A New Look into the Treatment of Small-Scale Drop Variability in Radiative Transfer

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Introduction

Drop size and drop spatial distribution determine photon-cloud interaction. The classical approach assumes that the number of drops of a given radius is proportional to volume with a drop-size-dependent coefficient of proportionality, the drop concentration, which is a volume-independent function of the spatial point. This assumption underlies the derivation of the drop size density distribution function, from which one can derive the extinction coefficient and scattering phase function, which are in turn input to the classical radiative transfer equation.

In general, drop size distributions depend on spatial scale. Liu et al. (2002) pointed out that there is a “saturation scale,” beyond which observed size distributions do not change much with further increases in averaging scale. The drop size distributions at the saturation or coarse scales maximize the spectral entropy and thus convey minimum information about drop distribution variation in space (Ash 1965). The use of such distributions in the radiative transfer equation, therefore, makes it insensitive to small-scale cloud heterogeneity. Another extreme is the case when drops are sampled at an infinitesimal scale. The drop distribution is then given by Dirac delta functions, which account for the total number of drops with specific radius and correspond to the minimum spectral entropy. Although information about drops conveyed by these distributions is maximized, the drop concentration can not be defined at this scale and thus can not be used in the radiative transfer equation. A Monte-Carlo technique seems to be the only way to account for the full amount of information conveyed by such distributions.

Drop size distributions are strongly scale dependent when the sampling scale is between the infinitesimal and saturation scales (Liu et al. 2002). This suggests that drop distributions at these scales convey a considerable amount of information on drop variability. The radiative transfer equation, in turn, aims to relate small-scale properties of the medium to the photon distribution in the entire medium. The question then arises, how essential is this information? The objective of this paper is to address this question.

Data Analysis

Two 10 min. samples of the cloud drop size distribution measured by the forward scattering spectrometer probe (FSSP) during the First ISCCP Regional Experiment (FIRE) (Albrecht et al. 1988) are used in our analysis. The measurements were taken on July 10, 1987, over the Pacific Ocean off San Diego (King et al. 1990). Two flight legs are used; each of them is about 50 km long at an altitude of 725 m in the middle of a 440 m thick marine stratocumulus cloud layer. The aircraft speed was 80 m/sec. Drops were accumulated over 1-sec time periods in order to form drop distributions consisting of 15 radius bins. The bin width is 2 μm; radii of the smallest and largest registered drop were 1.4 μm and 31.4 μm, respectively. In total, there were 1196 drop distributions; Figure 1 shows 100 consecutive ones. With the FSSP sample area ε_{min}^2 = 0.004 cm^2 (Liu and Hallett 1998) and accumulation distance l_{min} = 80 m, each measurement provides the distribution of drop sizes in a volume $v_{min}=l_{min} \times \varepsilon_{min}^2$ = 32 cm^3. We term the distance l_{min} and the volume v_{min} the smallest FSSP collectable linear and volume scales, respectively.

Let N(v,r) be the total number of drop distribution samples at a volume scale $v = l \times \varepsilon^2$ containing drops of radius r. For $\varepsilon = \varepsilon_{min}$ it can be evaluated from Figure 1 by counting nonempty radius bins in a column. Each such bin can contain more than one drop, but still adds only unity to N(v,r). We use N(v,r) to examine the emergence of drops as a function of scale v. At an infinitesimal scale, $v \to 0$, a bin has only one drop, so N(0,r) will be the total number of drops of radius r. In the case when samples represent every spatial point of the cloud, an "ideal" drop size distribution is given by Dirac delta functions $N(0,r)\delta(r-r^*)$ which account for the total number of drops N(0,r) with specific radius r^*. The drop concentration can not be defined in this case and thus can not be used in the radiative transfer equation. Increase in the volume scale v lowers the number N(v,r) of "nonempty bins" since nonempty bins can now contain more than one drop.

We define the average number of drops with radius r in nonempty bins as

$$\bar{n}(v,r) = \frac{\sum n(v,r)}{N(v,r)} \tag{1}$$

where the summation is performed over nonempty bins. Here n(v,r) > 0 is the number of drops with radius r in corresponding nonempty bin in a volume scale v. The average number of bins $\bar{n}(v,r)$ is required to restore the "ideal drop size distribution" from data sampled over a coarser scale, i.e., $N(0,r) = N(v,r) \cdot \bar{n}(v,r)$. Thus, one needs two variables, N(v,r) and $\bar{n}(v,r)$, to maximize the information content of data on drop sizes.

Next we degrade our FSSP data to coarser linear scales of $2l_{min}$, $3l_{min}$, …, $600l_{min}$ by joining two, three, etc. consecutive readings. Figure 2 shows variation in N(v,r) with respect to the linear scale l. Two situations can be clearly seen. First, beyond a linear scale of l_{sat} = 2300 m, variation in N(v,r) follows a power law with unit exponent; that is, the number N(v,r) of "nonempty" bins is inversely proportional to the linear scale l. At these km scales, differences in variability of the drops with different radii cannot

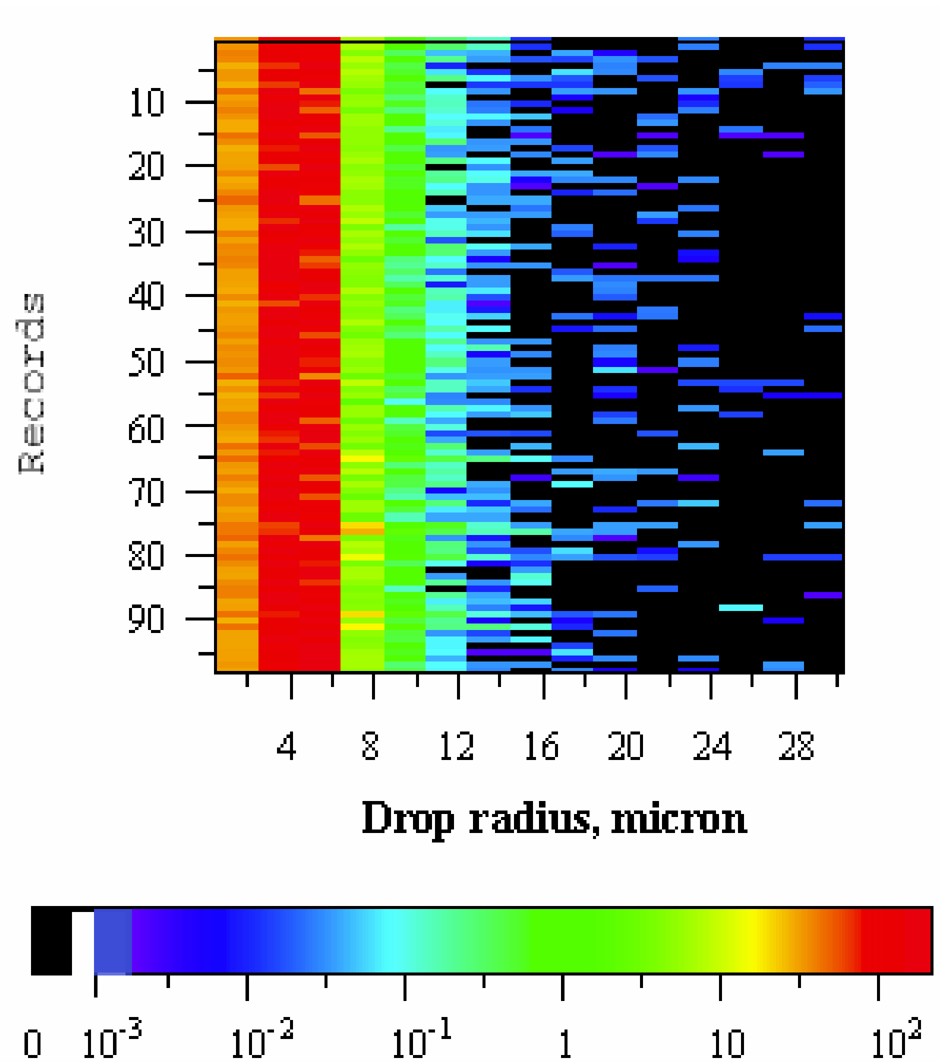

Figure 1. A fragment of 100 consecutive drop size distributions measured by the FSSP on July 10, 1987 (King et al. 1990). The measurements were taken from 8:44 to 8:54 and 9:31 to 9:41 Pacific daylight time (PDT) over the Pacific Ocean off San Diego. The aircraft speed was 80 m/sec and drops were accumulated over 1 sec time period. Drops above 31.4 μm are not registered. Each row corresponds to one record of the drop size distribution. Columns show counts of drops of a given radius in 80 m intervals along the flight leg. Note that at a linear scale of 80 m, 14-μm and larger drops do not appear in each sample.

be discerned. Thus, in our case 2300 m is Liu et al.'s (2002) saturation scale. Second, for linear scales between 80 m and about 2300 m, variation in N(v,r) also follows a power law, but with an exponent which is a function of drop size r, i.e.,

$$N(v,r) = m(V_L,D)\, v^{-D(r)}\,. \tag{2}$$

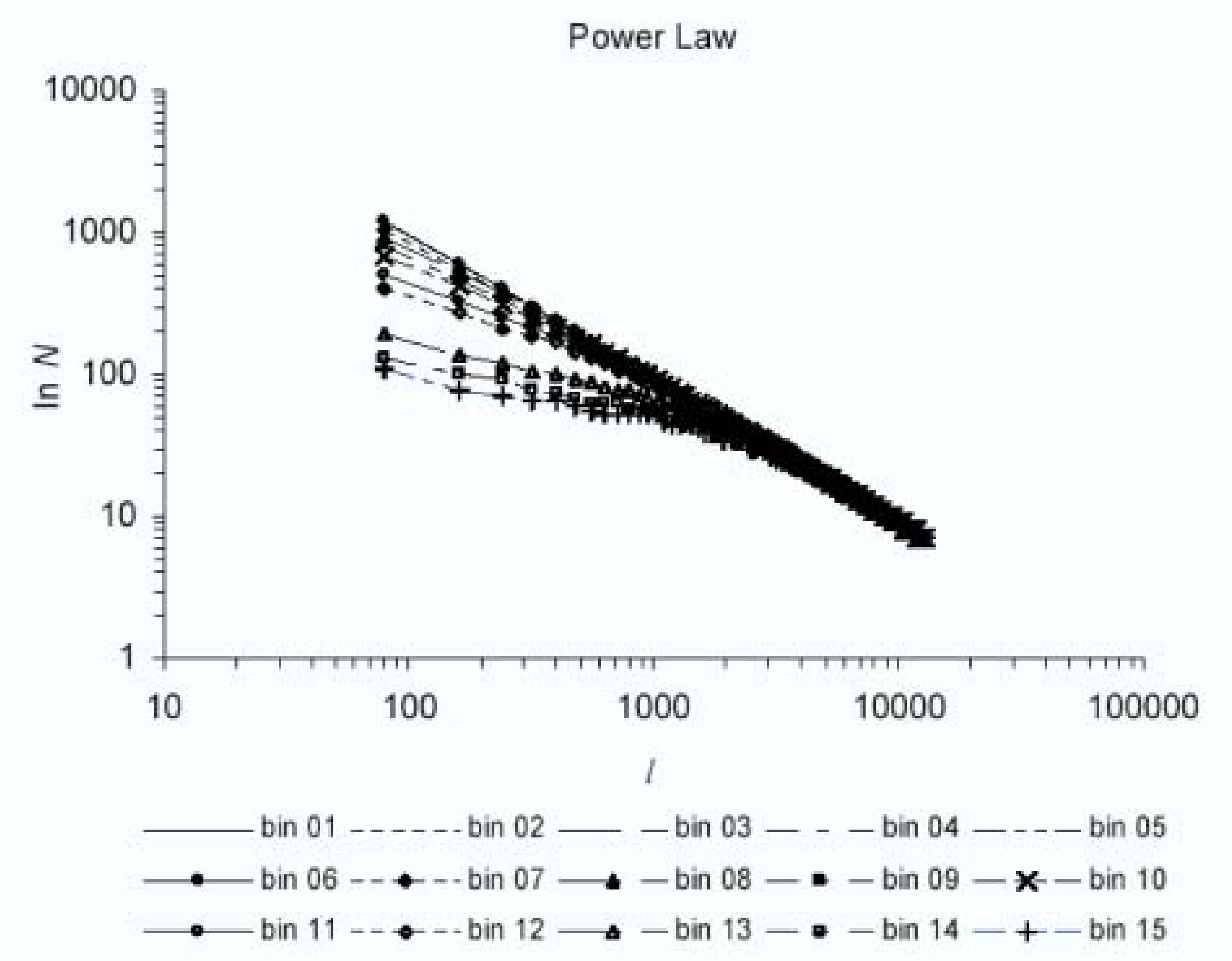

Figure 2. Scaling behavior of bin count N derived from data described in Figure 1. At linear scales between 80 m and 2300 m, variation in N(v,r) follows the power law with a drop size dependent exponent D(r), i.e., $N(v,r) = m(V_L,D)v^{-D(r)}$. Here $v = l \times \varepsilon^2_{min}$ where ε^2_{min} = 0.004 cm^2 is the FSSP sampling area, $m(V_L,D)$ is a coefficient which depends on the total volume $V_L = L\,\varepsilon^2_{min}$ in which drops are sampled, and L is the total length of the flight leg. Note that variation in the linear scale between 80 m and 2300 m corresponds to variation in the volume scale between 32 cm^3 and 920 cm^3.

Here $V_L = L \times \varepsilon^2_{min}$ is the total volume in which drops are sampled and L = 95,680 m is the total length of the flight leg. Function $m(V_L,D)$ (or more precisely $\ln[m(V_L,D)\,\varepsilon^{-2D(r)}_{min}]$) describes the interception of the vertical axis lnN by a straight line lnl corresponding to l <2300 m (see Figure 2). Figure 3a shows D(r) as a function of drop size r. At linear scales between 80 m and 2300 m, drops with r <14 μm follow a power law with exponent D(r) = 1. However, D(r) for larger drops falls below unity. Figure 3b complements Figure 3a with values of D(r) for drops larger than 30 μm. Note that this curve was derived from data collected at a different time, different site (Oklahoma vs. California), using different instrument (1D-C vs. FSSP) and sampling strategy. Both figures highlight an important behavior of

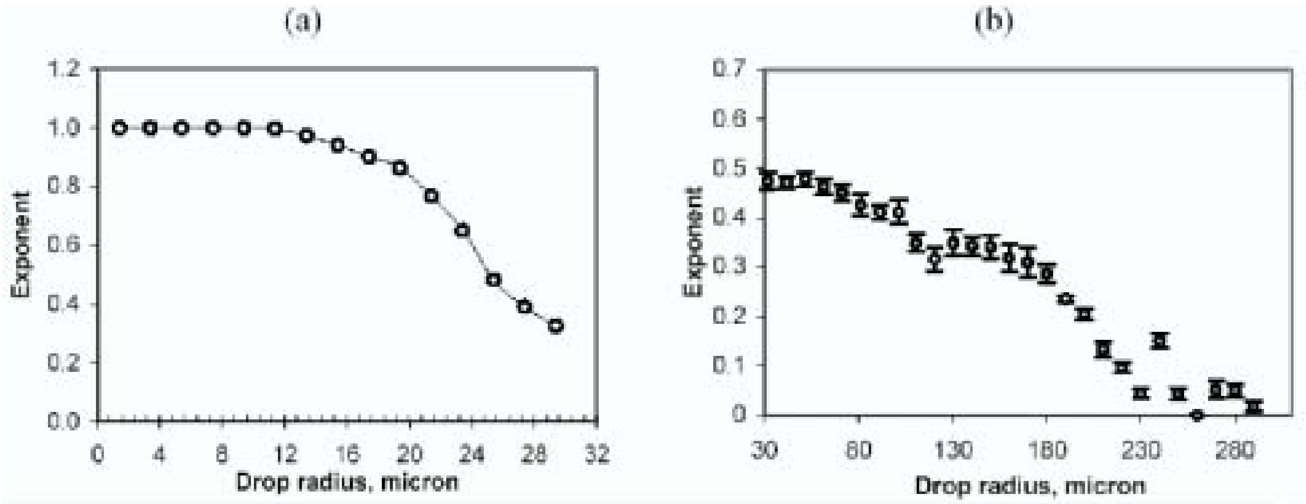

Figure 3. (a) Exponent D(r) as a function of drop radius in the range from 2 μm to 30 μm derived from data acquired during the flight on July 10, 1987, as part of FIRE field program is plotted. (b) Exponent as a function of drop radius in the range from 30 μm to 300 μm. This curve was derived from data collected by the 1D-C probe on board of the University of North Dakota Citation aircraft during the ARM Spring 2000 Intensive Operational Period, March, 2000. These data are publicly available at http://iop.archive.arm.gov/arm-iop/2000/sgp/cloud/poellot-citation/

D(r): for a given scale range, it is a non-increasing function with respect to the drop size. The exponent can be used to differentiate the emergence of drops with different sizes and thus data at these scales provide more information on drop size distribution. Finally, at infinitesimal scales, i.e., when l and ε are comparable with drop sizes, N(v,r) = N(0,r) does not depend on v and thus D(r) = 0. In the case of the ideal sampling strategy, information about drops conveyed by data is maximized.

Thus, our analysis of data suggests that: (1) distribution of drops at different linear scales follows a power law with a drop size dependent exponent; (2) the exponent is a non-increasing function with respect to the drop size; it varies between 0 and 1 and is associated with a certain spatial scale range; (3) the lower value of the exponent, the more information about drop size variability the data conveys.

Generalized Drop Concentration

We return to [Eq. (1)]. Note that the numerator in its right-hand side is the total number of drops, $N_{tot}(r)$, with radius r acquired during the flight and does not depend on scale. Substituting (2) into (1), one obtains

$$\bar{n}(v,r) = \frac{N_{tot}(r)}{m(V_L, D)} v^{D(r)}. \qquad (3)$$

It follows from [Eq. (3)] that if the drop concentration, c, is defined in a traditional way as the ratio of the mean number of drops to the volume they occupy,

$$c(r) = \frac{\overline{n}(v,r)}{v}, \tag{4}$$

then for those r that have D(r) <1 the ratio will depend on volume v! This violates the assumption of a drop concentration as a scale-independent function and requires a new definition of a drop concentration that is independent of scale.

Indeed, [Eq. (3)] can be rewritten as

$$\overline{n}(v,r) = \rho(r)v^{D(r)}, \tag{5}$$

where $\rho(r)$ is a generalized drop concentration (in number per $cm^{3D(r)}$) defined as

$$\rho(r) = \frac{N_{tot}(r)}{m(V_L, D)}. \tag{6}$$

where $V_L = 95.6 \cdot 10^5 cm \times 4 \cdot 10^{-3} cm^2 = 38{,}272\ cm^3$. Thus, for volumes between $v_{min} = 32\ cm^3$ (80 m × 0.004 cm^2) and $v_{sat} = 920\ cm^3$ (2300 m × 0.004 cm^2), the mean number of drops with radius r in a given volume is proportional to the drop dependent power of the volume.

Two variables are required to estimate the generalized drop concentration corresponding to the scales between v_{min} and v_{sat}. They are the total number, $N_{tot}(r)$, of drops with radius r acquired during the flight and the coefficient $m(V_L,D)$. The former does not depend on the scale and can directly be evaluated from data (see Figure 1). The latter can be specified either from the intercept of [Eq. (2)] in the log-log plane or from [Eq. (2)] given l, ε^2_{min}, D, and N(v,r), i.e.,

$$m(V_L,D) = N(v,r)v^{D(r)},\ v_{min} \le v < v_{sat}. \tag{7}$$

Figure 4 shows $m(V_L,D)$ as a function of r and the exponent D. For r <14 μm, D = 1 and $m(V_L,1)$ is equal to the total volume V_L in which drops were sampled. The generalized drop concentration $\rho(r)$ coincides with the classical one which is the mean number of drops in one cm^3. For r >14 μm, the generalized drop concentration shows the number of drops in $[cm^3]^{D(r)}$ and thus one cm^3 contains not $\rho(r)$ but $\rho(r)^{[1/D(r)]}$ such drops. Figure 5 shows values of $\rho(r)^{[1/D(r)]}$ and the drop concentrations, $\rho_{sat}(r)$, calculated using the classical definition, i.e., $\rho_{sat}(r) = N_{tot}(r)/V_L$. Note that the latter coincides with the drop concentration derivable from data acquired over scales coarser than the saturation scale v_{sat}. One can see from Figure 5 that the neglect of small-scale drop size variability can result in the underestimation of the mean number of drops in one cm^3.

Conclusions

Most of the existing cloud radiation models and conventional techniques of data processing assume that the number of drops with a given radius varies proportionally to volume. In other words, the number of drops in a volume follows a power law in volume with exponent equal to 1. The analysis of

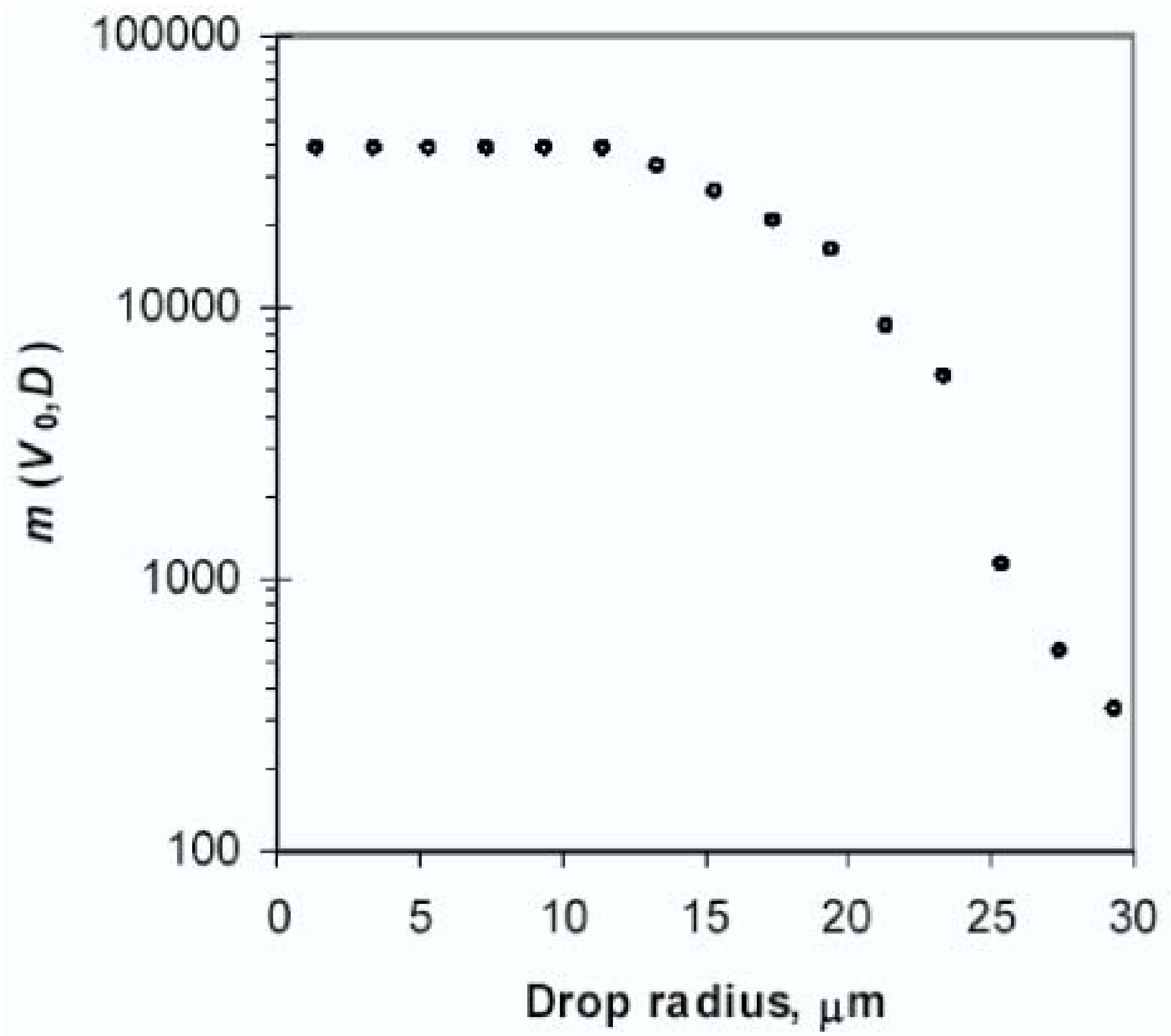

Figure 4. The dependence of the coefficient $m(V_L,D)$ on the drop radius derived from data described in Figure 1. For drop radii below 14 μm, D = 1 (see Figure 3) and $m(V_L,1) = V_L = 38{,}272\ cm^3$. For drop radii above 14 μm, D <1; the coefficient $m(V_L,D) < m(V_L,1)$ and its unit is expressed in “number of drops per cm^{3D}.

microphysical data on liquid water drop sizes acquired during the FIRE and the ARM Spring 2000 IOP (March 3, 2000) indicates that the power law has a drop-size dependent exponent. For abundant small drops (r ≤14 μm) present, the exponent is 1 as assumed in conventional approach. However, for rarer large drops (r >14 μm), the scaling exponents fall below unity for scales between the smallest FSSP sampling volume and a “saturation” scale. The assumption underlying the conventional approach, therefore, is not valid and the “traditional” drop concentration function is volume (or scale) dependent.

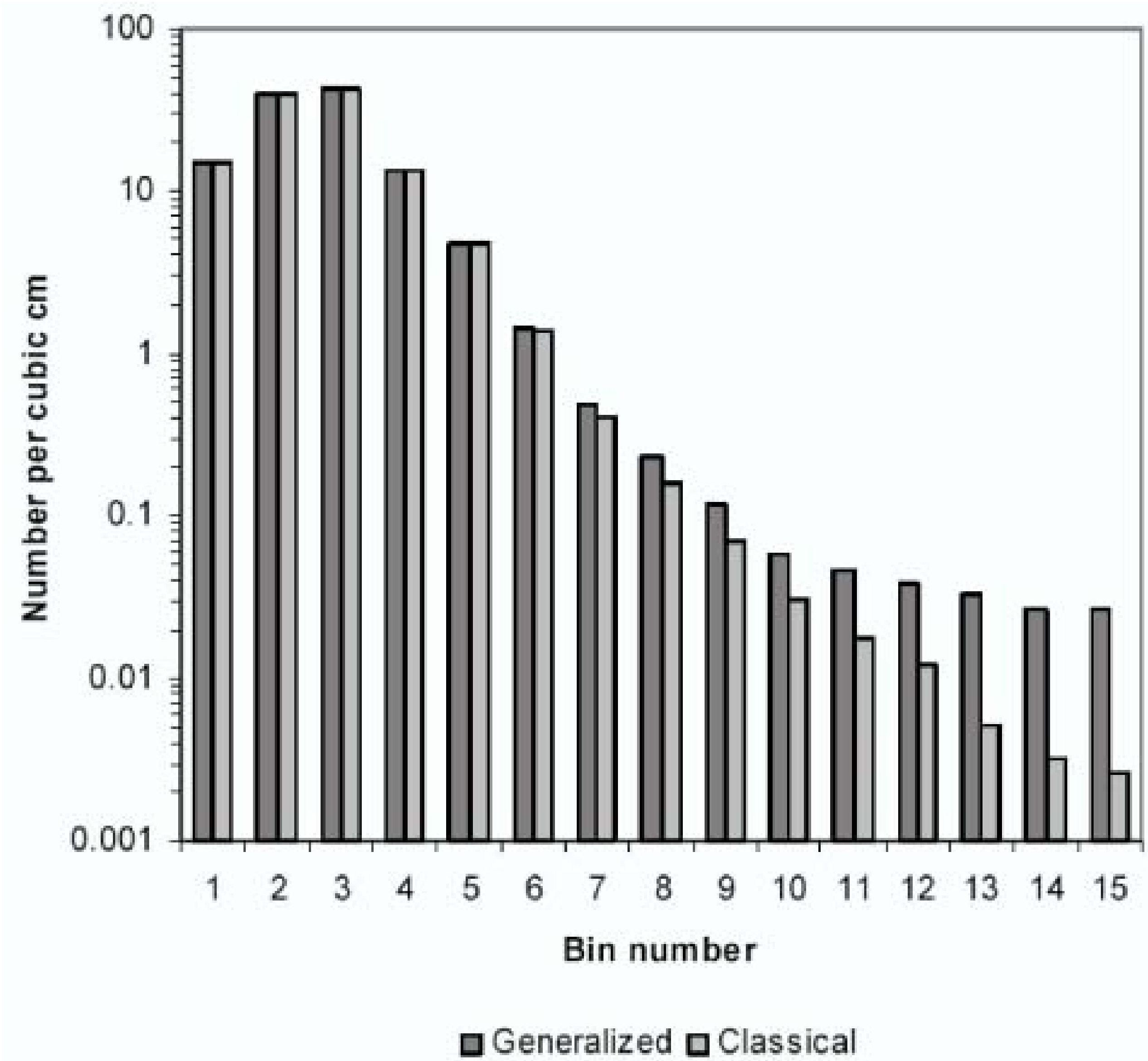

Figure 5. Drop concentration as a function of bin number (in number per cubic centimeter) derived from data collected above (legend "Classical") and below (legend "Generalized") the saturation scales. The neglect of small-scale drop size variability can result in the underestimation of the mean number of drops in one cubic centimeter.

It was found that, for sufficiently small volumes, the mean number of large drops in a given volume is proportional to the drop size dependent power of the volume. The drop size dependent coefficient of proportionality, or a generalized drop concentration, and the exponent are determined solely by the smallest FSSP sampling volume and are independent on the volume they occupy. Thus they can be used to parameterize the variability of drops at small scales. If the exponent is equal to 1, the generalized drop concentration coincides with the traditional one. If the exponent falls below unity, the traditional drop concentration becomes ill-defined, i.e., the ratio of the mean number of drops to the volume they occupy depends on the size of the volume. Its specification, therefore, requires averaging over a sufficiently coarse, or saturation, scale beyond which the drop concentration do not change much with further increases in averaging scale (Liu et al. 2002). Information on small-scale drop variability is lost in this case.

Although not demonstrated here, the neglect of small-scale drop size variability results in the underestimation of the mean number of drops in one cubic centimeter, leading to the systematical underestimation of the effect of large drops on the estimation of the cloud radiation regime. Furthermore, our calculations indicate that depending on cloud height, the neglect of small-scale drop size variability can result in a systematic underestimation of cloud optical depth (Knyazikhin et al. 2003).

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