Charm physics on the lattice with highly improved staggered quarks

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HPQCD collaboration

Work in collaboration with:

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Thanks: The MILC collaboration for making their configurations publicly available.

Outline

- Motivation.
- Staggered quarks.
  - HISQ (Highly improved staggered quarks.)
- Heavy quarks.
- Charmed systems: masses and decay constants.
- Outlook.

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Phys.Rev.Lett.100:062002,2008.

Motivation

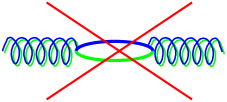
- Low-energy QCD is a strongly-coupled QFT. We need non-perturbative tools to deal with it.
  - Other strongly-coupled sectors BSM?
- Lattice QCD provides a non-perturbative definition of QCD. It also provides a quantitative calculational tool. And lately it is also becoming a precise tool.

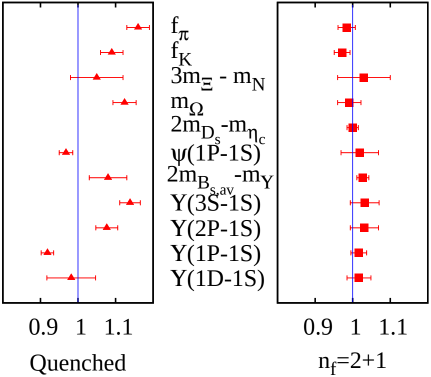
Goals

- To make precise calculations in QCD.
  - To test lattice field theory as a tool for studying strongly coupled field theories. (CLEO-c).
    - f_D, f_{D_s}
  - To calculate theoretical quantities needed in the analysis of experimental data, for example, in the determination of elements of the CKM matrix.
  - To further test QCD as the theory of strong interactions.
- To deepen our understanding of the physics of QCD, for example, confinement.

LQCD: Quenched vs Unquenched

- Fermions are numerically very hard to include.
- Ignore fermion pair production ⇒ quenched QCD.





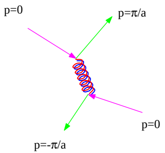
Plus the successful prediction of m_{B_c} (I. Allison et al).

(Some) systematic errors

- Finite volume: $m_\pi^{-1} \ll L$. In practice, $L \approx 2.5, 3fm$
- Finite lattice spacing: we need simulations at different values of a, to extrapolate to the continuum limit $a \to 0$.
  - To simulate at small values of a, while keeping the physical L constant is very expensive.
  - Typically, error $\propto a, a^2$
  - Improved actions (and operators) decrease the error, making the extrapolation from a given set of lattice spacings more precise.
- Chiral extrapolation: In practice, we are not able to simulate at physical values of the light quark masses $m_{u,d}$.
- Lattice spacing determination: Error in the determination of the lattice spacing in physical units (r_1).

Improved Staggered Quarks

- The staggered action describes 4 tastes (in 4D). The spectrum on the lattice has a multiplicity of states corresponding to the same continuum state. There are unphysical taste-changing interactions that lift the degeneracy between such states.
- These effects are lattice artifacts, of order a^2, and vanish in the continuum limit $a \to 0$. They involve at leading order the exchange of a gluon of momentum $q \approx \pi/a$.
- Such interactions are perturbative for typical values of the lattice spacing, and can be corrected systematically a la Symanzik.

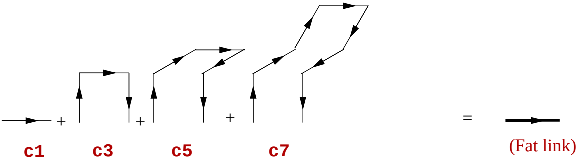


Smear the gauge field to remove the coupling between quarks and gluons with momentum π/a.

- In an unquenched simulation, $\sqrt[4]{\det}$. $\to$ "Rooting trick".

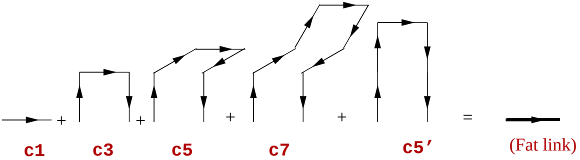
Improved Staggered Actions

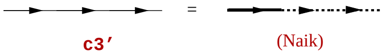
- FAT7(TAD)



Improved Staggered Actions

- ASQ(TAD)





(S. Naik, the MILC collaboration, P. Lepage.)

- Discretization errors $\approx \mathcal{O}(\alpha_s \mathrm{a}^2, \mathrm{a}^4)$.

Improved Staggered Actions

- HISQ

Two levels of smearing: first a FAT7 smearing on the original links, followed by a projection onto $SU(3)$, then a modified ASQ on these links.

$$\text{FAT7}\|_{SU(3)} \otimes \text{ASQ'}$$

(E.F., Q. Mason, C. Davies, K. Hornbostel, P. Lepage, H. Trottier.)

- Discretization errors $\approx \mathcal{O}(\alpha_s \mathrm{a}^2, \mathrm{a}^4)$.
- Substantially reduced taste-changing with respect to ASQTAD.

Heavy Quarks

- The discretization errors grow with the quark mass as powers of am.
- For a direct simulation, we need:

  $am_h \ll 1$ (heavy quarks)
  $La \gg m_\pi^{-1}$ (light quarks)

- Two scales. Difficult to do directly.
- Instead take advantage of the fact that m_h is large: $\Rightarrow$ effective field theory (NRQCD, HQET). Very successful for b quarks.

Charm Quarks

- The charm quark is in between the light and heavy mass regime.
- Quite light for an easy application of NRQCD.
- Quite large for the usual relativistic quark actions, $am_c \lesssim 1$.
- However, if we use a very accurate action (HISQ) and fine enough lattices (MILC), it is possible to get accurate results.
  - Errors for HISQ: $\mathcal{O}((am)^4, \alpha_s (am)^2)$.
  - Non-relativistic system: can be tuned for further suppression by factors of (v/c).
  - Can reduce the errors to the few percent level.
  - Simple: use the same action in the heavy and the light sector.
- We will use this action both for heavy-heavy and heavy-light systems $\Rightarrow$ consistency check.

Fixing the parameters

The free parameters in the lattice formulation are fixed by setting a set of calculated quantities to their measured physical values.

- Scale: lattice spacing a: Fixed through the upsilon ($b\bar{b}$) spectrum, $m_{\Upsilon(2S)} - m_{\Upsilon(1S)}$.
- Quark masses: $m_{u,d}, m_s, m_c$. Fixed by m_π, m_K, m_{η_c}.
- In the HISQ charm quark formulation: improvement parameter ϵ. Fixed by requiring relativistic dispersion relation, $c^2 = 1$.

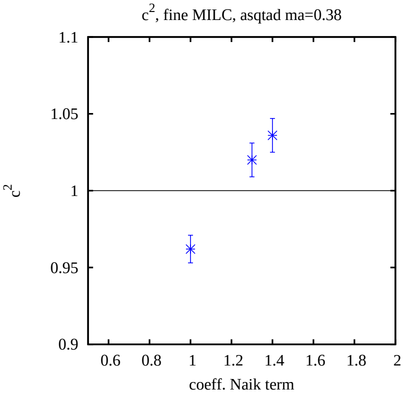
Configurations

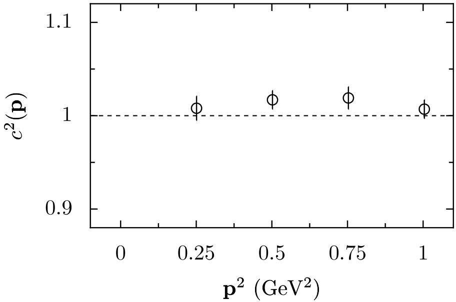
MILC ensembles: $2+1$ ASQTAD sea quarks: $(m_l,\ m_l,\ m_s)$

- Very coarse: a $\approx$ 0.16 fm, $16^3 x$ 48
  - $m_l = m_s/2.5,\ m_s/5$
  - Valence HISQ: $am_c = .85$
- Coarse: a $\approx$ 0.12 fm
  - $m_l = m_s/2,\ m_s/4$ $\quad 20^3 x$ 64
  - $m_l = m_s/8,$ $\quad 24^3 x$ 64
  - Valence HISQ: $am_c = .66$
- Fine: a $\approx$ 0.09 fm, $\quad 28^3 x$ 96.
  - $m_l = m_s/2.5,\ m_s/5$
  - Valence HISQ: $am_c = .43$.

c^2

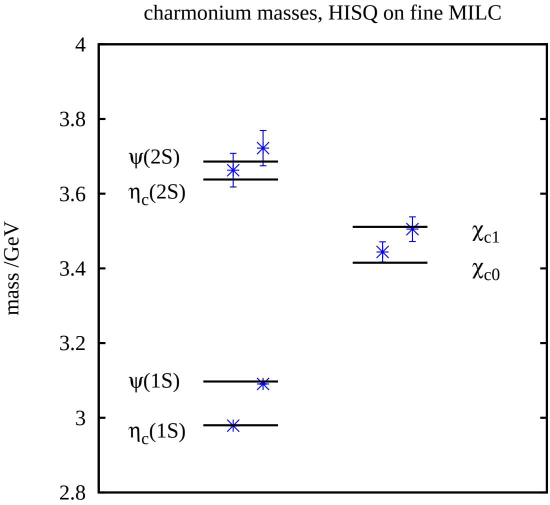
- We adjust the coefficient of the Naik term to have $c^2 = 1$. This further reduces the discretization errors by factors of $\frac{v}{c}$.





Masses

- We use the mass of the η_c to fix the mass of the charm quark.



Decay constants

- Meson decay constants:

$$\Gamma(P \to l\nu_l(\gamma)) = \frac{G_F^2|V_{ab}|^2}{8\pi} f_P{}^2 m_l^2 m_P \left(1 - \frac{m_l^2}{m_P^2}\right)^2$$

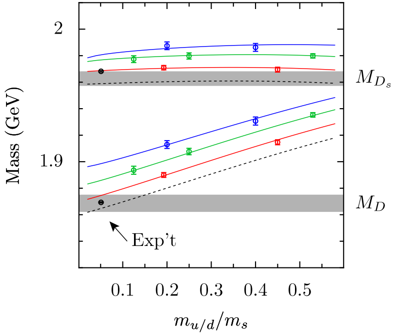
$$\langle 0|A^\mu|P(p)\rangle = f_P p_\mu$$

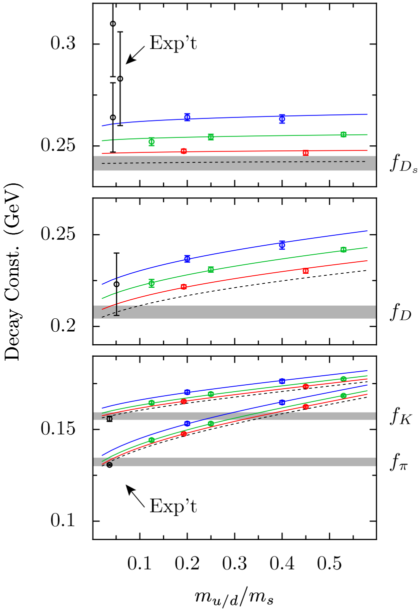
PCAC:

$$f_P m_P{}^2 = (m_a + m_b) < 0|\bar{a}\gamma_5 b|P >$$

- We do a simultaneous bayesian fit of the masses and decay constants to the chiral and continuum limits.
- Essentially the same calculation for f_π, f_K, f_D, f_{D_s}.

Masses and decay constants





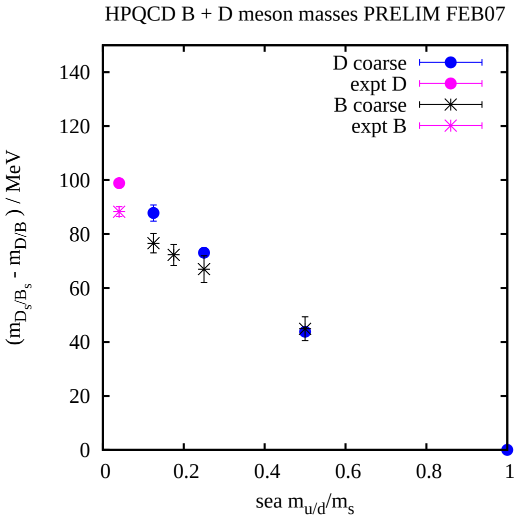
$m_{D_s} = 1.963(5)$ (exp. 1.968) GeV.
$m_D = 1.869(6)$ (exp. 1.869) GeV.

$$\frac{(2m_{D_s} - m_{\eta_c})}{(2m_D - m_{\eta_c})} = 1.249\ (14)$$

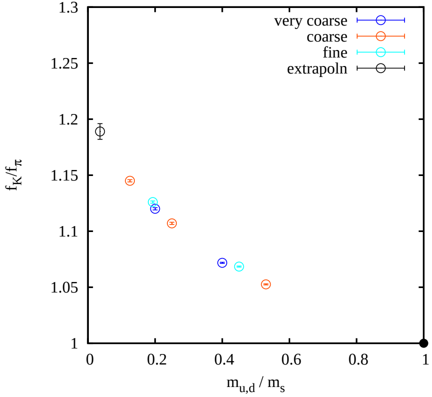
(exp. 1.260(2)) GeV

Mass differences

- We plot $m_{D_s}(m_l) - m_D(m_l)$ and $m_{B_s}(m_l) - m_B(m_l)$ as a function of the sea light quark mass, m_l.



Decay constants



$$f_\pi = 132(2) \text{ (Exp } 130.5(4)) \text{ MeV}$$
$$f_K = 157(2) \text{ (Exp } 156.0(8)) \text{MeV}$$

$$\frac{f_K}{f_\pi} = 1.189(7) \text{ (Exp } 1.196(6))$$

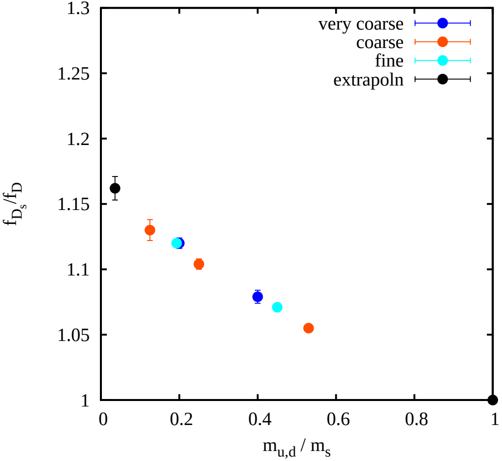
Using experimental leptonic branching fractions (KLOE)

$$V_{us} = 0.2262(13)(4)$$

This gives the unitarity relation

$$1 - V_{ud}^2 - V_{us}^2 - V_{ub}^2 = 0.0006(8)$$

Decay constants



$f_{D_s} = 241(3)$ MeV
$f_D = 208(4)$ (Exp 223(17)) MeV

Using experimental values from CLEO-c for μ decay:

$V_{cs} = 1.07(1)(7)$ (PDG : 0.96(9))

	r_1	stat	a^2	m_l	m_s evol	vol	isospin, QED	tot
% error f_{D_s}	1.0	0.6	0.5	0.3	0.3	0.1	0.0	1.3

Decay constants

$\frac{f_{D_s}}{f_D} = 1.162(9)$

Using experimental values from CLEO-c for μ decay:

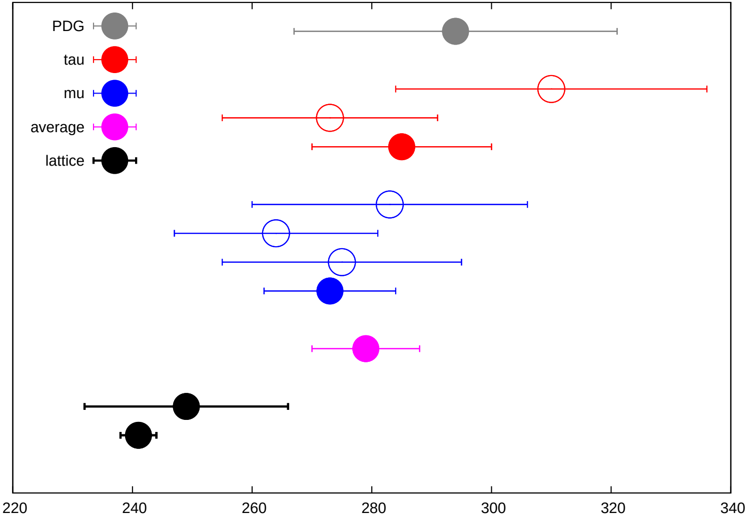
$$\frac{V_{cs}}{V_{cd}} = 4.42(4)(41)$$

Double ratios:

$$\frac{f_{D_s}/f_D}{f_K/f_\pi} = 0.977(10)$$

$$\frac{f_{B_s}/f_B}{f_{D_s}/f_D} = 1.03(3)$$

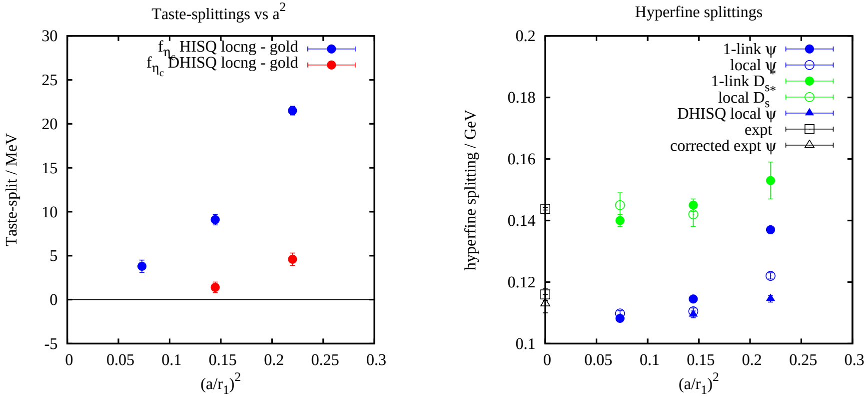
f_{D_s}



(Bogdan A. Dobrescu, Andreas S. Kronfeld, arXiv:0803.0512)

HISQ2 and hyperfine splitting

PRELIMINARY



Conclusions

- The use of a highly improved quark action and fine enough lattices provides a very good way of studying systems with charm quarks from first principles.
- We can calculate accurately a number of interesting quantities. At present all but f_{D_s} agree with experiment.

Outlook

- Direct determination of m_c from the lattice. Needs perturbative calculation (underway.) Accurate m_c/m_s.
- New method for the calculation of m_c (in collaboration with K. Chetyrkin et al, Karlsruhe.) combining continuum perturbation results for the moments of the η_c correlator with lattice data. Preliminary, work in progress.
- Leptonic decay width $\psi \to e^+e^-$. Known accurately from experiment ($\sim 2\%$).
- Semileptonic form factors: $D \to \pi l \nu, D \to K l \nu$