Validation of the Poisson Stochastic Radiative Transfer Model Against Cloud Cascade Models

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Background

Starting from a very simple stochastic cloud model by Mullamaa et al. (1972), several different stochastic models have been developed to describe radiative transfer regime in single-layer broken clouds (Kargin 1984; Titov 1990; Malvagi and Pomraning 1992; Barker et al. 1992; Malvagi et al. 1993; Kargin and Prigarin 1994; Prigarin and Titov 1996; Marshak et al. 1998; Prigarin et al. 1998, 2001; Evans et al. 1999, 2001). Recently Kassianov (2003a) generalized the Titov's (1990) stochastic model to multilayer clouds. Though the main applicability criterion of any stochastic cloud model is in its agreement with measured statistical characteristics for both cloud and radiation fields, there are yet very few examples of model validations (Lane 2002, Kassianov et al. 2003b).

The goal of the present study is to check the validity and applicability of the statistically homogeneous Poisson stochastic cloud model proposed by Titov (1990). This model (we will call it the "Poisson model") is completely determined by only three parameters: (1) cloud fraction N; (2) cloud optical depth τ (assumed to be constant for all cloud elements); and (3) aspect ratio $\gamma = H/D$ where H is the geometrical thickness of a cloud layer and D is the horizontal size of clouds. The earlier attempts to apply the Poisson model to experimental data (Titov 1990) showed that, in general, the radiative transfer processes in real broken clouds are well approximated by the model. Unfortunately, initially there was little data available for validation and even available measurements were incomplete. One of the main problems for the validation of the Poisson model is the ambiguity in estimation of the aspect ratio, one of the most important parameters of the model.

At this stage of validation, instead of real data we use realizations of a modified version of the fractionally integrated cascade model (Schertzer and Lovejoy 1987) with the modifications suggested by Marshak et al. (1998) to simulate broken cloudiness. We will call it the "cascade model". Each realization of the model has four well-defined parameters, easily estimated from real data: two of them come from a single-point statistics (mean optical depth, τ_{mean}, and standard deviation, or rather a direct function of it, p), one comes from a two-point statistics (scaling exponent β), and one is a cloud fraction, N. We assume that realizations of the cascade model represent real measurements. Mean radiative and cloud characteristics have been obtained by averaging over a set of cloud realizations (symbol ‹› will be used for the ensemble-averaged statistics).

We determine the aspect ratio $\gamma*$ by adjusting the mean direct solar radiation calculated by the Poisson model for oblique illumination to the corresponding values of the cascade model. There are two questions: (1) Since, in general, mean direct radiation is a function of solar zenith angle (SZA), can we find the aspect ratio $\gamma*$ that is appropriate for all SZA? (2) For the chosen aspect ratio $\gamma*$, how close will be the average values of albedo and diffuse transmittance for the cascade and Poisson models, respectively? The present study addresses these questions.

Statistical Characteristics of the Cascade Model

Statistical Characteristics of Clouds

Because for given parameters β and p, the cloud fraction varies from one realization to another, it is reasonable to use cloud fractions in statistical rather than in deterministic sense with its mean, <N>, standard deviation, σ_N, minimum, N_{min}, and maximum, N_{max}, as well as with its probability density function f(N). Cloud fraction statistics can be estimated from a sample of M cloud realizations; calculations showed that $M = 10^4$ cloud realizations are sufficient to adequately represent the N-statistics. Figure 1 illustrates examples of statistical characteristics of the cloud fraction. Note that going from one realization to another not only the cloud fraction varies but there are also changes in τ_{min} and τ_{max} for a fixed average value τ_{mean}.

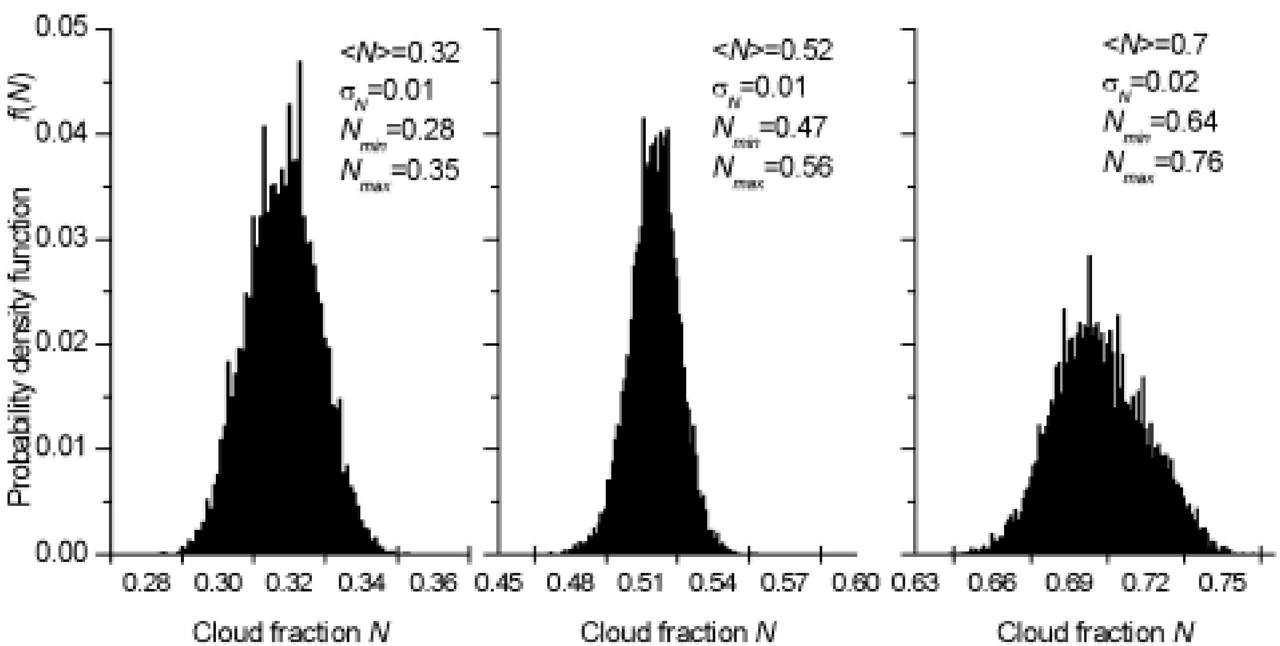

Figure 1. Statistical characteristics of cloud fraction *N* in a modified version of fractionally integrated cascade model for different values of <N>. Six cascades (L = 6) with spectral exponent $\beta = 5/3$, variance parameter p = 0.35 were used; the number of cloud realizations M = 10000.

Statistical Characteristics of Radiation

Let a unit solar flux be incident at the top of the cloud layer in direction $\vec{\Omega}_0 = (\theta_0, \varphi_0)$ where θ_0 and φ_0 are zenith and azimuth solar angles, respectively. For simplicity, we assume here an absorbing surface and conservative cloud droplet scattering; no aerosols are taken into account. Pixel sizes are taken to be 0.1 km × 0.1 km (with a modeled cloud field of 6.4 km × 6.4 km); periodic boundary conditions are also assumed, with the same average optical depth τ_{mean} for all realizations.

To calculate the radiative characteristics, for each cloud realization that represents a complex inhomogeneous three-dimensional medium, we used a Monte Carlo "maximal cross section" method (Marchuk et al. 1981). To get efficiently the radiative characteristics averaged over a number of realizations, we use the randomization procedure (Mikhailov 1986) that is based on introduction of an additional randomness. The optimum number of photon trajectories for each realization is usually selected from special numerical tests. Our calculations showed that the number of 10^4 trajectories is close to the optimum for the mean fluxes and their standard deviations and 10^5 trajectories for probability density functions. Figure 2 shows an example of the distributions of direct solar radiation S, diffuse transmittance Q_s, and albedo A, respectively; means, standard deviations, and variability ranges are added for completeness.

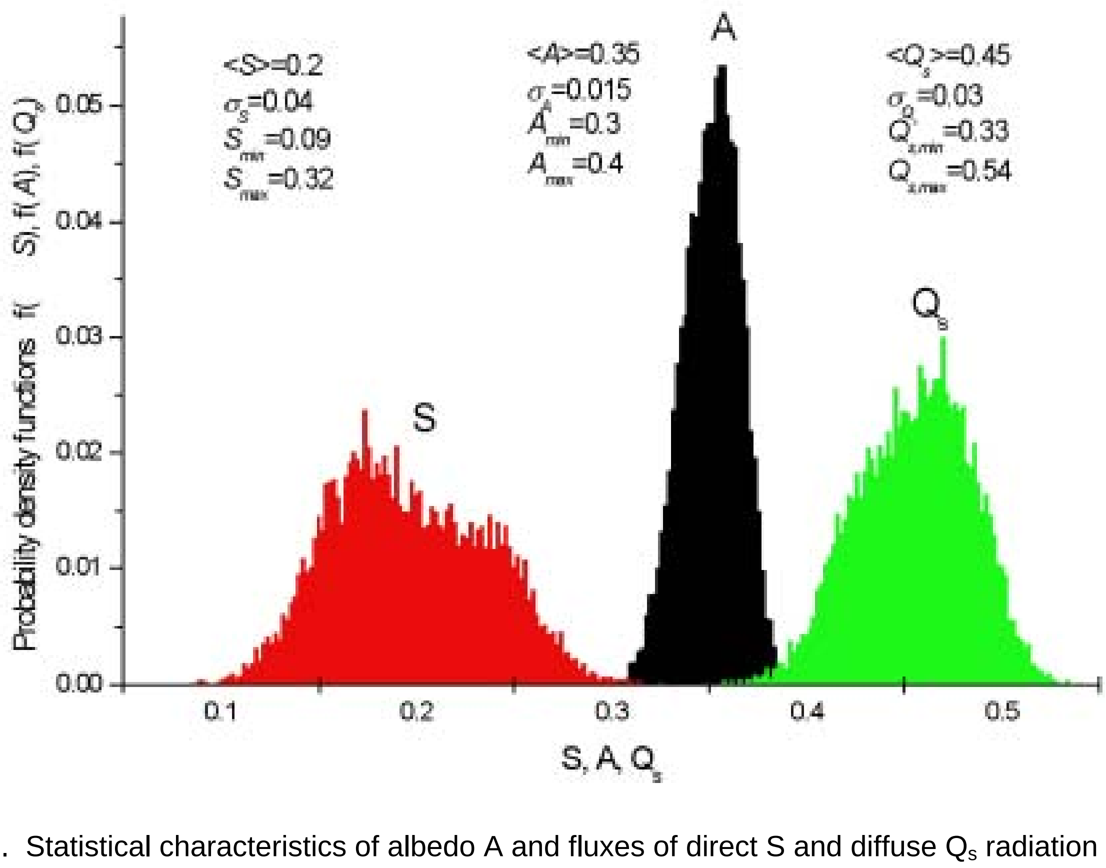

Figure 2. Statistical characteristics of albedo A and fluxes of direct S and diffuse Q_s radiation in the cascade model: L = 6, $\beta = 5/3$, <N>=0.515. Mean optical depth $\tau_{mean} = 13$, standard deviation $\sigma_\tau = 11.9$, pixel size 0.1 km×0.1 km, cloud thickness H = 1 km, and SZA $\theta_0 = 60°$.

Validation of the Poisson Broken Cloud Model

The main methodological aspect here is the development of a reasonable approach to specify the Poisson model parameters for validation of the mean radiative fluxes against realizations of the cascade model.

The Titov's (1990) approach permits efficient calculation of ensemble-averaged radiative characteristics $\langle R(\tau)\rangle_{pois}$ assuming that the cloud optical depth τ does not change from one realization to another. (Here R stands for S, Q_s, or A). We assume that the distribution of cloud optical depth can be well described by the gamma distribution (e.g., Barker et al. 1996)

$$p_\Gamma(\tau,\nu,\lambda)=\frac{1}{\Gamma(\nu)}(\lambda)^\nu\,\tau^{\nu-1}\exp(-\lambda\tau),\ \lambda=\frac{\nu}{\tau_{mean}} \tag{1}$$

where $\nu=\left(\frac{\tau_{mean}}{\sigma_\tau}\right)^2$ and $\sigma_\tau^2=\frac{\nu}{\lambda^2}$ is the standard deviation. To account for the variations in optical depth for the real clouds, we average $\langle R(\tau)\rangle_{pois}$ over the set of optical depth values using Eq. (1):

$$\left\langle R(\gamma,N,\theta_0)\right\rangle_{pois}=\int_0^\infty \rangle R(\tau,\gamma,N,\theta_0)\langle_{pois}\, p_\Gamma(\tau,\nu,\lambda)d\tau,\ R = S,\ A,\ Q_s. \tag{2}$$

The symbol $\rangle R\langle_{pois}$ is used to emphasize that the radiative characteristics are averaged over both the set of cloud realizations and cloud optical depths.

The other two required Poisson model parameters are the cloud fraction N, taken to be equal to <N>, and the aspect ratio, γ, chosen in such a way that, for N = <N>, the mean value of the direct radiation for the Poisson model, $\rangle S(\gamma,N,\theta_0)\langle_{pois}$ coincides with the direct radiation averaged over all realizations of the cascade model, $\langle S(N,\theta_0)\rangle$, i.e.,

$$\rangle S(\gamma,N,\theta_0)\langle_{pois}=\langle S(N,\theta_0)\rangle \tag{3}$$

Let us now take the statistical approach; we state and then verify the following two hypotheses.

Hypothesis 1. If for a given oblique SZA θ_0 >0 and a cloud fraction N = $\langle N\rangle$, the aspect ratio γ is determined from Eq. (3), the calculated average albedo $\rangle A(\gamma(\theta_0),N,\theta_0)\langle_{pois}$ and transmittance $\rangle Qs(\gamma(\theta_0),N,\theta_0)\langle_{pois}$ will be within the confidence intervals defined by the standard deviations of the cascade model,

$$\left\langle R(\gamma(\theta_0),N,\theta_0)\right\rangle_{pois}\in\left[\rangle R(N,\theta_0)\langle-\sigma_R(N,\theta_0),\ \rangle R(N,\theta_0)\langle+\sigma_R(N,\theta_0)\right],\ R = A,\ Q_s. \tag{4}$$

Assume that the SZA θ_0 is fixed. Since the mean flux of the direct radiation in the Poisson model for $\theta_0 = 0$ does not depend on γ, we will compare the mean values of the albedo and diffuse transmittance only for the oblique solar angles $10° \leq \theta_0 \leq 75°$. Our intense numerical calculations confirmed that, when γ is specified by (3), Eq. (4) is valid (see Table 1). Note that here, for specified $<N>$, τ_{mean}, and σ_τ, the γ value depends on θ_0, i.e., $\gamma = \gamma(\theta_0)$.

Table 1. Mean solar radiation fluxes calculated by the cascade and poisson models: $\langle N \rangle = 0.515$, $\tau_{mean} = 13$, $\sigma_\tau = 11.9$.

$\theta_0 = 60°$		$\theta_0 = 75°$	
Cascade Model	**Poisson Model** $\gamma = 1.56$, **D – 0.64 km**	**Cascade Model**	**Poisson Model** $\gamma = 2.04$, **D = 0.49 km**
$\langle S \rangle = 0.19$, $\sigma_S = 0.04$, $S \in (0.15, 0.23)$	$\rangle S \langle_{pois} = 0.191$	$\langle S \rangle = 0.047$, $\sigma_S = 0.05$, $S \in (0, 0.1)$	$\rangle S \langle_{pois} = 0.047$
$\langle A \rangle = 0.356$, $\sigma_A = 0.015$, $A \in (0.34, 0.37)$	$\rangle A \langle_{pois} = 0.348$	$\langle A \rangle = 0.517$ $\sigma_A = 0.03$, $A \in (0.49, 0.55)$	$\rangle A \langle_{pois} = 0.512$
$\langle Q_S \rangle = 0.454$	$\rangle Q_S \langle_{pois} = 0.461$	$\langle Q_S \rangle = 0.436$	$\rangle Q_S \langle_{pois} = 0.441$

Hypothesis 2. For a fixed cloud fraction $N = <N>$, there is a range of aspect ratios $\gamma \in [\gamma_{min}, \gamma_{max}]$ that is valid for any (reasonable) SZA $0 \leq \theta_0 \leq 75°$:

$$\rangle R(\gamma, N, \theta_0) \langle_{pois} \in \left[\langle R(N, \theta_0) \rangle - \sigma_R(N, \theta_0), \langle R(N, \theta_0) \rangle + \sigma_R(N, \theta_0) \right], \ R = S, A, Q_s. \tag{5}$$

To test the second hypothesis, we use the following approach. For given $<N>$, τ_{mean} and σ_τ, and fixed SZA $10° \leq \theta_0 \leq 75°$, we first calculate the mean direct radiation, $\langle S(N,\theta_0) \rangle$, and its root-mean-square deviation, $\sigma_S(N,\theta_0)$. Next, for the Poisson models, we select $\gamma_{min}(\theta_0)$ and $\gamma_{max}(\theta_0)$ in such a way that

$$\begin{aligned} \langle S(\gamma_{min}(\theta_0), N, \theta_0) \rangle_{pois} &= \rangle S(N, \theta_0) \langle + \sigma_S(N, \theta_0), \\ \langle S(\gamma_{max}(\theta_0), N, \theta_0) \rangle_{pois} &= \rangle S(N, \theta_0) \langle - \sigma_S(N, \theta_0) \end{aligned} \tag{6}$$

Finally, for chosen $\gamma_{min}(\theta_0)$ and $\gamma_{max}(\theta_0)$ we calculate the mean values of A and Q_s.

Figure 3 presents $\gamma_{min}(\theta_0)$ and $\gamma_{max}(\theta_0)$ for θ_0 varying in the range $10° \leq \theta_0 \leq 75°$. Evidently, there is a common region of the aspect ratios $(\gamma_{min}, \gamma_{max})$ for the entire angular range $10° \leq \theta_0 \leq 75°$. (In view of the weak dependence of the direct radiation in the Poisson model on parameter γ for $\theta_0 = 0$, it can be extended to $0° \leq \theta_0 \leq 75°$). This means that, in selection of parameter $\gamma \in (\gamma_{min}, \gamma_{max})$, for all *SZA* $0° \leq \theta_0 \leq 75°$ Eqs. (4) will be valid. For $<N>=0.515$ and $\tau_{mean} = 13$, γ_{min} and γ_{max} correspond approximately to the aspect ratio for $\theta_0 = 30°$ and are found to be 1.33 and 1.93, respectively. Qualitatively, this also applies to the other mean cloud fractions and mean cloud optical depths. It follows from these results that for a wide range of model input parameters ($0° \leq \theta_0 \leq 75°$, $6 \leq \tau_{mean} \leq 26$), there is a set of the aspect ratios around $\gamma^* \approx 5/3$, which can be used to calculate mean radiative fluxes in the Poisson cloud model with acceptable accuracy.

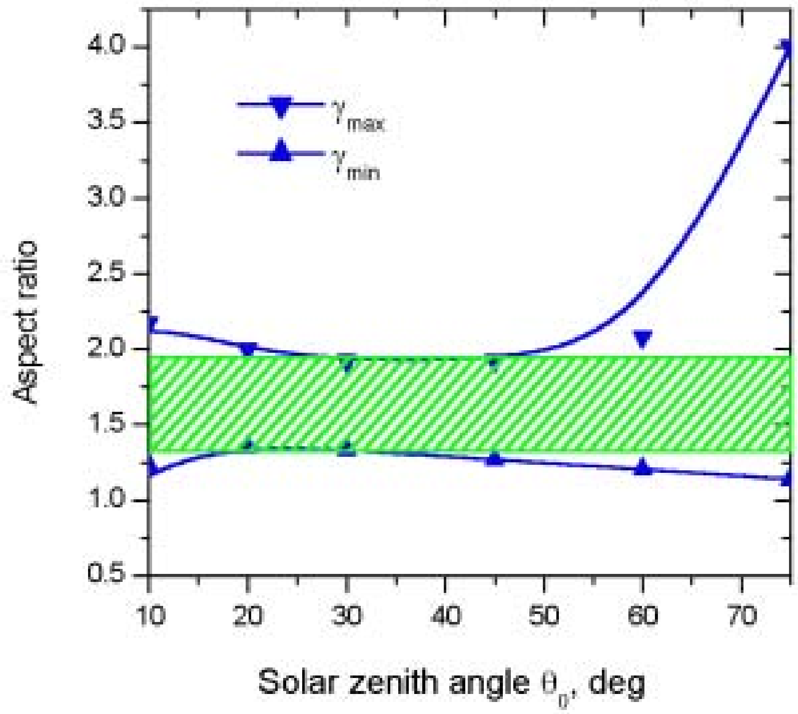

Figure 3. Variability range (γ_{min}, γ_{max}) of the aspect ratio γ that satisfies the condition: $\langle R(\gamma, N, \theta_0)\rangle_{pois} \in \left[\langle R(N,\theta_0)\rangle - \sigma_R(N,\theta_0), \langle R(N,\theta_0)\rangle + \sigma_R(N,\theta_0)\right]$, where *R* = *S*, *A*, or Q_s. The hatched region corresponds to those γ values that are common for the entire range of SZA $0 \leq \theta_0 \leq 75°$. Mean optical depth $\tau_{mean} = 13$, standard deviation $\sigma_\tau = 11.9$, *N* = 0.515

Summary and Plans for Future Work

The proposed approach allowed us to validate the stochastic Poisson model of broken clouds (Titov, 1990) against realizations of the cascade cloud model that served as a prototype of real measurements. The results of the validation test suggest that the Poisson cloud model can be successfully used to calculate the mean radiative properties of broken clouds. As soon as we know average cloud fraction and mean and variance of the in-cloud optical depth (assumed to be gamma distributed), we can estimate the average radiative transfer characteristic by setting the aspect ratio in the Poisson stochastic model to 5/3 for any reasonable SZAs. If, in addition, we know the direct radiation the aspect ratio can be determined more accurately from the condition of matching the mean direct radiative fluxes with those calculated from the Poisson cloud model.

In this study, the cloud cascade model determined the input parameters for the Poisson model. In a future, we plan to use cloud properties retrieved from ground-based observations at the ARM South Great Plains site: cloud-base and cloud top heights, cloud fraction and cloud optical depth. These data will be used to determine the input parameters for the Poisson model to validate it against the data from the ARM's shortwave radiometer archive.

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