Electroweak Model based on the Nonlinearly realized Gauge Group SU(2)XU(1).

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References

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- JHEP **0703** (2007) 065 [arXiv:hep-th/0701212].
- And more to come ...

Outline

Outline

Motivations

- One of the major open problems in QFT is the quest for the origin of the mass of elementary particles.
- Standard solution: the Higgs mechanism (linear realization of the gauge group on the scalar sector).
  1. Renormalizability, physical unitarity ($+$).
  2. Very good agreement with experimental data ($+$).
  3. No direct experimental evidence of the Higgs particle ($-$).
  4. Hierarchy problem ($-$).
- Alternative models that overcome the drawbacks of the SM: SUSY, TC, composite Higgs, extra dimensions ...

- We explore another possibility, i.e. that of managing the divergences of non p.c. renormalizable models.
- In a field theoretical model where the gauge group is realized nonlinearly on the scalar sector there are no fundamental scalar particles in the perturbative spectrum.
- Due to the presence of non-polynomial vertices in the tree level action the model is not p.c. renormalizable.
  1. How to subtract the divergences ?
  2. How many physical parameters are there ?
  3. Is the model unique ?

Outline

Notations

- We will consider a $SU(2) \times U(1)$ gauge group and denote by $A_\mu = A_{a\mu}\frac{\tau_a}{2}$ and B_μ the gauge connections. τ_a are the Pauli matrices.
- The nonlinear sigma model field Ω is an element of the $SU(2)$ group, which is parameterized in terms of the coordinate fields ϕ_a as follows:

$$\Omega = \frac{1}{v_D}(\phi_0 + i\tau_a\phi_a) \quad \text{with} \quad \phi_0^2 + \phi_a^2 = v_D^2 \,,$$

where $v_D = v^{(D/2-1)}$ is a mass scale.
- We introduce a $SU(2)$ flat connection (its field strength vanishes)

$$F_\mu = i\Omega\partial_\mu\Omega^\dagger = F_{a\mu}\frac{\tau_a}{2} \,.$$

$SU(2)_L$ transformations

- Under a local $SU(2)$ left transformation $U_L = \exp\left(ig_2\,\alpha_a^L\,\frac{\tau_a}{2}\right)$ one gets

$$\Omega'' = U_L\Omega\,, \qquad F''_\mu = U_L F_\mu U_L^\dagger + iU_L\partial_\mu U_L^\dagger\,,$$
$$B''_\mu = B_\mu\,, \qquad A''_\mu = U_L A_\mu U_L^\dagger + iU_L\partial_\mu U_L^\dagger\,.$$

- The $SU(2)$ gauge symmetry is nonlinearly realized on the fields ϕ_a,

$$\delta_2\phi_a = \frac{g_2}{2}\phi_0\alpha_a^L + \frac{g_2}{2}\epsilon_{abc}\phi_b\alpha_c^L\,,$$

$U(1)_Y$ transformations

- Under a local $U(1)$ right transformation $V_R = \exp\left(ig_1\,\alpha^R\,\frac{\tau_3}{2}\right)$ one gets

$$\Omega' = \Omega\, V_R^\dagger\,, \qquad F'_\mu = F_\mu + i\Omega\, V_R^\dagger \partial_\mu V_R\, \Omega^\dagger\,,$$
$$A'_\mu = A_\mu\,, \qquad B'_\mu = B_\mu + iV_R\partial_\mu V_R^\dagger\,.$$

- In passing we note that also the $U(1)$ symmetry is nonlinearly realized

$$\delta_1\phi_1 = \frac{g_1}{2}\phi_2\,\alpha^R\,,\quad \delta_1\phi_2 = -\frac{g_1}{2}\phi_1\,\alpha^R\,,\quad \delta_1\phi_3 = -\frac{g_1}{2}\phi_0\,\alpha^R\,.$$

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Bleaching

- It is possible to trivialize the $SU(2)$ gauge symmetry by introducing the "bleached" field, a_μ,

$$a_\mu = \Omega^\dagger (A_\mu - F_\mu)\Omega = \Omega^\dagger A_\mu \Omega - i\partial_\mu \Omega^\dagger \Omega \,.$$

- By construction a_μ is invariant under SU(2), while it transforms as a connection (plus a piece in the adjoint representation) under U(1)

$$a''_\mu = a_\mu \,, \quad a'_\mu = V_R a_\mu V_R^\dagger + i V_R \partial_\mu V_R^\dagger \,.$$

- Finally we can trivialize also the abelian gauge symmetry

$$w_\mu = a_\mu - B_\mu \frac{\tau_3}{2} \quad \text{so that}$$
$$w''_\mu = w_\mu \,, \quad w'_\mu = V_R w_\mu V_R^\dagger \,.$$

- The above relations allow us to introduce a charged bleached field, $w^{\pm}_{\mu} = \frac{1}{\sqrt{2}}\left(w_{1\mu} \mp i w_{2\mu}\right)$, and a neutral bleached field, $w_{3\mu}$, satisfying

$$\delta_1 w^{\pm}_{\mu} = \pm i g_1 \, \alpha^R \, w^{\pm} \, , \quad \delta_1 w_{3\mu} = 0 \, .$$

- There are many possible local monomials allowed on the basis of symmetry arguments. All of them can enter the tree-level lagrangian with independent parameters.
- Just to mention some of them. We have two gauge mass terms

$$M^2 \, w^- \cdot w^+ \, , \quad \frac{1+\kappa}{2} M^2 \, w_3^2 \, .$$

- One can also construct higher dimension operators such as:

$$(w^- \cdot w^+)^n\,, \quad (w_3^2)^m\,.$$

- Then there are all possible electrically neutral monomials containing $w^{\pm}$, w_3 and derivatives thereof.

$$\begin{aligned} &\partial_\mu w_\nu^- \, \partial^\mu w^{+\nu}\,, \quad \partial w^- \cdot \partial w^+\,, \\ &\partial_\mu w_{3\nu} \, \partial^\mu w_3^\nu\,, \quad \partial w_3 \cdot \partial w_3\,, \\ &\partial^\mu w_3^\nu \, w_\mu^- \, w_\nu^+\,, \quad \partial^\mu w^{+\nu} \, w_\mu^- \, w_{3\nu} \ldots \end{aligned}$$

Inclusion of Fermions

- We introduce the left-handed doublets and the right-handed singlets

$$L_i^L = \begin{pmatrix} \nu_i^L \\ l_i^L \end{pmatrix} \,, \quad Q_i^L = \begin{pmatrix} u_i^L \\ d_i^L \end{pmatrix} \,, \quad l_i^R \,, \quad u_i^R \,, \quad d_i^R \,.$$

- We can bleach the left-handed doublets

$$\widetilde{\Psi}^L = \Omega^\dagger \, \Psi^L \,.$$

- Both components of the bleached fermions are separately invariant under $SU(2)$, thus their hypercharge coincides with the electrical charge.

- Again on the basis of symmetry arguments, we have too many allowed interaction terms. There are the fermion mass terms,

$$m^l_{jk}\overline{\tilde{l}^L}_j\, l^R_k + \mathrm{h.c.}\,,\quad m^u_{jk}\overline{\tilde{u}^L}_j\, u^R_k + \mathrm{h.c.}\,,\quad m^d_{jk}\overline{\tilde{d}^L}_j\, d^R_k + \mathrm{h.c.}\ .$$

- There are kinetic terms,

$$i\overline{\widetilde{\nu}^L}_j \partial\!\!\!/\, \widetilde{\nu}^L_j\,,\quad i\overline{\widetilde{l}^L_i}\widetilde{D\!\!\!/}\,\widetilde{l}^L_i\,,\quad i\overline{l^R_i}\widetilde{D\!\!\!/}\, l^R_i \ldots$$

where $\widetilde{D}$ denotes the covariant derivative w.r.t. B_μ only.

- There are couplings with the gauge bosons,

$$h^L_{jk}\overline{\widetilde{u}^L}_j w\!\!\!/^+\widetilde{d}^L_k + \mathrm{h.c.}\,,\quad h^R_{jk}\overline{u^R}_j w\!\!\!/^+ d^R_k + \mathrm{h.c.}\,,\quad g^L_{jk}\overline{\widetilde{u}^L}_j w\!\!\!/_3\widetilde{u}^L_k \ldots$$

Outline

Local Functional Equation

- The D-dimensional classical action of the NLSM,

$$\Gamma^{(0)}[\vec{\phi}, \vec{J}, K_0] = \frac{v_D^2}{8} \int d^D x \left(F_{a\mu} - J_{a\mu} \right)^2 + \int d^D x \, K_0 \phi_0 \, ,$$

  is invariant under local left multiplication provided that $\vec{J}$ transforms as a background connection.
- Enforcing the invariance of the path integral Haar measure under local left multiplication we obtain

$$-\partial^\mu \frac{\delta\Gamma}{\delta J_a^\mu} - g\epsilon_{abc} J_b^\mu \frac{\delta\Gamma}{\delta J_c^\mu} + g\frac{1}{2}\epsilon_{abc}\phi_c \frac{\delta\Gamma}{\delta\phi_b} + g^2 \frac{1}{2}\phi_a K_0 + \frac{1}{2}\frac{\delta\Gamma}{\delta K_0}\frac{\delta\Gamma}{\delta\phi_a} = 0 \, .$$

Hierarchy

- All the amplitudes involving at least one ϕ field are fixed once those involving only insertions of the flat connection and of the source of the nonlinear constraint,K_0, are given.

Example

At first order in the loopwise expansion we take one derivative of the local functional equation w.r.t. J_b^ν and then w.r.t. ϕ_c.

$$-\partial^\mu \frac{\delta \Gamma^{(1)}}{\delta J_a^\mu J_b^\nu} + \frac{v_D}{2} \frac{\delta \Gamma^{(1)}}{\delta \phi_a \delta J_b^\nu} = 0\,,$$

$$-\partial^\mu \frac{\delta \Gamma^{(1)}}{\delta J_a^\mu \phi_c} + \frac{v_D}{2} \frac{\delta \Gamma^{(1)}}{\delta \phi_a \delta \phi_c} = 0\,.$$

The weak power-counting bound

- At every order of perturbation theory there is a finite number of divergent ancestor amplitudes (i.e. involving F and/or J, but no ϕ).
- The superficial degree of divergence of a 1-PI amplitude with n loops, N_J insertions of F and N_{K_0} insertions of K_0 is

$$\delta = (D-2)n + 2 - N_J - 2N_{K_0} \, .$$

- At fixed n, δ becomes negative with a finite number of insertions. Notice, however, that this number of required insertions grows with n.

Outline

Subtraction Strategy

- In order to obtain a consistent and predictive theory out of the set of Feynman rules of a model where the gauge group is nonlinearly realized we proposed to:
  1. Write down the most general action compatible with the symmetry requirements and the wpc criterion (i.e. finite number of divergent ancestor amplitudes).
  2. Subtract only the pole parts of properly normalized ancestor amplitudes. Finite renormalizations are not allowed since the corresponding invariants cannot be reinserted back at tree-level without violating either the wpc or the symmetries.

wpc at work

- We show on a simple example how the wpc can reduce the number of allowed monomials.

Example

Consider a pure Y-M theory with gauge group $SU(2)$.
The allowed monomials on the basis of symmetry considerations are

$$\partial_\mu a_\nu \partial^\mu a^\nu \,, \quad \partial a \cdot \partial a \,, \quad \epsilon_{abc} \partial_\mu a_{a\nu} a^\mu_b a^\nu_c \,, \quad (a^2)^2 \,, \quad a_{a\mu} a^\mu_b a_{a\nu} a^\nu_b \,.$$

They all contains a dangerous vertex with two A's, two ϕ's and two derivatives which gives rise to infinitely many divergent ancestor amplitudes.

Example

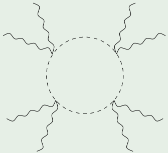


Figure: A weak power-counting violating graph.

The only safe linear combination of them is

$$\partial_\mu a_\nu \partial^\mu a^\nu - \partial a \cdot \partial a + 2\epsilon_{abc}\partial_\mu a_{a\nu} a^\mu_b a^\nu_c$$
$$+\frac{1}{2}(a^2)^2 - \frac{1}{2} a_{a\mu} a^\mu_b a_{a\nu} a^\nu_b = \frac{1}{4} G_{a\mu\nu}[A] G^{\mu\nu}_a [A]\,.$$

Tree-level lagrangian

- By imposing the validity of the wpc one is left with

$$
\begin{aligned}
\mathcal{L} \;=\; \Lambda^{(D-4)}\Big[& -\frac{1}{4}G_{a\mu\nu}[A]G_a^{\mu\nu}[A] - \frac{1}{4}F_{\mu\nu}[B]F^{\mu\nu}[B] \\
& + M^2\, w^- \cdot w^+ + \frac{1+\kappa}{2}M^2\, w_3^2 \\
& + i\overline{L^L}_j \not{D}\, L_j^L + i\overline{Q^L}_j \not{D}\, Q_j^L \\
& + i\,\overline{l_j^R}\, \not{D}\, l_j^R + i\overline{u_j^R}\, \not{D}\, u_j^R + i\overline{d_j^R}\, \not{D}\, u_j^R \\
& - h_{jk}^l \overline{\widetilde{l^L}}_j\, l_k^R - h_{jk}^u \overline{\widetilde{u^L}}_j\, u_k^R - h_{jk}^d \overline{\widetilde{d^L}}_j\, d_k^R \Big] \,,
\end{aligned}
$$

where Λ is a mass scale introduced to define the theory in D dimensions.

Custodial Symmetry

- The model is unique in the sense that it is the most general one which is locally invariant under $SU(2) \times U(1)$ and at the same time it satisfies the wpc criterion.
- We still have two independent gauge mass terms , which means that the ratio of the gauge boson masses is not fixed by the Weinberg angle only.
- For $g_1 = 0$ and $\kappa = 0$ we have a $SU(2)$ custodial symmetry.
- For $g_1 = 0$ and $\kappa \neq 0$ the custodial symmetry is broken along the τ_3 direction.

Gauge Mass Terms in the SM

- The bleaching procedure can be worked out also in the linearly realized theory.
- Let Φ be the usual Higgs doublet. We consider a 2×2 matrix field $H = \left(\Phi^c, \Phi\right)$ whose $SU(2)$ transformation is

$$H'' = U_L H \,.$$

- We can construct a $SU(2)$-invariant matrix field,

$$h_\mu = H^\dagger D_\mu H = h_{0\mu}\mathbb{I} + h_{a\mu}\frac{\tau_a}{2}\,.$$

- The bleached field h_μ is in a one-to-one correspondence with the gauge fields.

$$h_\mu|_{\phi_a=0} = A_\mu - B_\mu \delta_{a3} \,.$$

There are three allowed gauge mass terms on the basis of symmetry

$$h_0^2 \,,\; h_3^2 \,,\; h_1^2 + h_2^2 \,,$$

but only one linear combination of them gives rise to a power-counting renormalizable lagrangian

$$h_0^2 + \vec{h}^2 = \frac{1}{2} \mathrm{Tr}\Big[\big(D_\mu H\big)^\dagger D_\mu H \Big] \,.$$

Fermionic Sector

- The couplings of the fermions to the gauge bosons are those of the SM. (Every possible anomalous coupling is forbidden by the wpc).
- There are no fermion-Higgs couplings, $\bar{\psi}\psi H$, but there are non-polynomial couplings of the fermions with the unphysical scalars, $\bar{\psi}\psi\sqrt{v_D^2 - \vec{\phi}^2}$.
- The effective degree of divergence of the fermions in the wpc bound is 1 instead of $3/2$.

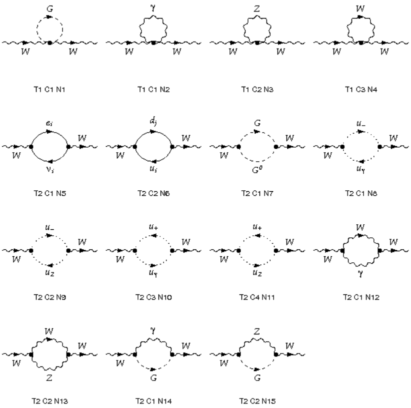
Outline

Physical Unitarity

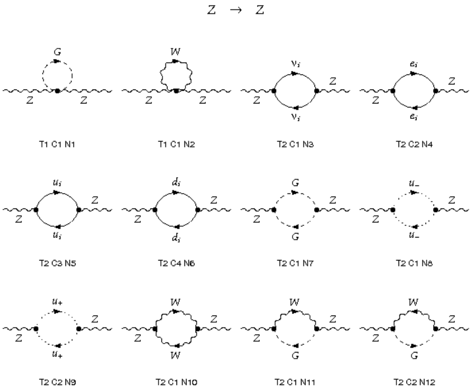
- The 't Hooft gauge can be worked out (D. B., R. Ferrari and A. Quadri, arXiv:0712.1410).
- In the Landau gauge the unphysical modes stay massless. In this way we have checked the cancellation of the unphysical contributions to the $W^{\pm}$ and Z self-masses, for arbitrary values of κ (physical unitarity).
- The photon stays massless, i.e. the photon self-energy vanishes on its mass-shell.
- The self-masses of the gauge particles are gauge independent.

W self-energy

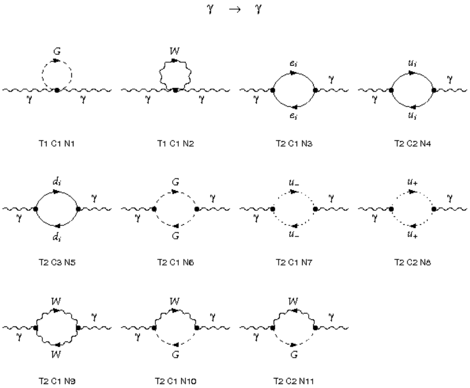
$W \rightarrow W$



Z self-energy



Photon self-energy



Outline

Free Parameters

- In the bosonic sector our model has four free parameter (g_1, g_2, M, κ), while in the SM there are only three of them $(\kappa = 0)$, but then there is the Higgs mass.
- The free parameters of the fermionic sector coincide with those of the SM, in particular there are the fermion masses and the CKM matrix.
- A change in the scale Λ cannot be compensated by a shift in the tree-level parameters. Thus also Λ has to be kept as an additional parameter to be fitted against experiments.

The fit strategy

- The aim of the fit is to assess the impact of the second mass parameter on the radiative corrections.
- We include the self-energy corrections only, no vertex correction (QED,QCD,EW) is taken into account.
- $\alpha_{EM}(0)$ and G_μ are used to fix g_1 and g_2.
- Massless fermions (except for $m_t = 174.2 GeV$) are considered.
- $M_W = g_2 M$, κ, Λ are fitted over the leptonic asymmetries.

Results

- Tree level fit

$$\chi^2 = 3824.4\,,\quad M_W = (79.02 \pm 0.02)GeV\,,\quad \kappa = 0.0353 \pm 0.0004\,. \qquad (1)$$

- One-loop fit

$$\chi^2 = 10.2\,,\quad M_W^{(0)} = (77.541 \pm 0.004)GeV\,,\quad \kappa = 0.0107 \pm 0.0001\,,\quad \Lambda = (242.6 \pm 0.5)GeV\,. \qquad (2)$$

Observ.	Exp.value	Tree-level	χ^2_{0L}	One-loop	χ^2_{1L}
M_W (GeV)	80.450	79.044	1782.5	80.414	0.71
	80.392				
M_Z (GeV)	91.1876	91.188	0.0006	91.188	0.002
A^e_{FB}	0.0145	0.0356	71.03	0.0169	0.922
A^μ_{FB}	0.0169	0.0356	206.26	0.0169	0
A^τ_{FB}	0.0188	0.0356	97.31	0.0169	1.25
s^2	0.2324	0.2224	68.86	0.2311	3.27
	0.2238				
A_e	0.15138	0.2178	1248.94	0.1501	0.85
	0.1544				
	0.1498				
A_μ	0.142	0.2178	25.52	0.1501	0.29
A_τ	0.136	0.2178	324.90	0.1501	2.98
	0.1439				

Summary

- The nonlinearly realized EW model can be symmetrically subtracted to all orders of perturbation theory.
- The tree-level action is unique and depends on a finite number of free parameters. At variance with the SM there are two gauge mass terms.
- The second mass parameters has a sizeable effect on the one-loop radiative corrections.

Outlook

- Is there a renormalization group equation for the proposed subtraction scheme?
- Does this model become nonperturbative around 1 TeV (providing a way to unitarize $W - W$ scattering) ?
- Is it possible to generalize the proposed subtraction scheme to other groups (e.g. $SU(5)$) ?