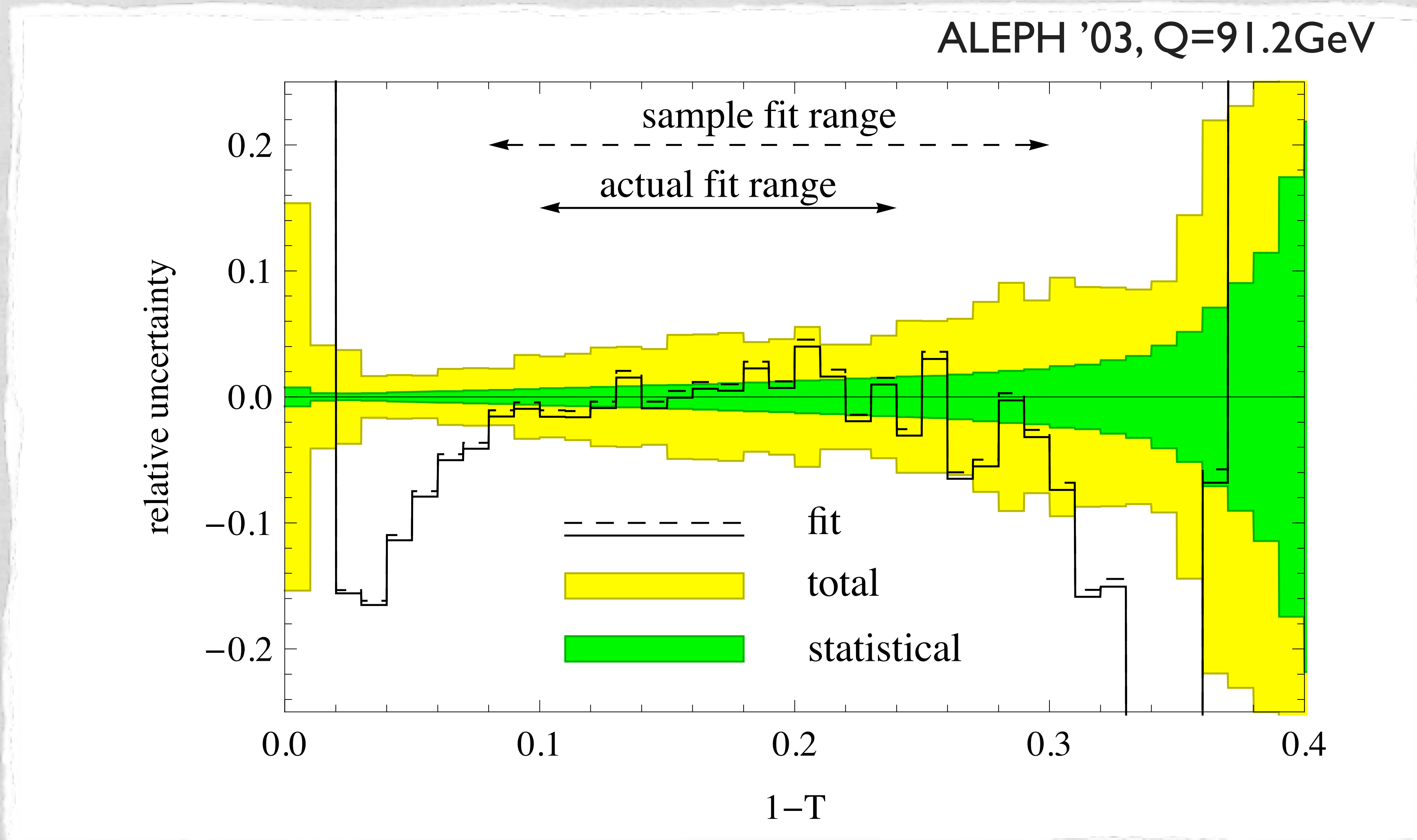
ERROR BAND METHOD

Jones, Ford, Salam Stenzel & Wicke '03; adopted by ALEPH and OPAL



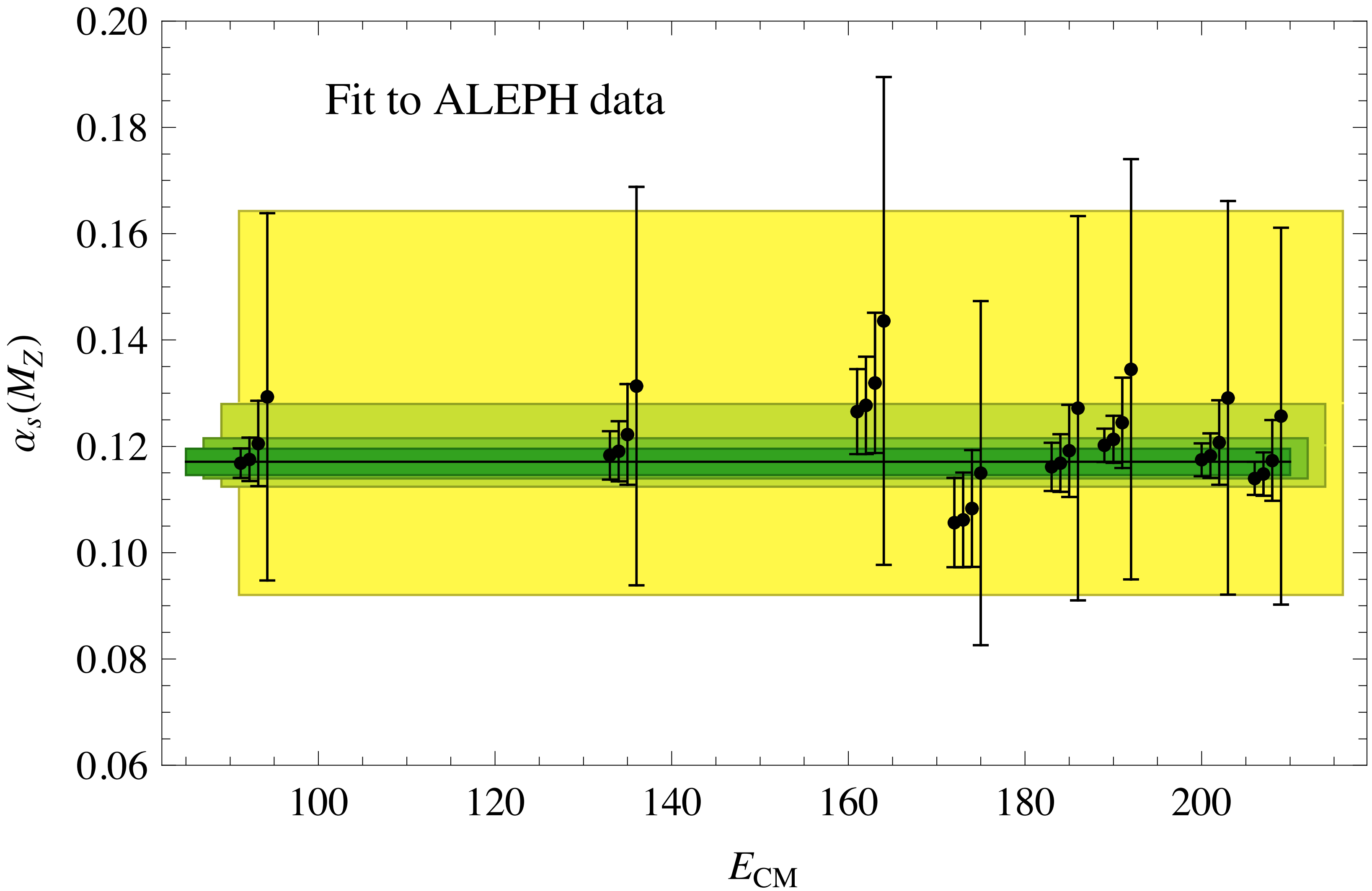
- Perform χ^2-fit to the data, extract best-fit value of α_s. Calculate maximum deviation from default distribution: "error band".
- To get theoretical uncertainty, calculate max. and min. α_s for which theoretical distribution lies inside the error band.

EXPERIMENTAL UNCERTAINTY



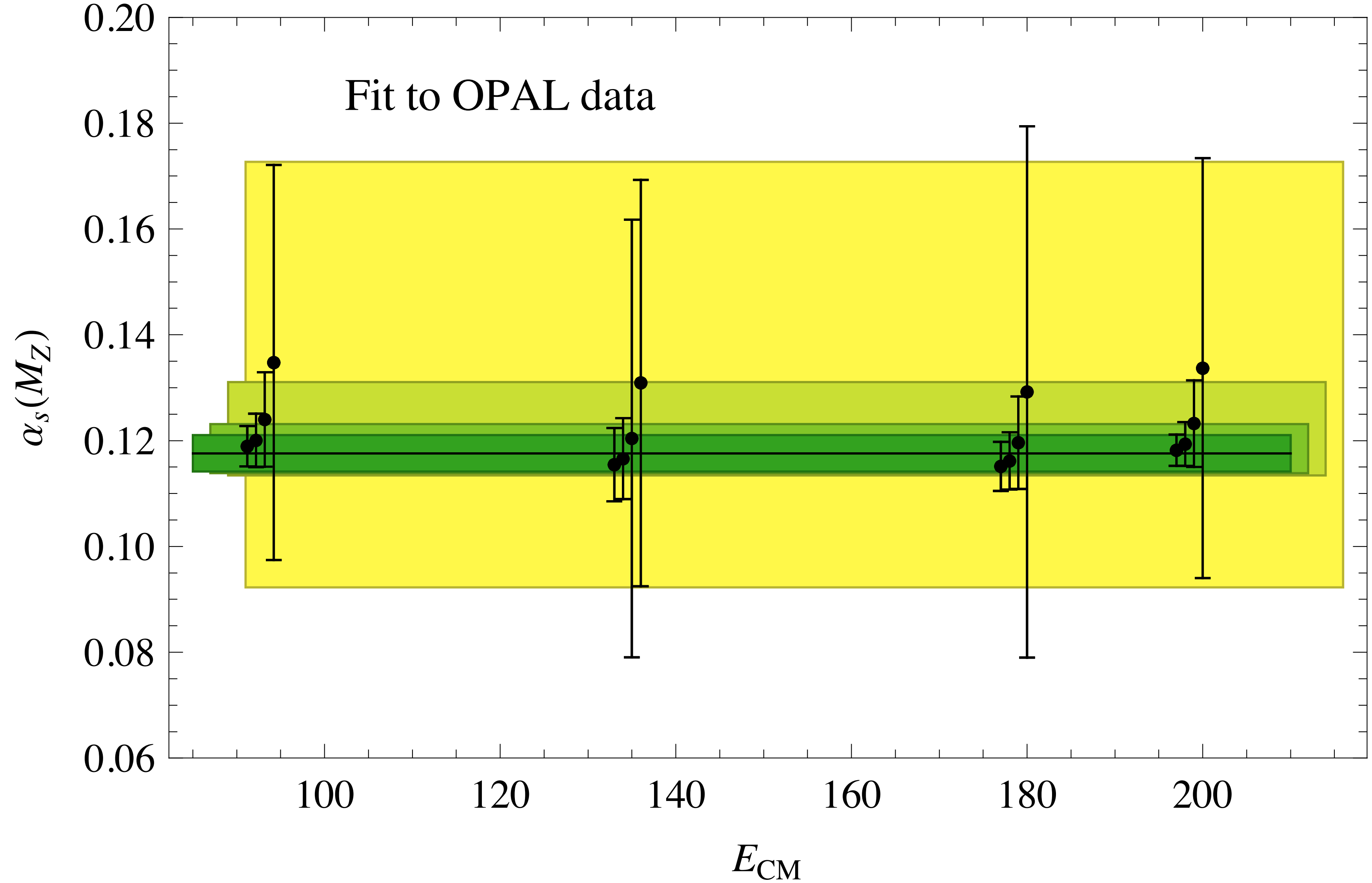
- OPAL '05 and ALEPH '03 give results for binned thrust distributions. Do not provide correlations.
- Put only stat. err. in our χ^2-fit. For each Q, use same fit ranges as exp. paper and use their systematic uncertainties.

RESULT: ALEPH



order	α_s	total err	stat err	pert. err	α_s (LEP 1)	tot.err (LEP 1)
first	0.1281	0.0361	0.0023	0.0360	0.1293	0.0345
second	0.1202	0.0078	0.0014	0.0074	0.1205	0.0080
third	0.1178	0.0038	0.0010	0.0032	0.1175	0.0041
fourth	0.1171	0.0025	0.0009	0.0015	0.1168	0.0028

RESULT OPAL

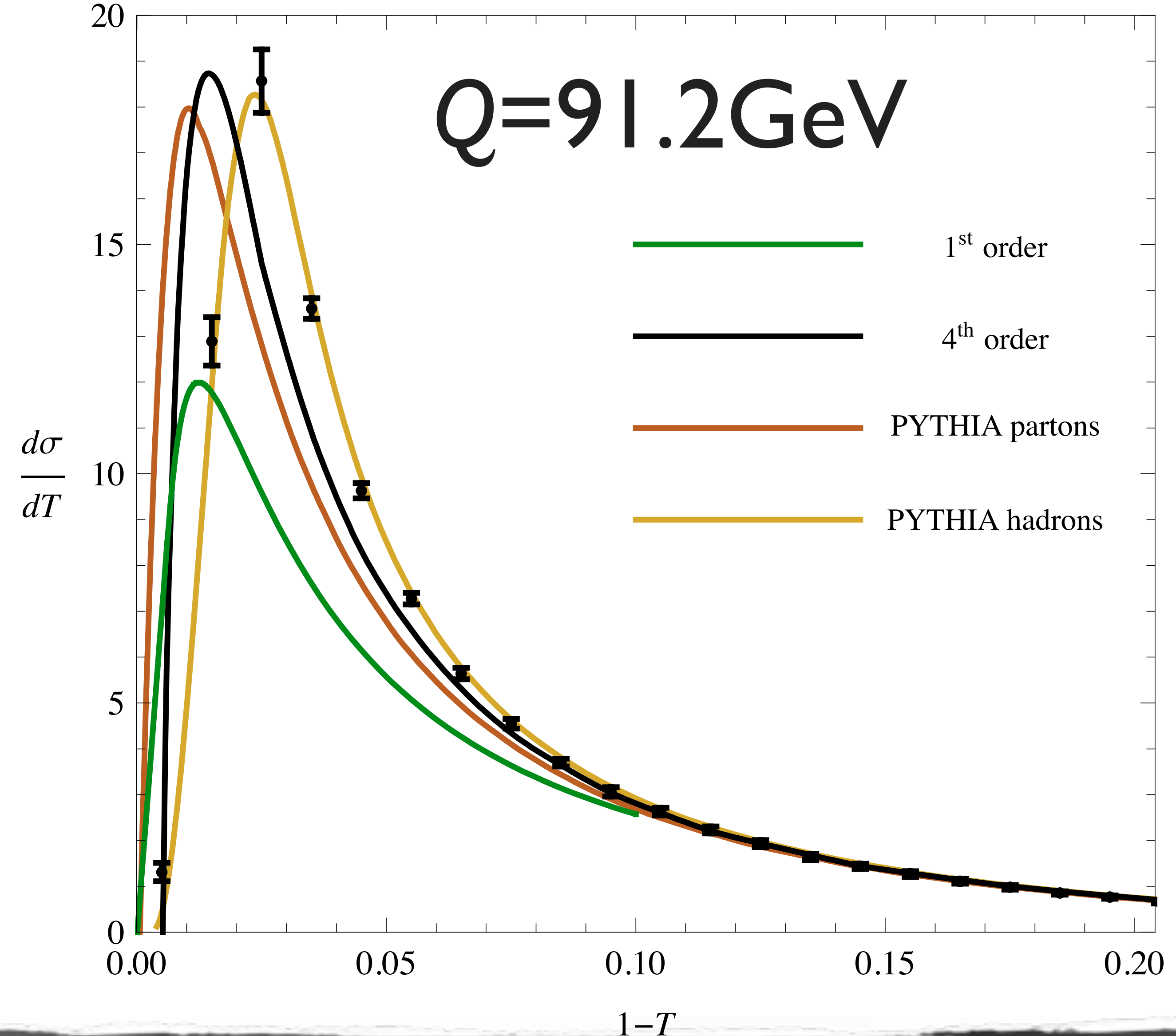


order	α_s	total err	stat err	pert. err	α_s (LEP 1)	tot.err. (LEP 1)
first	0.1325	0.0402	0.0016	0.0400	0.1348	0.0373
second	0.1223	0.0088	0.0012	0.0082	0.1240	0.0089
third	0.1185	0.0046	0.0009	0.0033	0.0120	0.0051
fourth	0.1176	0.0034	0.0008	0.0016	0.1189	0.0038

POWER CORRECTIONS

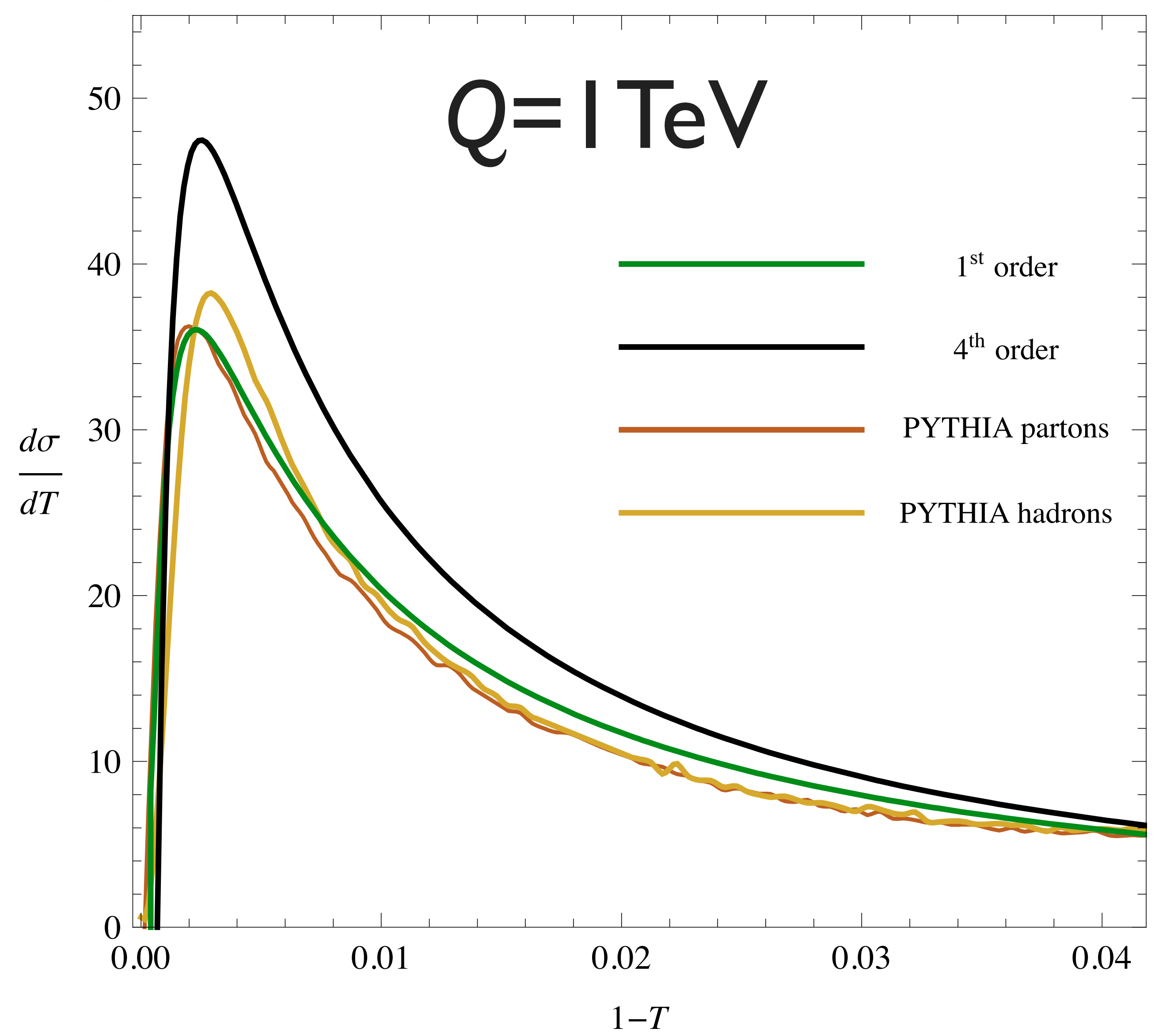
- So far, we have not included 1/Q power corrections:
  - finite b-quark mass effects ≈ +1.5% at LEP I
    - calculated perturbatively, e.g. using NLO event generator by Nason and Oleari.
    - could perform resummation for this part, using SCET, but presumably not worth it.
  - hadronisation ~ -1.5% at LEP I
    - estimated using Pythia to calculate transfer matrix
    - uncertainty is estimated by comparing Pythia to Herwig and Ariadne: 2.5% at LEP I. Now the dominant uncertainty!
    - Our precise perturbative prediction can and should be used to study hadronisation effects in more detail, using also lower energy data.

COMPARISON WITH PYTHIA



- hadronic Pythia agrees perfectly with the ALEPH data
- partonic Pythia does much better than NLL

I TEV LEPTON COLLIDER



- Partonic Pythia now looks much more NLL like.
- Will need to retune (or redesign) the shower.
  - Can tune partonic shower to our theoretical prediction.

FUTURE WORK

- study of power corrections
- hand over the prediction to experiments, so that the fit to data can be redone including all correlations
- other 2-jet event shapes Bauer, Fleming, Lee and Sterman arXiv:0801.4569
- 3-jet event shapes
- $pp \to 2$ jets, ...
- $\bar{p}p \to \bar{t}\, t$, ...
- relation to / implementation into MCs? Bauer and Schwartz ’06; Bauer, Tackmann, Thaler ’08
- ...

SUMMARY

- Have used effective field theory methods to resum thrust distribution to N^3LL.
  - Traditional method works only up to NLL.
- Logarithmically enhanced contributions dominate. Have evaluated all singular terms at α_s^3.
  - Strong check of NNLO calculation of $e^+e^- \rightarrow 3$ jets.
- Used result to determine α_s from a fit to LEP data.
  - Value agrees well with low energy determinations.
  - Most precise determination of α_s in high energy scattering. Perturbative uncertainty is 1.5%, below hadronisation uncertainties.