Light-quark baryon spectroscopy with staggered fermions

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Rooted staggered QCD

- Applications of rooted staggered QCD (Davies *et al*. [HPQCD, UKQCD, MILC, Fermilab], 2004)
  - heavy-light decay constants and form factors (Aubin *et al*. [Fermilab, MILC, HPQCD], 2005)
  - strong coupling (Mason *et al*. [HPQCD], 2005)
  - quark masses (Aubin *et al*. [HPQCD, MILC, UKQCD], 2004)

- Unresolved theoretical questions (Kronfeld 2007 and refs therein)
  - 4 lattice species, or *tastes*, per physical flavor
  - Fourth root of fermion determinant
  - Taste $SU(4)_T$ symmetry is broken on the lattice

- Calculations of experimentally well-known quantities are valuable cross-checks of rooted staggered QCD

Staggered baryon spectroscopy

- Use rooted staggered QCD to extract masses of lightest octet and decuplet baryons in isospin limit

- Complete control of systematic errors
  - Extrapolation to physical quark masses
  - Continuum limit
  - Infinite volume limit

- Presence of valence taste quantum numbers complicates continuum limit
- Spin-taste violations split and mix baryon spectrum

- Need to account for taste quantum numbers and spin-taste violations

Effective field theory approach

- Symanzik's effective continuum action describes taste degrees of freedom and spin-taste violations in terms of continuum quarks and gluons (Lee and Sharpe, 1999; Lepage, 1999; Aubin and Bernard, 2003; Sharpe and Van de Water, 2004; Bernard, Golterman, Shamir, 2007)

$$S_{\text{eff}} = S'_{\text{QCD}} + a^2 S_6 + a^4 S_8 + \ldots$$

- Valence sector of rooted staggered QCD contains 12 light quarks

- Chiral perturbation theory corresponding to Symanzik's effective continuum action describes tastes and spin-taste violations in terms of hadrons of rooted staggered QCD (Bernard, 2006; Bernard, Golterman, Shamir, 2007)
  - Extrapolation to physical quark masses
  - Continuum limit
  - Infinite volume limit

Staggered heavy baryon chiral perturbation theory

- Incorporate lattice spacing in power counting: $\Lambda_{\rm QCD} a \sim \frac{m_\pi}{\Lambda_\chi} \sim \frac{\Lambda_{\rm QCD}}{m_B}$
- Map operators of Symanzik action to operators of heavy baryon χPT
- Hadrons of chiral theory transform in irreducible representations of $SU(12)_f$

$$\begin{aligned} \text{meson } \mathbf{8} \quad &\Rightarrow \quad \text{meson } \mathbf{143} \\ \text{baryon } \mathbf{8_M} \quad &\Rightarrow \quad \text{baryon } \mathbf{572_M} \\ \text{baryon } \mathbf{10_S} \quad &\Rightarrow \quad \text{baryon } \mathbf{364_S} \end{aligned}$$

Where are the physical octet and decuplet?

Identifying physical baryons

- If rooted staggered QCD is correct, then in the continuum limit:
  - Dynamics of sea correct (fourth root works)
  - Taste $SU(4)_T$ is restored
    - Taste violations (taste changing interactions) vanish, taste quantum numbers are conserved
    - All tastes in valence sector are physically equivalent
    - Tastes are like extra flavors that play no dynamical role—sterile labels

- Staggered baryons composed of quarks of a single taste correspond to physical states in the continuum limit.

- Testing this picture and its consequences means testing rooted staggered QCD.

Flavor-taste basis

- Disentangle flavor $SU(3)_F$ and taste $SU(4)_T$ quantum numbers:

$$
\begin{aligned}
SU(12)_f &\supset SU(3)_F \times SU(4)_T \\
\mathbf{572_M} &\rightarrow (\mathbf{10_S},\ \mathbf{20_M}) \oplus (\mathbf{8_M},\ \mathbf{20_S}) \oplus (\mathbf{8_M},\ \mathbf{20_M}) \\
&\quad \oplus (\mathbf{8_M},\ \mathbf{\bar{4}_A}) \oplus (\mathbf{1_A},\ \mathbf{20_M}) \\
\mathbf{364_S} &\rightarrow (\mathbf{10_S},\ \mathbf{20_S}) \oplus (\mathbf{8_M},\ \mathbf{20_M}) \oplus (\mathbf{1_A},\ \mathbf{\bar{4}_A})
\end{aligned}
$$

- In the continuum limit, all members of a given taste multiplet are degenerate

- All $\mathbf{20_S}$ baryons correspond to physical states

Continuum symmetry

- Continuum symmetry is larger than taste alone $m_x = m_y = \hat{m},\ m_z = m_s$

$$M = \begin{pmatrix} \hat{m}I_4 & 0 & 0 \\ 0 & \hat{m}I_4 & 0 \\ 0 & 0 & m_s I_4 \end{pmatrix} = \begin{pmatrix} \hat{m}I_8 & 0 \\ 0 & m_s I_4 \end{pmatrix}$$

$$\Rightarrow SU(8)_{x,y} \times SU(4)_z \supset SU(4)_T$$

- Baryons transforming within a given irrep of continuum symmetry group are physically equivalent

Continuum irreps

$$SU(12)_f \supset SU(8)_{x,y} \times SU(4)_z$$

$$\mathbf{364_S} \quad \rightarrow \quad (\mathbf{120_S},\ \mathbf{1}) \oplus (\mathbf{36_S},\ \mathbf{4}) \oplus (\mathbf{8},\ \mathbf{10_S}) \oplus (\mathbf{1},\ \mathbf{20_S})$$

$$\begin{aligned} \mathbf{572_M} \quad \rightarrow \quad & (\mathbf{168_M},\ \mathbf{1}) \oplus (\mathbf{28_A},\ \mathbf{4}) \oplus (\mathbf{36_S},\ \mathbf{4}) \oplus \ldots \\ & (\mathbf{8},\ \mathbf{6_A}) \oplus (\mathbf{8},\ \mathbf{10_S}) \oplus (\mathbf{1},\ \mathbf{20_M}) \end{aligned}$$

- Deduce correspondence between continuum irreps and physical states by locating single-taste baryons in each continuum irrep

Correspondence with physical baryons

$$SU(12)_f \supset SU(8)_{x,y} \times SU(4)_z$$

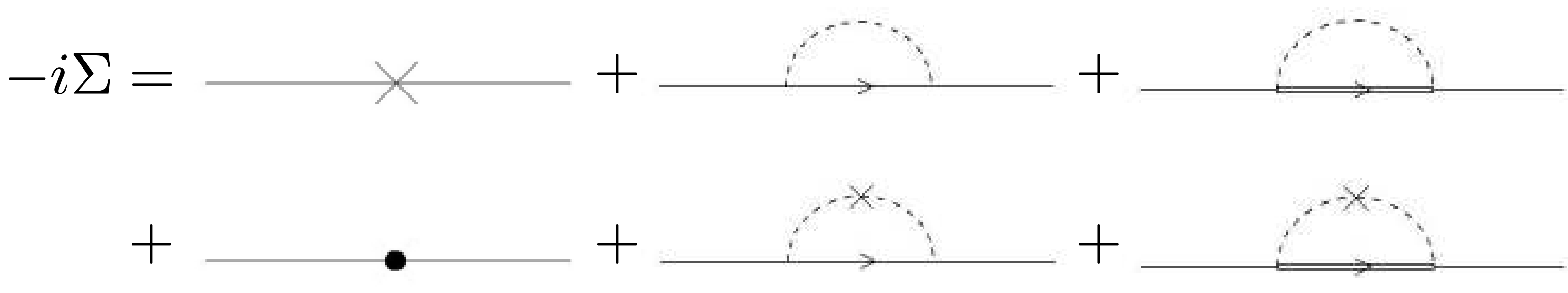
$$\begin{array}{ccl} & & \qquad \Delta \qquad\qquad\quad \Sigma^* \qquad\qquad\quad \Xi^* \qquad\qquad\quad \Omega \\ \mathbf{364_S} & \rightarrow & (\mathbf{120_S},\ \mathbf{1}) \oplus (\mathbf{36_S},\ \mathbf{4}) \oplus (\mathbf{8},\ \mathbf{10_S}) \oplus (\mathbf{1},\ \mathbf{20_S}) \end{array}$$

$$\begin{array}{ccl} & & \qquad N \qquad\qquad\quad \Lambda \qquad\qquad\quad \Sigma \\ \mathbf{572_M} & \rightarrow & (\mathbf{168_M},\ \mathbf{1}) \oplus (\mathbf{28_A},\ \mathbf{4}) \oplus (\mathbf{36_S},\ \mathbf{4}) \oplus \ldots \\ & & (\mathbf{8},\ \mathbf{6_A}) \oplus (\mathbf{8},\ \mathbf{10_S}) \oplus (\mathbf{1},\ \mathbf{20_M}) \\ & & \quad \Lambda_s \qquad\qquad\quad \Xi \qquad\qquad\quad N_s \\ & & (1400) \qquad\qquad\qquad\qquad (1600) \end{array}$$

- All irreps but two correspond to physical states
- Exceptions are degenerate with partially quenched baryons; continuum symmetry forbids their mixing with physical subspace

Masses of $(\mathbf{10_S},\ \mathbf{20_M})$ nucleons in SχPT

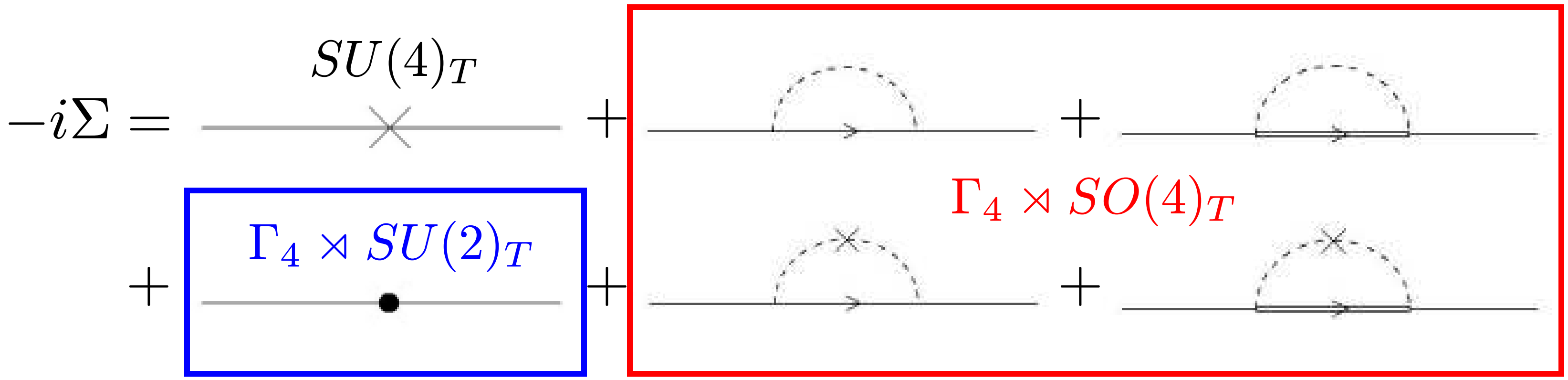
- Self-energy for each member of $\mathbf{10_S}$ is 20 x 20 matrix in baryon taste space ~ $\mathbf{20_M}$ of $SU(4)_T$



$$
\begin{aligned}
M &= const. + m_q + a^2 \\
&+ (m_q + a^2)^{3/2} + a^2(m_q + m_q^{1/2}a + a^2)^{1/2} \\
&+ \Delta(m_q + a^2 + \Delta^2)\ln(m_q + a^2) + \ldots
\end{aligned}
$$

Masses of $(\mathbf{10_S}, \mathbf{20_M})$ nucleons in SχPT

- Taste breaking occurs in two stages:



$$SU(4)_T \supset \Gamma_4 \rtimes SO(4)_T \supset \Gamma_4 \rtimes SU(2)_T$$

$$\mathbf{20_M} \rightarrow \mathbf{12} \oplus \mathbf{4} \oplus \mathbf{4} \rightarrow \mathbf{8} \oplus \mathbf{4} \oplus \mathbf{4} \oplus \mathbf{4}$$

- Taste violations
  - lift continuum degeneracies
  - introduce off-diagonal elements in mass matrix

Degeneracies and Mixings of $(\mathbf{10_S},\ \mathbf{20_M})$ nucleons: Loops

- Loops break taste: $SU(4)_T \supset \Gamma_4 \rtimes SO(4)_T$

$$\mathbf{20_M} \rightarrow \mathbf{12} \oplus \mathbf{4} \oplus \mathbf{4}$$

- Loops are degenerate ~ irreps of remnant taste, $\Gamma_4 \rtimes SO(4)_T$
- States with the same conserved quantum numbers mix

$$\begin{pmatrix} a & 0 & c & 0 & \cdots & & & \\ 0 & a & 0 & -c & & & & \\ c & 0 & b & 0 & & & & \\ 0 & -c & 0 & b & & & & \\ \vdots & & & & a & 0 & c & 0 \\ & & & & 0 & a & 0 & -c \\ & & & & c & 0 & b & 0 \\ & & & & 0 & -c & 0 & b \end{pmatrix}$$

- To third order, these loop contributions have been explicitly calculated

Degeneracies and Mixings of $(\mathbf{10_S},\ \mathbf{20_M})$ nucleons: Analytic $\mathcal{O}(a^2)$ terms

- Taste breaking: $SU(4)_T \supset \Gamma_4 \rtimes SU(2)_T$

$$\mathbf{20_M} \rightarrow \mathbf{8} \oplus \mathbf{4} \oplus \mathbf{4} \oplus \mathbf{4}$$

- Contributions are degenerate ~ irreps of remnant taste, $\Gamma_4 \rtimes SU(2)_T$
- States with the same conserved quantum numbers mix

$$\begin{pmatrix} a & 0 & d & 0 & e & 0 & \cdots \\ 0 & a & 0 & -d & 0 & -e & \\ d & 0 & b & 0 & f & 0 & \\ 0 & -d & 0 & b & 0 & f & \\ e & 0 & f & 0 & c & 0 & \\ 0 & -e & 0 & f & 0 & c & \\ \vdots & & & & & & \ddots \end{pmatrix}$$

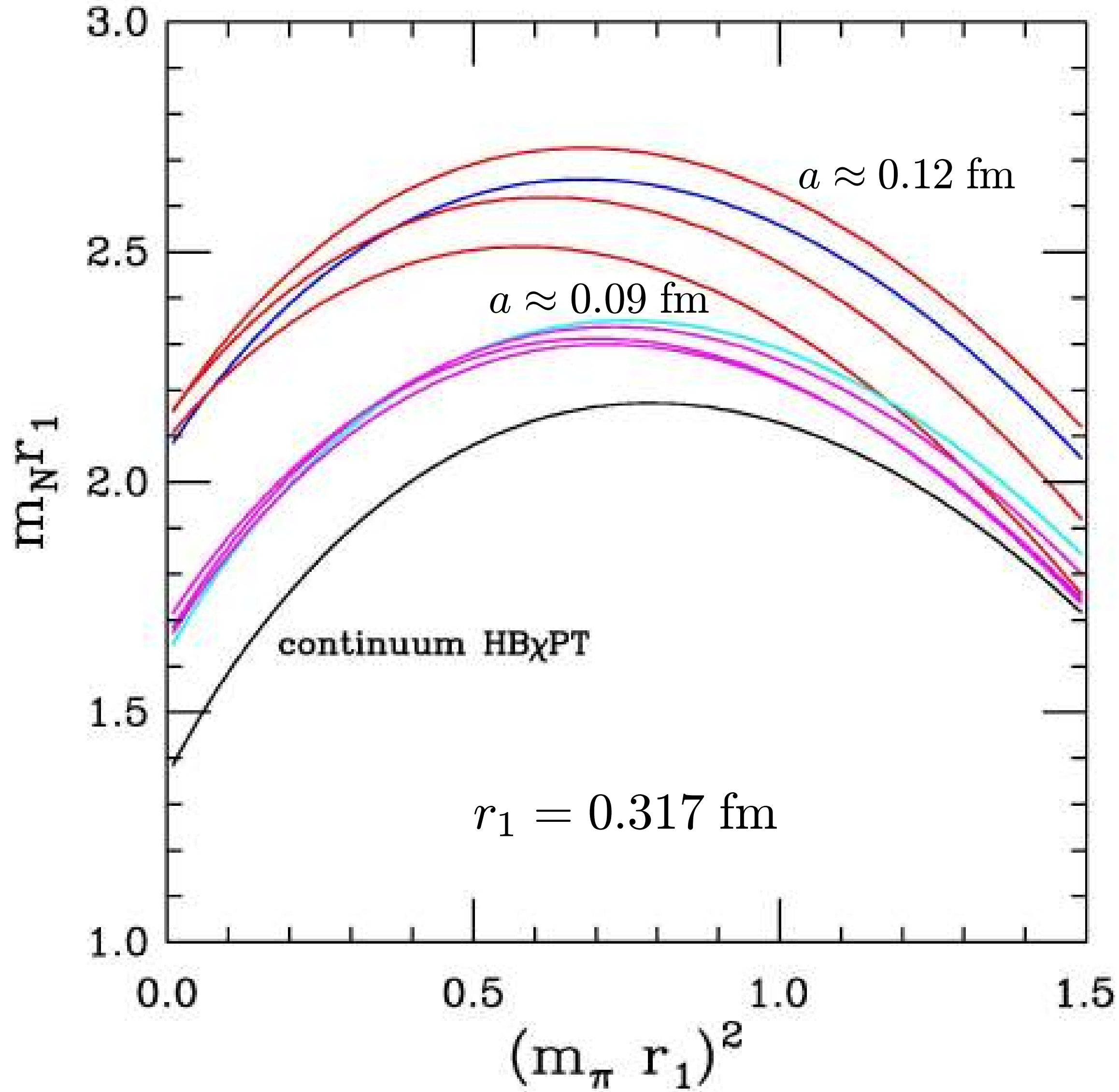
- These tree-level contributions have been calculated

$\mathcal{O}(m_q^{3/2})$ chiral forms for $(\mathbf{10_S}, \mathbf{20_M})$ nucleons

$SU(4)_T \supset \Gamma_4 \rtimes SU(2)_T$

$\mathbf{20_M} \rightarrow \mathbf{8} \oplus 3(\mathbf{4})$

- At larger quark mass
  - forms $\sim -m_\pi^3$
  - cut-off reg?
- Taste violations split $\mathbf{20_M}$
  - $\sim$ 10-40 MeV
  - excited states close to ground state
- Operators for each state?



Operators for $(xxx,\ \mathbf{20_M})$ nucleons

- operators ~ irreps of geometrical time slice group (GTS): **8**, **8′**, **16**
  (Golterman and Smit, 1984)

$$SU(2)_S \times SU(4)_T \rightarrow \text{GTS}$$

$$(\tfrac{\mathbf{1}}{\mathbf{2}},\ \mathbf{20_M}) \rightarrow \mathbf{16} \oplus 3(\mathbf{8})$$

⇒ operators ~ **8** of GTS create three states in the $\mathbf{20_M}$

⇒ operators ~ **16** of GTS create one state in the $\mathbf{20_M}$

Connecting operators with chiral forms

- In the context of spin, consider taste breaking in SχPT:

$$SU(2) \times SU(4)_T \supset SU(2) \times [\Gamma_4 \rtimes SU(2)_T]$$

$$(\tfrac{\mathbf{1}}{\mathbf{2}}, \mathbf{20_M}) \rightarrow (\tfrac{\mathbf{1}}{\mathbf{2}}, \mathbf{8}) \oplus 3(\tfrac{\mathbf{1}}{\mathbf{2}}, \mathbf{4})$$

- Decompose spin-taste irreps of the chiral forms into irreps of GTS:

$$SU(2) \times [\Gamma_4 \rtimes SU(2)_T] \supset \mathrm{GTS}$$

$$\begin{aligned} (\tfrac{\mathbf{1}}{\mathbf{2}}, \mathbf{8}) &\rightarrow \mathbf{16} \\ (\tfrac{\mathbf{1}}{\mathbf{2}}, \mathbf{4}) &\rightarrow \mathbf{8} \end{aligned}$$

$\Rightarrow$ operators ~ **8** of GTS create the three **4** s of $\Gamma_4 \rtimes SU(2)_T$

$\Rightarrow$ operators ~ **16** of GTS create the **8** of $\Gamma_4 \rtimes SU(2)_T$

Degeneracies and Mixings

- Spin-taste violations break continuum irreps into lattice irreps
- Lattice symmetry governs degeneracies, mixings at nonzero lattice spacing

$$SU(2)_S \times SU(8)_{x,y} \times SU(4)_z \supset SU(2)_I \times \mathrm{GTS}$$

$$\begin{array}{ccl} (\frac{\mathbf{3}}{\mathbf{2}},\ \mathbf{36_S},\ \mathbf{4}) & \to & 3(\mathbf{1},\ \mathbf{8})_{-1} \oplus 3(\mathbf{1},\ \mathbf{8'})_{-1} \oplus 7(\mathbf{1},\ \mathbf{16})_{-1} \oplus \ldots \\ \Sigma^* & & (\mathbf{0},\ \mathbf{8})_{-1} \oplus (\mathbf{0},\ \mathbf{8'})_{-1} \oplus 5(\mathbf{0},\ \mathbf{16})_{-1} \end{array}$$

$\Rightarrow$ 20 lattice energy levels degenerate with Σ^* in continuum limit

Lattice irreps per continuum irrep

- Decompose remaining continuum irreps into lattice irreps
- Many lattice irreps correspond to each continuum irrep

$$SU(2)_S \times SU(8)_{x,y} \times SU(4)_z \supset SU(2)_I \times \mathrm{GTS}$$

$$\mathbf{364_S} \quad \to \quad (\mathbf{120_S},\ \mathbf{1}) \oplus (\mathbf{36_S},\ \mathbf{4}) \oplus (\mathbf{8},\ \mathbf{10_S}) \oplus (\mathbf{1},\ \mathbf{20_S})$$

Δ 13 $\quad \Sigma^*$ 20 $\quad \Xi^*$ 13 $\quad \Omega^-$ 7

N 12 $\quad \Lambda$ 12 $\quad \Sigma$ 12

$$\mathbf{572_M} \quad \to \quad (\mathbf{168_M},\ \mathbf{1}) \oplus (\mathbf{28_A},\ \mathbf{4}) \oplus (\mathbf{36_S},\ \mathbf{4}) \oplus \ldots$$

$$(\mathbf{8},\ \mathbf{6_A}) \oplus (\mathbf{8},\ \mathbf{10_S}) \oplus (\mathbf{1},\ \mathbf{20_M})$$

Λ_s 5 $\quad \Xi$ 7 $\quad N_s$ 4

Mixing at nonzero lattice spacing

- Spin-taste violations mix corresponding members of same type of lattice irrep—baryons with same (conserved) lattice quantum numbers

$$\overset{\Sigma^*}{(\tfrac{\mathbf{3}}{\mathbf{2}},\ \mathbf{36_S},\ \mathbf{4})} \quad\rightarrow\quad 3(\mathbf{1},\ \mathbf{8})_{-1}\oplus 3(\mathbf{1},\ \mathbf{8'})_{-1}\oplus 7(\mathbf{1},\ \mathbf{16})_{-1}\oplus\ldots$$
$$(\mathbf{0},\ \mathbf{8})_{-1}\oplus(\mathbf{0},\ \mathbf{8'})_{-1}\oplus 5(\mathbf{0},\ \mathbf{16})_{-1}$$

$$\overset{\Lambda}{(\tfrac{\mathbf{1}}{\mathbf{2}},\ \mathbf{28_A},\ \mathbf{4})} \quad\rightarrow\quad 4(\mathbf{1},\ \mathbf{8})_{-1}\oplus(\mathbf{1},\ \mathbf{16})_{-1}\oplus 4(\mathbf{0},\ \mathbf{8})_{-1}\oplus 3(\mathbf{0},\ \mathbf{16})_{-1}$$
$$\underset{\Sigma}{(\tfrac{\mathbf{1}}{\mathbf{2}},\ \mathbf{36_S},\ \mathbf{4})} \quad\rightarrow\quad 4(\mathbf{1},\ \mathbf{8})_{-1}\oplus 3(\mathbf{1},\ \mathbf{16})_{-1}\oplus 4(\mathbf{0},\ \mathbf{8})_{-1}\oplus(\mathbf{0},\ \mathbf{16})_{-1}$$

- There is a 9-dimensional mixing matrix for each member of the $(\mathbf{0}, \mathbf{16})_{-1}$

- Operators with quantum numbers of a given member of $(\mathbf{0}, \mathbf{16})_{-1}$
  create all 9 corresponding states: 3 states ~ Λ, 1 Σ, 5 states ~ Σ^*

Implications for practical calculations

- Rooted staggered QCD contains many baryons corresponding to each physical baryon (taste ~ flavor in continuum limit)
- At nonzero lattice spacing, spin-taste violations lift degeneracies within continuum multiplets and introduce mixing (~ 10-40 MeV)
- Irreducible operators create states in corresponding lattice irreps

- In general, accounting for splitting and mixing of the spectrum at nonzero *a* promises to be difficult (labor and numerics)
  – Compute and fit correlation matrices of mixing operators
  – Calculate and diagonalize corresponding matrices in chiral theory

- Special cases in which splitting and mixing is less problematic
  – Sufficiently improved lattice action (e.g., HISQ) squashes spin-taste violations beneath statistical uncertainties
  – Sufficiently large lattice spacing might be numerically easier

Examine spectra of operators

- Are there any operators whose spectra are not split and mixed by spin-taste violations? ~ "optimal" operators

Operator irreps (for $a \neq 0$)	States created/mixed
$(\frac{\mathbf{3}}{\mathbf{2}},\ \mathbf{8})_0$	$3\ N$ and $2\ \Delta$
$(\frac{\mathbf{3}}{\mathbf{2}},\ \mathbf{16})_0$	$1\ N$ and $3\ \Delta$
$(\frac{\mathbf{1}}{\mathbf{2}},\ \mathbf{8})_0$	$5\ N$ and $1\ \Delta$
$(\frac{\mathbf{1}}{\mathbf{2}},\ \mathbf{8}')_0$	$1\ \Delta$
$(\mathbf{1},\ \mathbf{8}')_{-1}$	$3\ \Sigma^*$
$(\mathbf{1},\ \mathbf{16})_{-1}$	$1\ \Lambda$, $3\ \Sigma$, and $7\ \Sigma^*$
$(\mathbf{0},\ \mathbf{8}')_{-1}$	$1\ \Sigma^*$
$(\mathbf{0},\ \mathbf{16})_{-1}$	$3\ \Lambda$, $1\ \Sigma$, and $5\ \Sigma^*$
$(\frac{\mathbf{1}}{\mathbf{2}},\ \mathbf{8})_{-2}$	$4\ \Xi$, $4\ \Lambda_s$, and $3\ \Xi^*$
$(\frac{\mathbf{1}}{\mathbf{2}},\ \mathbf{8}')_{-2}$	$3\ \Xi^*$
$(\mathbf{0},\ \mathbf{8})_{-3}$	$3\ N_s$ and $2\ \Omega$
$(\mathbf{0},\ \mathbf{8}')_{-3}$	$2\ \Omega$

Examine spectra of operators

- Operators ~ **(½, 8′)**$_0$ create a lattice state ~ Δ;
- Operators ~ **(0, 8′)**$_{-1}$ create a lattice state ~ Σ^*

$$m_x = m_y = \hat{m},\ m_z = m_s$$

Operator irreps (for $a \neq 0$)	States created/mixed
$(\frac{\mathbf{3}}{\mathbf{2}},\ \mathbf{8})_0$	3 N and 2 Δ
$(\frac{\mathbf{3}}{\mathbf{2}},\ \mathbf{16})_0$	1 N and 3 Δ
$(\frac{\mathbf{1}}{\mathbf{2}},\ \mathbf{8})_0$	5 N and 1 Δ
$(\frac{\mathbf{1}}{\mathbf{2}},\ \mathbf{8'})_0$	1 Δ
$(\mathbf{1},\ \mathbf{8'})_{-1}$	3 Σ^*
$(\mathbf{1},\ \mathbf{16})_{-1}$	1 Λ, 3 Σ, and 7 Σ^*
$(\mathbf{0},\ \mathbf{8'})_{-1}$	1 Σ^*
$(\mathbf{0},\ \mathbf{16})_{-1}$	3 Λ, 1 Σ, and 5 Σ^*
$(\frac{\mathbf{1}}{\mathbf{2}},\ \mathbf{8})_{-2}$	4 Ξ, 4 Λ_s, and 3 Ξ^*
$(\frac{\mathbf{1}}{\mathbf{2}},\ \mathbf{8'})_{-2}$	3 Ξ^*
$(\mathbf{0},\ \mathbf{8})_{-3}$	3 N_s and 2 Ω
$(\mathbf{0},\ \mathbf{8'})_{-3}$	2 Ω

Partial quenching

- We are free to vary valence quark masses and sea quark masses independently

$$\langle \bar{B} B \rangle = \frac{1}{Z} \int \mathcal{D}\mathcal{U} \ \sum (\not{D}_{\rm stag} + m_x)^{-1} (\not{D}_{\rm stag} + m_y)^{-1} \times$$
$$\times (\not{D}_{\rm stag} + m_z)^{-1} \ [\det(\not{D}_{\rm stag} + m_{u,d,s})]^{1/4} \ e^{-S_g[\mathcal{U}]}$$

- We have considered

$$m_x = m_y = \hat{m} \quad \text{and} \quad m_z = m_s$$

- Now consider

$$m_x = m_y = m_s \quad \text{and} \quad m_z = \hat{m}$$

Reconsider the spectrum

- Valence sector symmetries unchanged everywhere in decompositions
- Pattern of degeneracies and mixings unchanged in continuum and on lattice

$$SU(2)_S \times SU(8)_{x,y} \times SU(4)_z \supset SU(2)_I \times \mathrm{GTS}$$

- Continuum masses of lattice irreps change

$$\begin{array}{ccc} \Delta & \rightleftarrows & \Omega \\ \Sigma^* & \rightleftarrows & \Xi^* \\ \Sigma & \rightleftarrows & \Xi \\ N & \rightleftarrows & N_s \\ \Lambda & \rightleftarrows & \Lambda_s \end{array}$$

Recall spectra of operators

- Operators ~ **(½, 8′)$_0$** create a lattice state ~ Δ;
- Operators ~ **(0, 8′)$_{-1}$** create a lattice state ~ Σ^*

$$m_x = m_y = \hat{m},\ m_z = m_s$$

Operator irreps (for $a \neq 0$)	States created/mixed
$(\frac{\mathbf{3}}{\mathbf{2}},\ \mathbf{8})_0$	3 N and 2 Δ
$(\frac{\mathbf{3}}{\mathbf{2}},\ \mathbf{16})_0$	1 N and 3 Δ
$(\frac{\mathbf{1}}{\mathbf{2}},\ \mathbf{8})_0$	5 N and 1 Δ
$(\frac{\mathbf{1}}{\mathbf{2}},\ \mathbf{8'})_0$	1 Δ
$(\mathbf{1},\ \mathbf{8'})_{-1}$	3 Σ^*
$(\mathbf{1},\ \mathbf{16})_{-1}$	1 Λ, 3 Σ, and 7 Σ^*
$(\mathbf{0},\ \mathbf{8'})_{-1}$	1 Σ^*
$(\mathbf{0},\ \mathbf{16})_{-1}$	3 Λ, 1 Σ, and 5 Σ^*
$(\frac{\mathbf{1}}{\mathbf{2}},\ \mathbf{8})_{-2}$	4 Ξ, 4 Λ_s, and 3 Ξ^*
$(\frac{\mathbf{1}}{\mathbf{2}},\ \mathbf{8'})_{-2}$	3 Ξ^*
$(\mathbf{0},\ \mathbf{8})_{-3}$	3 N_s and 2 Ω
$(\mathbf{0},\ \mathbf{8'})_{-3}$	2 Ω

Change masses of valence quarks

- Operators ~ **(½, 8′)$_0$** create a lattice state ~ Ω;
- Operators ~ **(0, 8′)$_{-1}$** create a lattice state ~ Ξ^*

$$m_x = m_y = m_s,\ m_z = \hat{m}$$

Operator irreps (for $a \neq 0$)	States created/mixed
$(\frac{\mathbf{3}}{\mathbf{2}},\ \mathbf{8})_0$	3 N_s and 2 Ω
$(\frac{\mathbf{3}}{\mathbf{2}},\ \mathbf{16})_0$	1 N_s and 3 Ω
$(\frac{\mathbf{1}}{\mathbf{2}},\ \mathbf{8})_0$	5 N_s and 1 Ω
$(\frac{\mathbf{1}}{\mathbf{2}},\ \mathbf{8'})_0$	1 Ω
$(\mathbf{1},\ \mathbf{8'})_{-1}$	3 Ξ^*
$(\mathbf{1},\ \mathbf{16})_{-1}$	1 Λ_s, 3 Ξ, and 7 Ξ^*
$(\mathbf{0},\ \mathbf{8'})_{-1}$	1 Ξ^*
$(\mathbf{0},\ \mathbf{16})_{-1}$	3 Λ_s, 1 Ξ, and 5 Ξ^*
$(\frac{\mathbf{1}}{\mathbf{2}},\ \mathbf{8})_{-2}$	4 Σ, 4 Λ, and 3 Σ^*
$(\frac{\mathbf{1}}{\mathbf{2}},\ \mathbf{8'})_{-2}$	3 Σ^*
$(\mathbf{0},\ \mathbf{8})_{-3}$	3 N and 2 Δ
$(\mathbf{0},\ \mathbf{8'})_{-3}$	2 Δ

Avoiding splittings and mixings

- Choosing appropriate quark masses and operators allows us to extract masses of decuplet and nucleon w/o accounting for splittings and mixings
- Only possible with operators ~ irreps of lattice symmetry group
- Operators in all lattice irreps have been constructed

Valence quark masses	Operator irreps	States created/mixed
$m_x = m_y = \hat{m},\ m_z = m_s$	$(\frac{\mathbf{1}}{\mathbf{2}},\ \mathbf{8'})_0$ $(\mathbf{0},\ \mathbf{8'})_{-1}$	$1\ \Delta$ $1\ \Sigma^*$
$m_x = m_y = m_s,\ m_z = \hat{m}$	$(\frac{\mathbf{1}}{\mathbf{2}},\ \mathbf{8'})_0$ $(\mathbf{0},\ \mathbf{8'})_{-1}$	$1\ \Omega$ $1\ \Xi^*$
$m_x = m_y = m_z = \hat{m}$	$(\mathbf{1_A},\ \mathbf{16})$	$1\ N$ and $1\ \Delta$
$m_x = m_y = m_z = m_s$	$(\mathbf{1_A},\ \mathbf{16})$	$1\ N_s$ and $1\ \Omega$

Constructing operators with $SU(3)_F$ quantum numbers

- Consider objects that are
  - composed of three staggered fields ~ fundamental irrep of GTS (**8**)
  - color singlets
  - fermions—completely anti-symmetric under simultaneous perms of all indices:
    - color - *abc*
    - GTS - *ABC*
    - flavor - *ijk*
- Leads us to consider the object (cf. Golterman and Smit, 1984):

$$_{ijk}\tilde{B}_{ABC} \equiv \sum_{\mathbf{x},\, x_k \text{ even}} \tfrac{1}{6}\epsilon_{abc} D_A \chi_i^a(\mathbf{x}) D_B \chi_j^b(\mathbf{x}) D_C \chi_k^c(\mathbf{x})$$

$$_{ijk}\tilde{B}_{ABC} =_{jki} \tilde{B}_{BCA} =_{kij} \tilde{B}_{CAB} =_{jik} \tilde{B}_{BAC} =_{ikj} \tilde{B}_{ACB} =_{kji} \tilde{B}_{CBA}$$

Exploiting $SU(N)$

- GTS is a proper subgroup of $SU(8)$
- Embed GTS in $SU(8)$ such that the fundamental of $SU(8)$ transforms as the fundamental of GTS:

$$SU(8) \supset \mathrm{GTS}$$
$$\mathbf{8} \to \mathbf{8}$$

- Then we recognize the symmetric irrep of $SU(24)$:

$$\Rightarrow \tilde{B} \sim \mathbf{2600_S}$$

- Decompose this irrep to obtain operators with definite $SU(3)$:

$$SU(24) \supset SU(3)_F \times SU(8)$$
$$\mathbf{2600_S} \quad \to \quad (\mathbf{10_S},\, \mathbf{120_S}) \oplus (\mathbf{8_M},\, \mathbf{168_M}) \oplus (\mathbf{1_A},\, \mathbf{56_A})$$

Operators $\sim$ $SU(3)_F \times \mathrm{GTS}$

- Decompose the *SU*(8) irreps under GTS:

$$
\begin{aligned}
(\mathbf{10_S},\, \mathbf{120_S}) &\rightarrow 5(\mathbf{10_S},\, \mathbf{8}) \oplus 2(\mathbf{10_S},\, \mathbf{8'}) \oplus 4(\mathbf{10_S},\, \mathbf{16}) \\
(\mathbf{1_A},\, \mathbf{56_A}) &\rightarrow 3(\mathbf{1_A},\, \mathbf{8}) \oplus 2(\mathbf{1_A},\, \mathbf{16}) \\
(\mathbf{8_M},\, \mathbf{168_M}) &\rightarrow 6(\mathbf{8_M},\, \mathbf{8}) \oplus (\mathbf{8_M},\, \mathbf{8'}) \oplus 7(\mathbf{8_M},\, \mathbf{16})
\end{aligned}
$$

- These operators could be used to extract the masses of all the lightest spin-1/2 and spin-3/2 baryons

$$
\begin{aligned}
SU(24) &\supset SU(2)_S \times SU(12)_f \\
\mathbf{2600_S} &\rightarrow (\tfrac{\mathbf{3}}{\mathbf{2}},\, \mathbf{364_S}) \oplus (\tfrac{\mathbf{1}}{\mathbf{2}},\, \mathbf{572_M})
\end{aligned}
$$

Summary and Outlook

- Relationship between staggered baryons and physical spectrum suggests additional nontrivial tests of rooted staggered QCD
- Irreducible lattice operators for staggered baryons constructed; certain of these promising for extraction of nucleon and decuplet masses (Bailey, 2007)
- Baryon sector of staggered chiral perturbation theory developed to control extrapolation to physical quark masses and continuum limit (Bailey, 2008)
- Analysis of spectrum and operator spectra can be extended to excited light-quark baryons, heavy-light-light baryons, heavy-light mesons, . . .