Lattice Study of the Conformal Window in QCD-Like Theories

George Fleming
Ethan Neil
TA

arXiv:0712.0609, → PRL

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LSD Collaboration

Lattice Strong Dynamics

J. C. Osborn

Argonne National Laboratory

R. Babich, R. C. Brower, M. A. Clark, C. Rebbi, D. Schaich

Boston University

M. Cheng, T. Luu, R. Soltz, P. M. Vranas

Lawrence Livermore National Laboratory

T. Appelquist, G. T. Fleming, E. T. Neil

Yale University

Beyond the Standard Model

Conformal or Near-Conformal Behavior in the IR:

Dynamical Electroweak Symmetry Breaking. (Walking Technicolor)

ADS/CFT

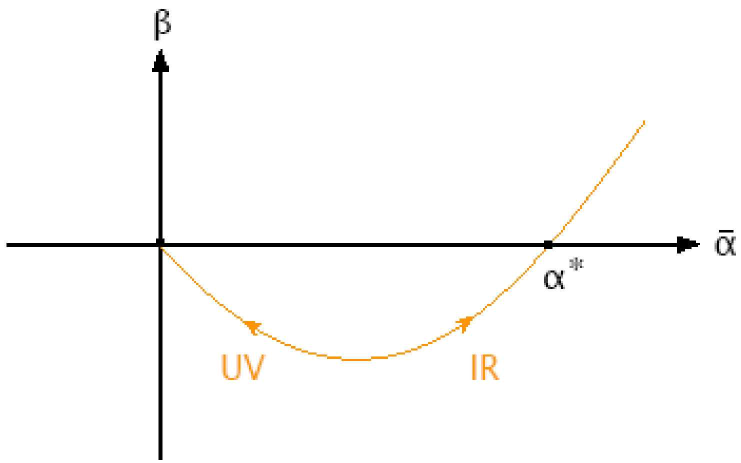
SUSY Flavor Hierarchies (Nelson & Strassler 2000/01)

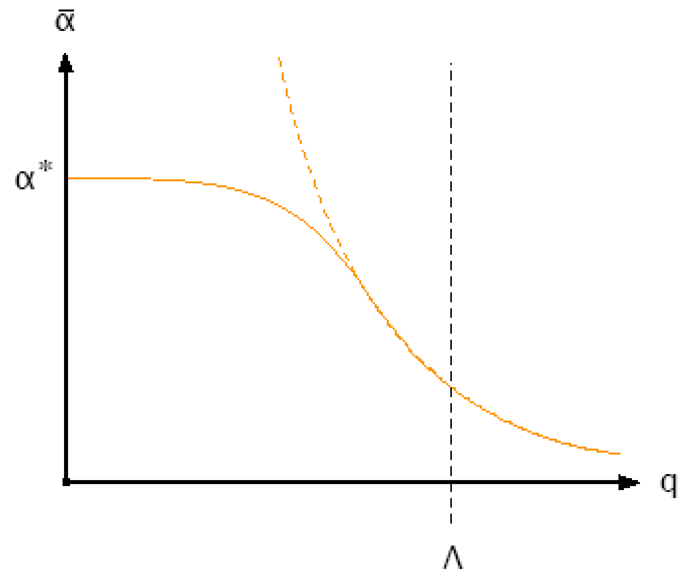
For an asymptotically free theory, an IR fixed point can emerge already in the two-loop β function, depending on the number of fermions N_f

Gross and Wilczek, antiquity
Caswell, 1974
Banks and Zaks, 1982
Many Others

Reliable if the number of fermions is very close to the number at which asymptotic freedom is lost

Cartoons





α^* increases as N_f decreases.

Should be a range of N_f where IR fixed point exists, not necessarily accessible in PT. (This is known in certain SUSY theories.)

Possibilities

(1) $\alpha^* < \alpha_c^*$ $(N_f > N_{fc})$

Conformal IR behavior (Non-abelian coulomb phase).

(2) $\alpha^* > \alpha_c^*$ $(N_f < N_{fc})$ Chiral symmetry breaking, confinement

(3) $\alpha^* \gtrsim \alpha_c^*$ $(N_f \lesssim N_{fc})$ (fine tuning?)

If the transition is continuous, breaking scale << Λ ,

$\Rightarrow$ Walking at intermediate scales.

Questions

1. Value of N_{fc}?

2. Order of the phase transition?

3. Physical states below and near the transition?

4. Implications for EW precision studies? (The S parameter etc)?

5. Implications for the LHC?

N_{fc} in SU(N) QCD

- Degree-of-Freedom Inequality (Cohen, Schmaltz, TA 1999). Fundamental rep:

$$N_{fc} \leq \; 4\,N[1 - 1/18N^2 + \ldots]$$

- Gap-Equation Studies, Instantons: $N_{fc} \cong 4\,N$

- Lattice Simulation (Iwasaki et al, Phys Rev D69, 014507 2004):

$$6 < N_{fc} < 7 \qquad \text{For } N = 3$$

N_{fc} in SUSY SU(N) QCD

Degree of Freedom Inequality:

$$N_{fc} \leq (3/2)\, N$$

Seiberg Duality: $N_{fc} = (3/2)\, N$!!

Weakly coupled magnetic dual in the vicinity of this value

Some Quasi-Perturbative Studies of the Conformal Window in QCD-like Theories

1. Gap – Equation studies in the mid 1990s

2. V. Miransky and K. Yamawaki hep-th/9611142 (1996)

3. E. Gardi, G. Grunberg, M. Karliner hep-ph/9806462 (1998)

4. E. Gardi and G. Grunberg JHEP/004A/1298 (2004)

   "The IRFP is perturbative in the entire conformal window"

5. Kurachi and Shrock, hep-ph/0605290

6. H. Terao and A. Tsuchiya arXiv:0704.3659 [hep-ph] (2007)

Lattice-Simulation Study of the Extent of the Conformal window in an SU(3) Gauge Theory with Dirac Fermions in the Fundamental Representation

Previous Lattice Work with Many Light Fermions

1. Brown et al (Columbia group) Phys. Rev. D12, 5655 (1992)
   $N_f = 8$
2. Damgaard, Heller, Krasnitz and Oleson, hep-lat/9701008
   $N_f = 16$
3. R. Mahwinney, hep/lat/9701030(1) $(N_f \to 4)$, Nucl.Phys.Proc.Suppl.83:57-66,2000. e-Print: hep-lat/0001032

4. C. Sui, Flavor dependence of quantum chromodynamics. PhD thesis, Columbia University, New York, NY, 2001. UMI-99-98219

5. Iwasaki et al, Phys. Rev, D69, 014507 (2004)

Focus: Gauge Invariant and Non-Perturbative Definition of the Running Coupling Deriving from the Schroedinger Functional of the Gauge Theory

ALPHA Collaboration: Luscher, Sommer, Weisz, Wolff, Bode, Heitger, Simma, …

Using Staggered Fermions as in

U. Heller, Nucl. Phys. B504, 435 (1997)
Miyazaki & Kikukawa

$O(a^2)$ Chiral Breaking $\Longrightarrow$ Remaining Continuous Chiral Symmetry

Focus on N_f = Multiples of 4:

16: Perturbative IRFP

12: IRFP "expected", Simulate

8 : IRFP uncertain , Simulate

4 : Confinement, ChSB

The Shroedinger Functional

- Transition amplitude from a prescribed state at t=0 to one at t=T (Dirichlet BC).
- Euclidean path integral with Dirichlet BC in time and periodic in space (L) to describe a constant chromoelectric background field.

$$Z\left[W,\zeta,\bar{\zeta};W',\zeta',\bar{\zeta}'\right]=$$
$$\int\left[DUD\chi D\bar{\chi}\right]e^{-S_G(W,W')-S_F(W,W',\zeta,\bar{\zeta},\zeta',\bar{\zeta}')}$$

Abelian Boundary Fields

$$W_k(x) = diag\left(e^{i\phi_1/L}, e^{i\phi_2/L}, e^{i\phi_3/L}\right),$$
$$W_k'(x) = diag\left(e^{i\phi_1/L}, e^{i\phi_2/L}, e^{i\phi_3/L}\right).$$

$$\phi_1 = -\frac{\pi}{3} + \eta, \quad \phi_2 = -\tfrac{1}{2}\eta, \quad \phi_3 = -\frac{\pi}{3} + \tfrac{1}{2}\eta,$$

$$\phi_1' = -\pi - \eta, \quad \phi_2' = \frac{\pi}{3} + \tfrac{1}{2}\eta, \quad \phi_3' = \frac{2\pi}{3} + \tfrac{1}{2}\eta.$$

- Constant chromoelectric background field of strength $\frac{1}{L}$
- Can set $m_f = 0$

Schroedinger Functional (SF) Running Coupling on Lattice

Define: $$\frac{1}{\overline{g}^2(L,T)} \equiv \frac{-1}{k}\frac{\partial}{\partial \eta}\log Z\big|_{\eta=0},$$

$$= \frac{1}{g_0^{\,2}} + 0(1) + 0\left(g_0^{\,2}\right) + \ldots$$

Response of system to small changes in the background field.

$$k = 12\left(\frac{L}{a}\right)^2\left[\sin\left(\frac{2\pi a^2}{3LT}\right) + \sin\left(\frac{\pi a^2}{3LT}\right)\right]$$

SF Running Coupling

Then, to remove the O(a) bulk lattice artifact

$$\frac{1}{\bar{g}^2(L)} = \frac{1}{2}\left[\frac{1}{\bar{g}^2(L, L-a)} + \frac{1}{\bar{g}^2(L, L+a)}\right]$$

Depends on only one scale L

Look for conformal symmetry (IRFP) <u>at</u> the box scale L

Loop Expansion

$$L\frac{\partial}{\partial L}\bar{g}^2(L) = \beta\left(\bar{g}^2(L)\right) = b_1\bar{g}^4(L) + b_2\bar{g}^6(L) + b_3\bar{g}^8(L) + \ldots.$$

$$b_1 = \frac{2}{(4\pi)^2}\left(11 - \frac{2}{3}N_f\right), \qquad b_2 = \frac{2}{(4\pi)^4}\left(102 - \frac{38}{3}N_f\right)$$

$$b_3 = b_3^{\overline{MS}} + \frac{b_2 c_2}{2\pi^2} - \frac{b_1(c_3 - c_2)}{8\pi^2}$$

$$b_3^{\overline{MS}} = \frac{1}{(4\pi)^6}\left[\frac{2857}{2} - \frac{5033}{18}N_f + \frac{325}{54}N_f^2\right]$$

$$c_2 = 1.256 + 0.04N_f$$

$$c_3 = c_2^2 + 1.20 + 0.14N_f - 0.03N_f^2$$

Loop Expansion

$N_f = 16$ IRFP at $g^{*2}_{SF} = 0.47$ $\left(\frac{\bar{g}^2}{4\pi^2} \approx .01\right)$

$N_f = 12$ IRFP at $g^{*2}_{SF} = 5.18$ $\left(\frac{\bar{g}^2}{4\pi^2} \approx .13\right)$

$N_f \leq 8$ No perturbative IRFP

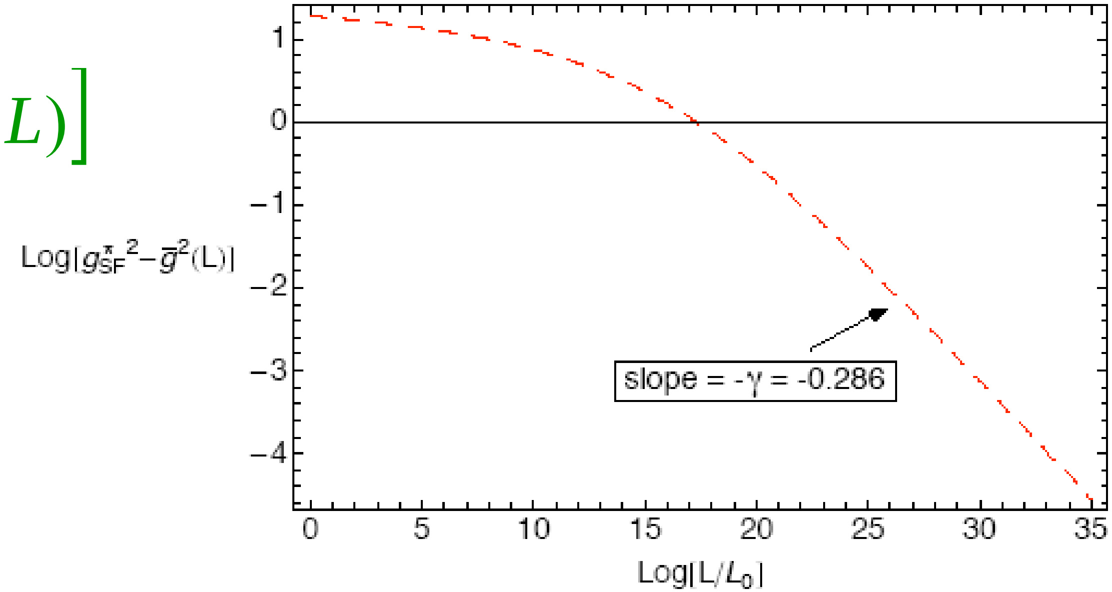
Loop Expansion

Linearize β near the $N_f = 12$ IRFP

$$\beta(\bar{g}^2(L)) \cong \gamma\left[g_{SF}^{*2} - \bar{g}^2(L)\right]$$

Then:

$$\bar{g}^2(L) \xrightarrow{L\to\infty} g_{SF}^{*2} - \frac{const}{L^{\gamma}}$$



Lattice Simulations

MILC Code (Heller)

Staggered Fermions

$$N_f = 8, 12$$

Range of Lattice Couplings g_0^2 (= $6/\beta$) and Lattice Sizes $L/a \rightarrow 20$

O(a) Lattice Artifacts due to Dirichlet Boundary Conditions

Lattice Simulations

An observable of correlation length $R << L$ can be independently estimated $(L/R)^3$ times on a single lattice. $dS/d\eta$ can be estimated only once per lattice so more independent lattices must be generated.

Locality of action means short distances are decorrelated faster than longer distances by the update algorithm, so computer must run longer to produce another independent lattice configuration for estimating $dS/d\eta$.

Excursions - at strong coupling the system can tunnel into other classical minima. Low excursion frequency means that we have to run even longer to average over them properly!

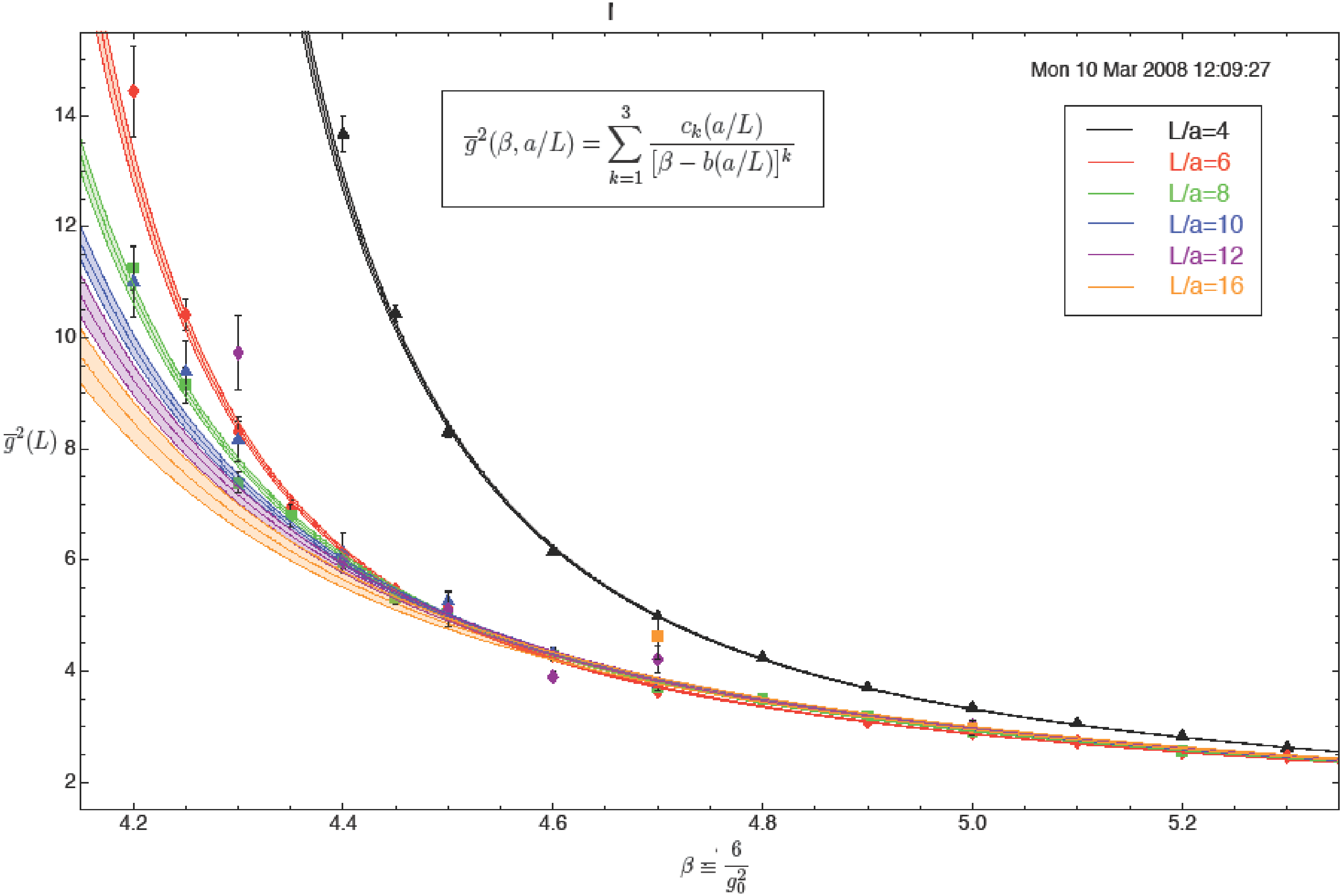
Bottom Line: Schrödinger functional calculations require an order of magnitude more computer time than other lattice calculations on similar volumes.

$N_f = 12$ Data

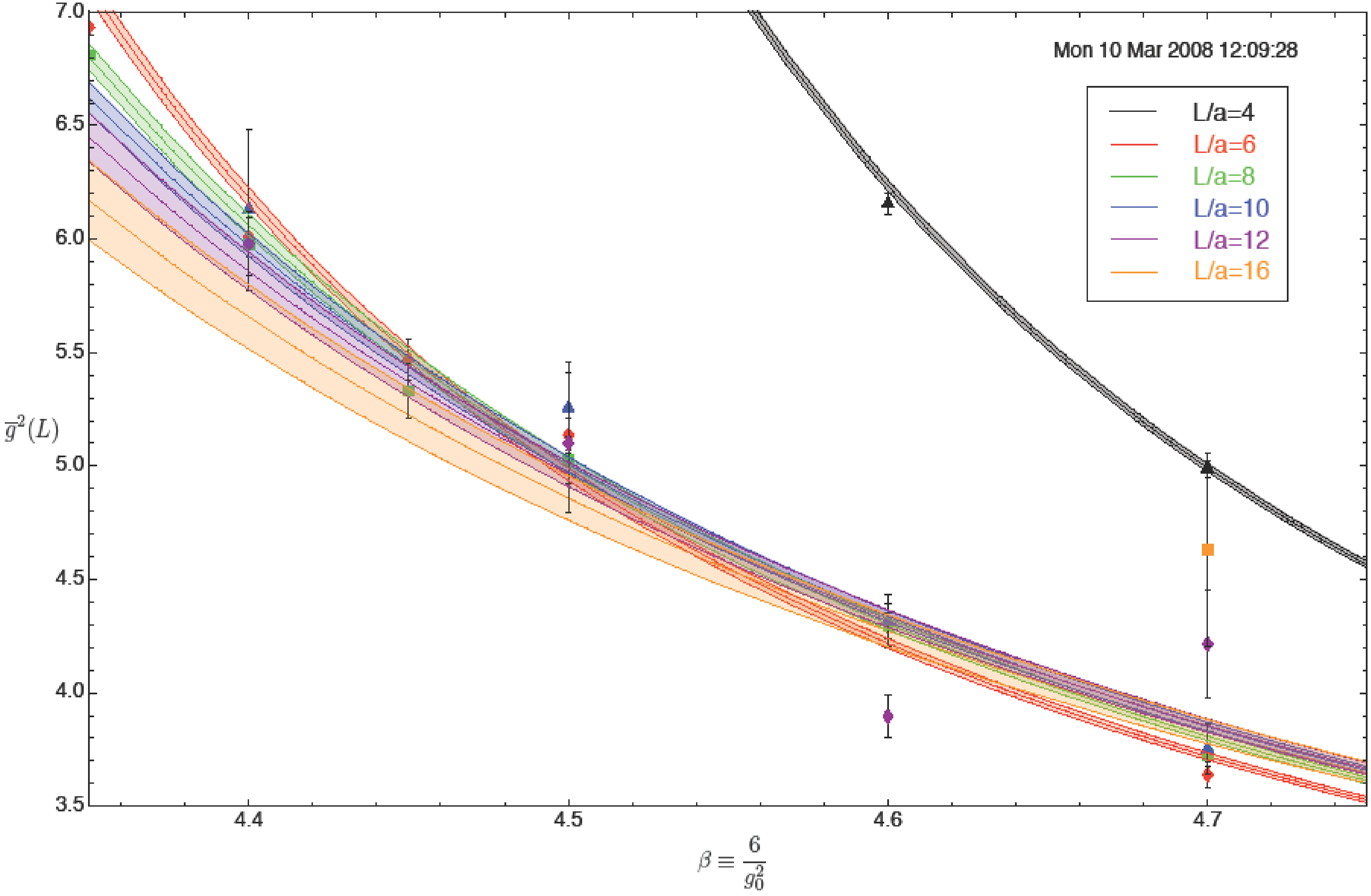
Bold results are recently updated.
Blue (Red) numbers indicate the number of trajectories collected for $T = L + 1$ $(L - 1)$.

$\bar{g}^2(L)$							L/a							
β	4	N_{traj}	6	N_{traj}	8	N_{traj}	10	N_{traj}	12	N_{traj}	16	N_{traj}	20	N_{traj}
4.1			65(14)	82K 82K	24.8(1.8)	28K 38K								
4.15			26.6(1.3)	81K 81K	15.80(68)	81K 81K								
4.18			**17.50(84)**	42K 52K	**11.48(74)**	13K 18K								
4.2			14.44(82)	82K 82K	11.26(41)	82K 82K	**11.50(60)**	40K 54K						
4.25	**70.3(5.2)**	162K 162K	10.42(29)	81K 81K	9.15(32)	81K 81K	**9.39(57)**	50K 50K						
4.3	29.56(99)	162K 162K	8.32(17)	82K 82K	7.40(19)	82K 82K	**8.51(40)**	60K 65K	**9.40(64)**	51K 58K	**8.29(74)**	5K 7K		
4.35	18.99(40)	162K 162K	6.93(14)	82K 82K	6.81(20)	82K 82K								
4.4	13.66(32)	162K 162K	6.008(87)	82K 82K	5.98(14)	82K 82K	6.13(35)	25K 37K	5.96(35)	40K 50K				
4.45	10.44(15)	162K 162K	5.36(11)	82K 82K	5.33(12)	82K 82K								
4.5	8.296(79)	162K 162K	5.137(71)	96K 96K	**5.026(101)**	84K 98K	**5.26(19)**	55K 45K	**4.955(239)**	45K 53K				
4.6	6.155(48)	162K 162K	4.214(78)	82K 82K	4.298(96)	82K 82K	**4.44(11)**	80K 79K	**3.896(93)**	41K 57K	**4.04(15)**	19K 17K	**2.93(13)**	2K 2K
4.7	4.986(36)	162K 162K	3.839(55)	82K 80K	**3.728(50)**	86K 88K	3.752(109)	21K 24K	**3.962(171)**	46K 36K	**4.749(423)**	52K 54K		
4.8	4.244(28)	162K 162K	3.483(46)	82K 82K	3.511(57)	82K 82K								
4.9	3.704(20)	162K 162K	3.092(26)	82K 82K	3.193(44)	82K 82K								
5.0	3.30(4)	122K 41K	2.899(27)	54K 40K	2.916(34)	54K 90K	2.998(58)	41K 41K	**3.020(73)**	44K 47K	**2.971(148)**	27K 28K		
5.1	3.059(34)	41K 41K	2.731(33)	36K 52K										
5.2	**2.832(24)**	41K 41K	2.545(12)	36K 56K	2.568(26)	54K 90K								
5.3	2.622(17)	41K 41K	2.468(29)	36K 56K										
5.4	**2.483(19)**	41K 41K	2.323(19)	36K 56K	2.311(1)	42K 42K								
5.5	2.342(12)	41K 41K	2.196(10)	36K 56K										
5.6	**2.237(12)**	41K 41K	2.098(5)	36K 56K	2.118(1)	42K 42K								
5.7	2.126(9)	41K 41K	2.016(5)	36K 56K										

$N_f = 12$ Data with Fits



Blow Up



Mon 10 Mar 2008 12:09:28
L/a=4
L/a=6
L/a=8
L/a=10
L/a=12
L/a=16
7.0
6.5
6.0
5.5
5.0
4.5
4.0
3.5
$\bar{g}^2(L)$
4.4
4.5
4.6
4.7
$\beta \equiv \frac{6}{g_0^2}$

Renormalization Group (Step Scaling)

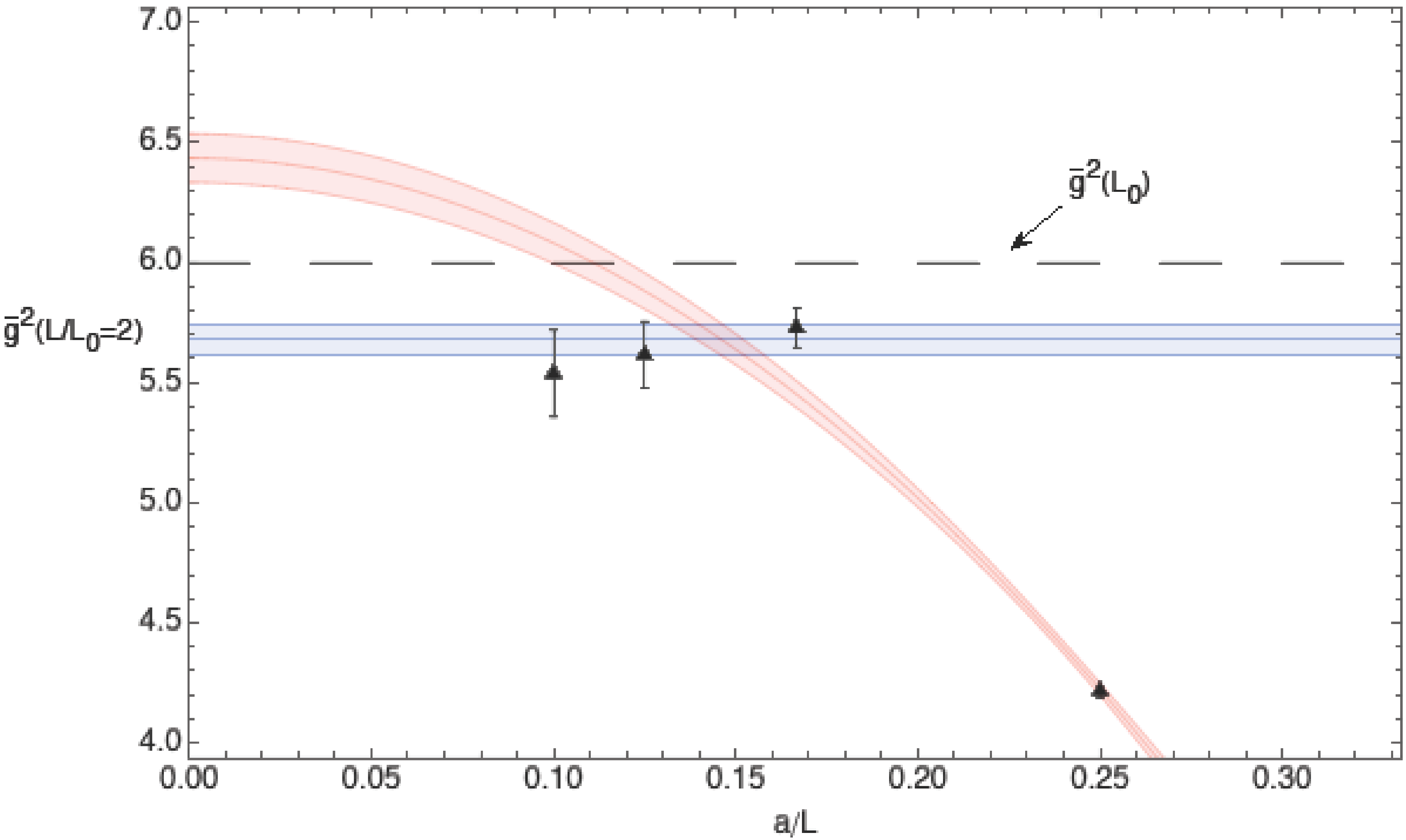
$$\bar{g}^2\left(g_0^2, \frac{a}{L}\right) = \bar{g}^2\left(\bar{g}^2(L_0), \frac{L}{L_0}, \frac{a}{L_0}\right)$$

$$g_0^2 \xrightarrow{a \to 0} 1/\ln\left(L_0/a\right)$$

$$\xrightarrow{a/L_0 \to 0} \bar{g}^2\left(\bar{g}^2(L_0), \frac{L}{L_0}\right) \equiv \bar{g}^2\left(\frac{L}{L_0}\right) \qquad \left(\frac{L}{L_0} = 2\right)$$

$$\bar{g}^2\left(\bar{g}^2(L), \frac{L'}{L}, \frac{a}{L}\right) \xrightarrow{\frac{a}{L} \to 0} \bar{g}^2\left(\frac{L'}{L}\right) \qquad \left(\frac{L'}{L} = 2\right)$$

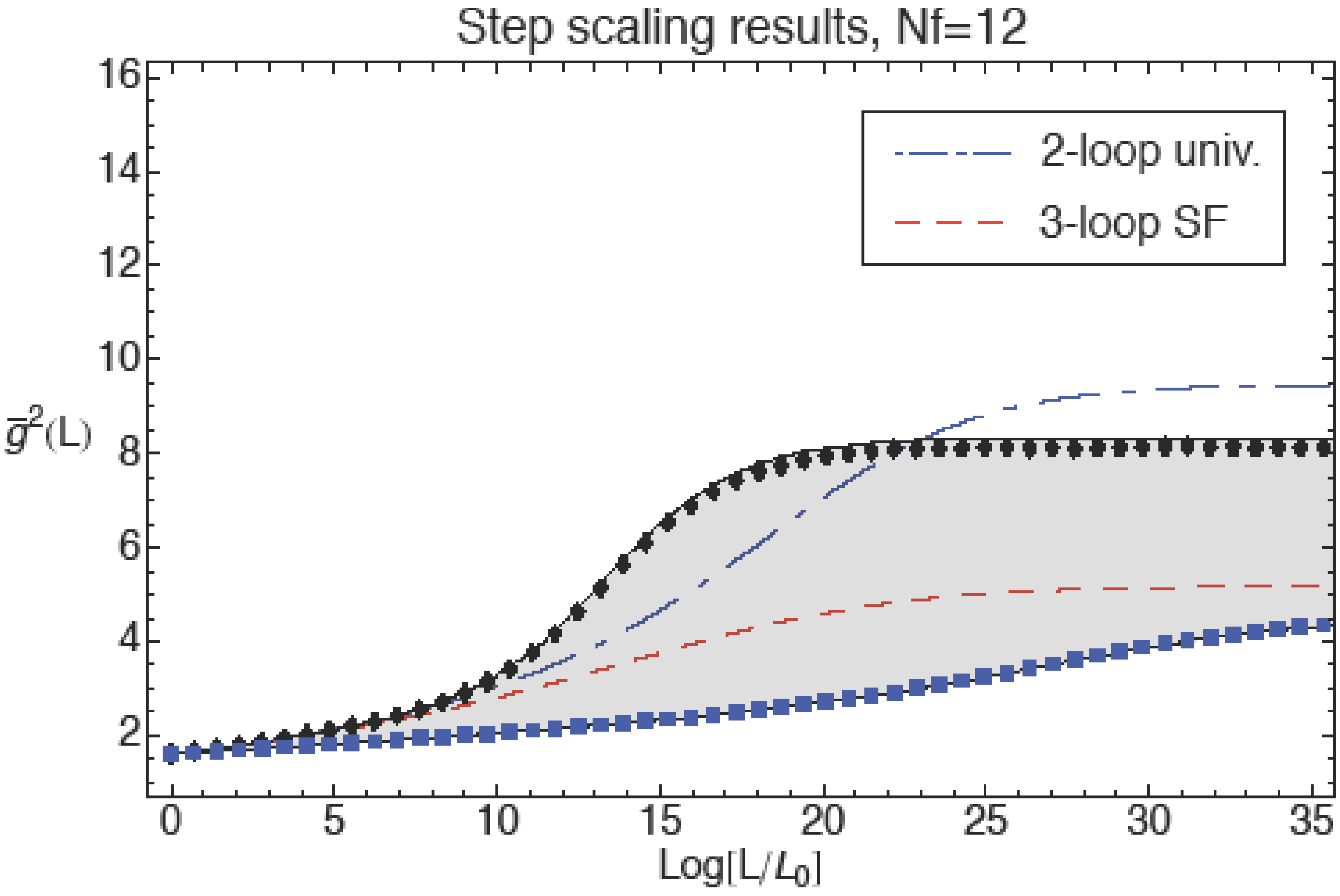
N_f=12 Extrapolation Curve



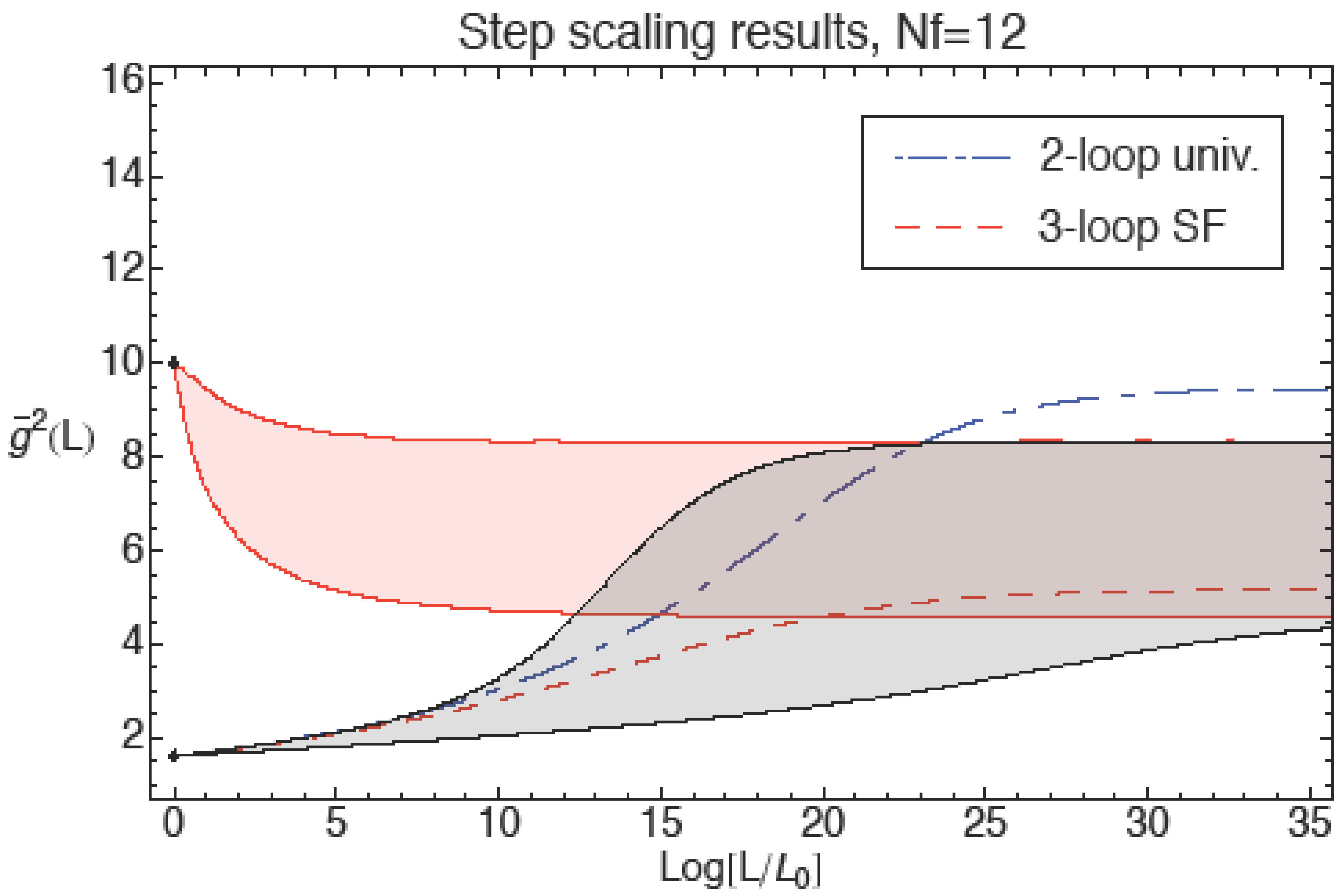
Systematic Error:

1. Interpolating curve at large L
2. Extrapolation, o(a) terms

N_f = 12 Continuum Running



N_f=12 Running From Above



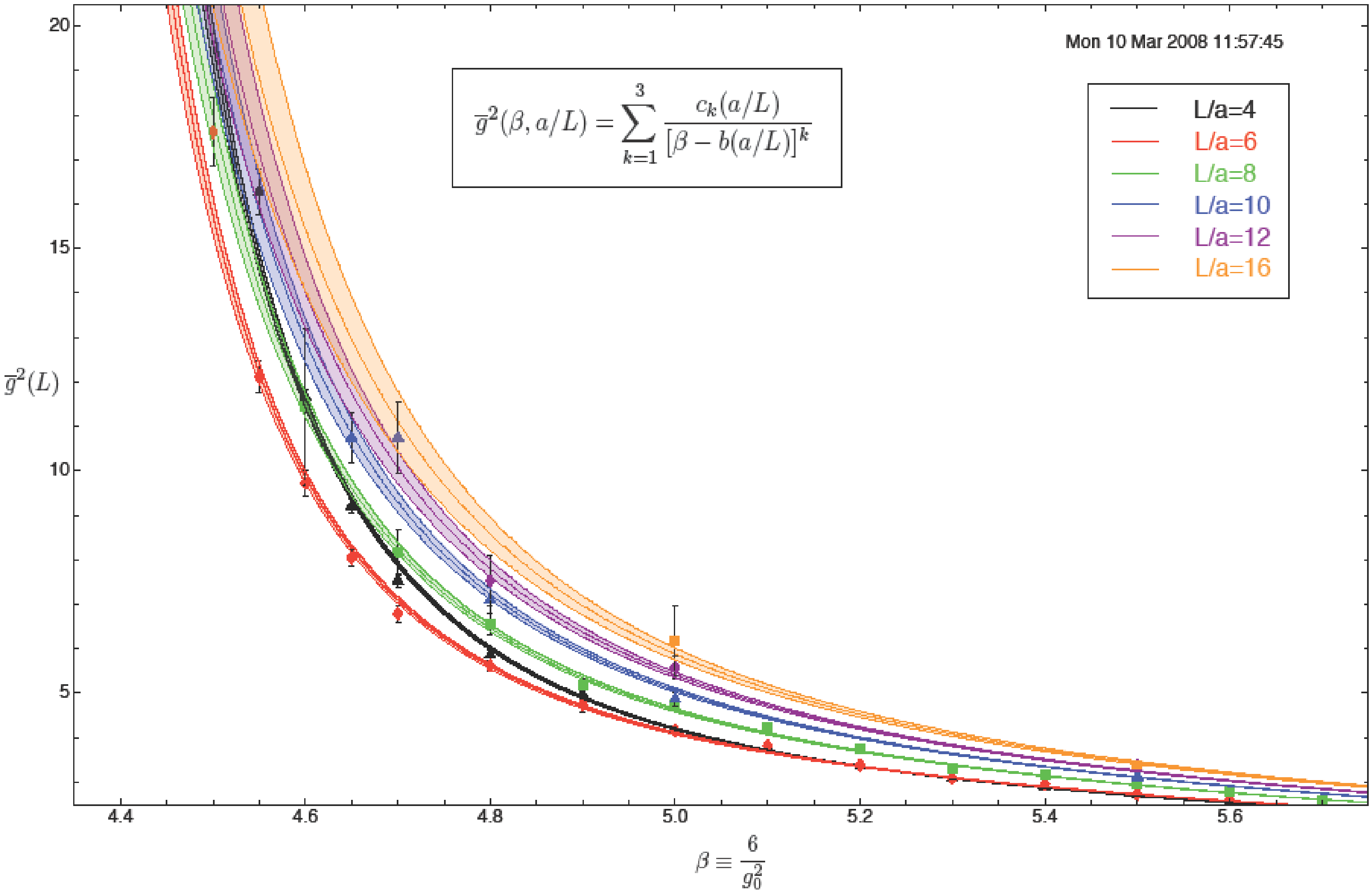
$N_f = 8$ Data

Table 1: Schrödinger functional running coupling $\bar{g}^2(L)$ for SU(3), N_f=8

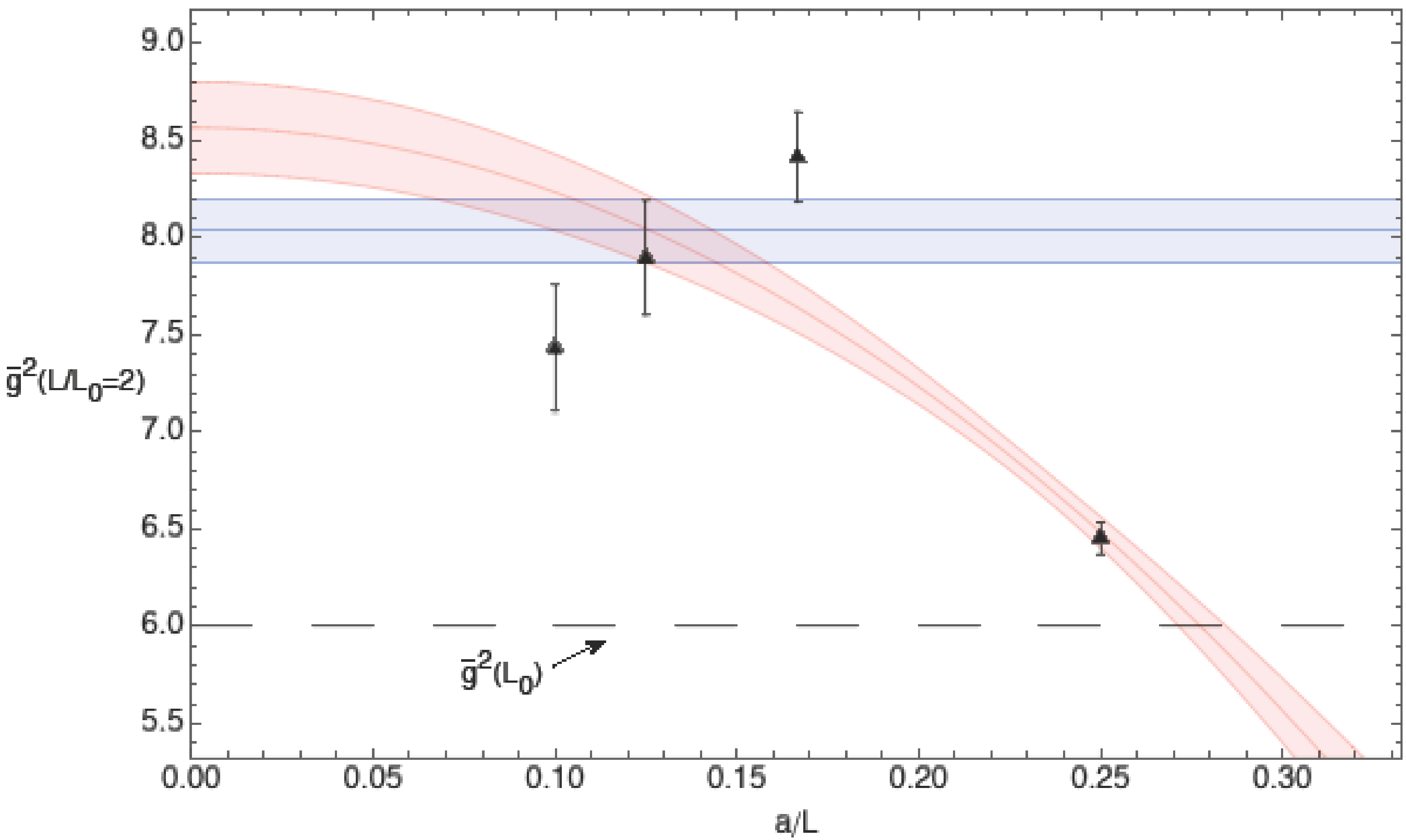
β	L=4	$N_{\rm traj}$	L=6	$N_{\rm traj}$	L=8	$N_{\rm traj}$	L=10	$N_{\rm traj}$	L=12	$N_{\rm traj}$	L=16	$N_{\rm traj}$	L=20	$N_{\rm traj}$
4.45	41.4(2.0)	82000 82000	37.4(2.4)	28600 61900										
4.5	23.87(57)	82000 82000	*17.75(78)*	60000 48000	35.4(4.1)	26000 30000								
4.55	16.28(53)	82000 82000	12.35(45)	82000 56000	*20.8(2.0)*	31000 13000								
4.6	11.66(16)	82000 82000	9.73(30)	82000 82000	11.4(1.8)	81000 81000	*15.9(1.4)*	32000 21000	16.2(2.1)	10500 10000				
4.65	9.21(15)	82000 82000	8.04(19)	82000 82000	*9.40(40)*	27000 29000	10.58(59)	34000 33000	*16.0(1.4)*	29000 11000				
4.7	7.52(14)	42000 42000	6.79(19)	42000 42000	8.17(50)	41000 41000	10.74(80)	31000 26000			10.9(2.2)	900 800		
4.8	5.86(11)	42000 42000	5.62(12)	42000 42000	6.55(25)	41000 42000	7.10(31)	31000 22000	7.26(55)	16500 16500	8.04(65)	8500 8500		
4.9	4.966(84)	42000 42000	4.75(18)	42000 42000	5.18(14)	42000 42000								
5.0	4.157(65)	42000 42000	4.160(87)	40500 40500	4.75(13)	40500 40500	*4.90(14)*	30000 41000	5.58(25)	40500 40500	6.18(78)	27000 28750		
5.1	3.753(40)	41000 41000	3.813(67)	41000 41000	4.226(94)	41000 41000								
5.2	3.386(39)	41000 41000	3.387(37)	41000 41000	3.756(71)	41000 41000								
5.3	3.103(22)	41000 41000	3.087(27)	41000 41000	3.307(53)	59000 60000								
5.4	2.891(26)	41000 41000	2.958(38)	41000 41000	3.163(40)	41000 41000								
5.5	2.735(24)	41000 41000	2.731(21)	41000 41000	2.977(38)	40500 40500	3.113(52)	41000 41000	3.374(73)	40500 40500	3.361(73)	25750 25500		
5.6	2.571(21)	41000 41000	2.599(18)	41000 41000	2.794(31)	41000 41000								
5.7	2.406(11)	41000 41000	2.442(11)	41000 41000	2.592(15)	41000 41000								
5.8	2.2829(97)	41000 41000	2.322(10)	41000 41000	2.494(15)	41000 41000								

Blue (Red) numbers indicate the number of trajectories collected for T=L+1 (L-1). Sat Mar 8 01:08:06 EST 2008

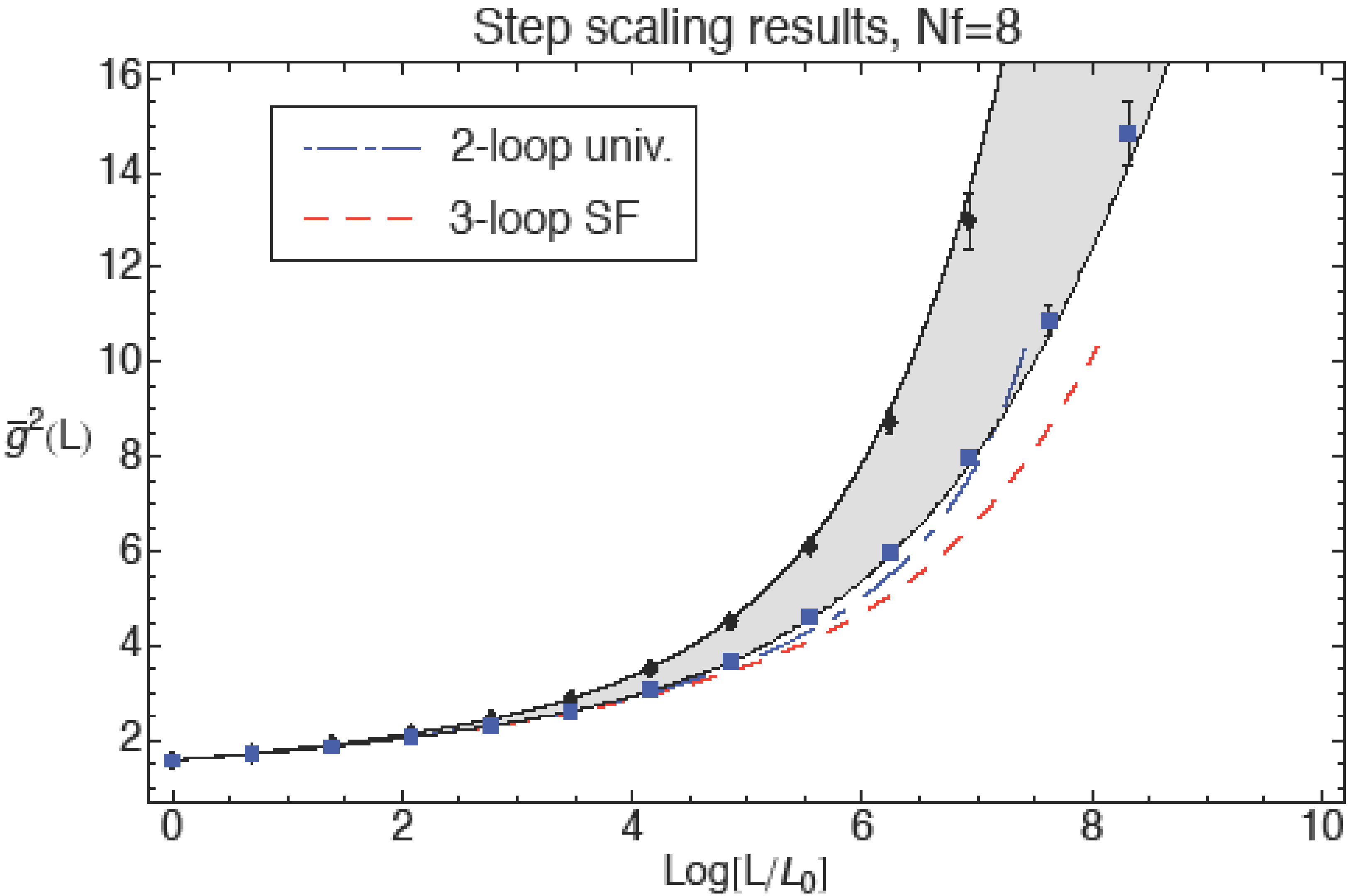
$N_f = 8$ Data with Fits



N_f=8 Extrapolation Curve



N_f = 8 Continuum Running



N_f = 8 Features

1. No evidence for IRFP or even inflection point up through $\bar{g}^2(L) \approx 15$.

2. Exceeds rough estimate $\left(\alpha_c^* / \pi \approx 1/4\right)$ of strength required to break chiral symmetry, and therefore produce confinement. Must be confirmed by direct lattice simulations.

3. Rate of growth exceeds 3 loop perturbation theory.

4. Behavior similar to quenched theory [ALPHA N.P. Proc. Suppl. 106, 859 (2002)] and N_f=2 theory [ALPHA, N.P. B713, 378 (2005)], but slower growth as expected.

Conclusions

1. First lattice evidence that for an SU(3) gauge theory with N_f Dirac fermions in the fundamental representation

   $8 < N_{fc} < 12$

2. N_f=12: Relatively weak IRFP

3. N_f=8: Confinement and chiral symmetry breaking – in disagreement with Iwasaki et al

Employing the Schroedinger functional running coupling defined at the box boundary L

Things to Do

1. Refine the simulations at N_f = 8 and 12 and examine other values such as N_f=10.

2. Study the phase transition as a function of N_f.

3. Consider other gauge groups and representation assignments for the fermions

4. Examine physical quantities such as the static potential (Wilson loop)

5. Examine chiral symmetry breaking directly:
<ψ ψ> at zero temperature

6. Apply to BSM Physics. Is S naturally small as $N_f \to N_{fc}$ due to approximate parity doubling?

$$S(m_{H,ref}) = 4\int_0^\infty \frac{ds}{s}\left\{[\mathrm{Im}\,\Pi_{VV}(s) - \mathrm{Im}\,\Pi_{AA}(s)] - \frac{1}{48\pi}\left[1-\left(1-\frac{m_{H,ref}}{s}\right)^3 \theta\left(s-m_{H,ref}^2\right)\right]\right\}$$

Includes the contribution of the [N_f^2 - 1 - 3] pseudo-Nambu-Goldstone bosons present in the model.