Toward a Unified Formulation of the Kessler-Type Autoconversion Parameterizations

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Introduction

Rain is initiated in liquid water clouds by collision and coalescence of cloud droplets wherein larger droplets with higher settling velocities collect smaller droplets and become embryonic raindrops. Kessler (1969) proposed a simple parameterization that linearly relates the autoconversion rate to the cloud liquid water content, and this parameterization has been widely used in cloud-related modeling studies because of its simplicity. However, this simple parameterization leaves much to be desired, because it is well known that the autoconversion rate is a function of not only the liquid water content but also the cloud droplet number concentration and the spectral dispersion of the cloud droplet size distribution. Over the last several decades, much effort has been devoted to improving the original Kessler parameterization by including the effect of the droplet concentration as well as liquid water content (Manton and Cotton 1977; Tripion and Cotton 1980; Liou and Ou 1989; Baker 1993).

However, these Kessler-type parameterizations, for the most part, lack a physical foundation and the relationships between the various parameterizations are not clear. The effect of the spectral dispersion (defined as the ratio of the standard deviation to the mean radius of the cloud droplet size distribution) is not explicitly accounted for. The primary purpose of this work is to show that these seemingly different parameterizations can be derived from the same formalism by applying the generalized mean value theorem for integrals to the general collection equation. This unified formulation clearly reveals both the approximations assumed by the different parameterizations and the connections between them. A new parameterization is derived that eliminates various incorrect and/or unnecessary assumptions inherent in the existing parameterizations and includes the relative dispersion of the cloud droplet size distribution as a dependent variable.

Kessler-Type Parameterizations for Autoconversion Rate

Without loss of generality, all of the Kessler-type parameterizations can be written as

$$P = cLH\left(y - y_c\right), \qquad (1)$$

where P is the autoconversion rate in g cm-3 s^{-1}, c is an empirical coefficient in s^{-1} (hereafter conversion coefficient), and L is the cloud liquid water content in g cm^{-3}. The Heaviside step function $H(y - y_c)$ is introduced to describe a threshold yc (hereafter threshold coefficient) below which the autoconversion is negligibly small. The meaning of y is different in different parameterizations; for example, y represents the cloud liquid water content (LWC) in the original Kessler parameterization, whereas it represents the mean volume radius in the Manton and Cotton expression, and the mean radius of the fourth moment in the parameterizations of Liou and Ou (1989), Baker (1993) and Boucher et al. (1995).

Re-Examination of the Previous Kessler-Type Parameterizations

Here we derive the various existing Kessler-type parameterizations by applying the generalized mean value theorem (Spiegel 1992) to the general collection equation (Pruppacher and Klett 1997). This derivation also reveals the assumptions and connections associated with these parameterizations. Table 1 summarizes the results.

Table 1. Summary of the Kessler-Type autoconversion parameterizations
$P = cLH(y - y_c) = \alpha N^{-1/3} L^{7/3} H(y - y_c)$

Parameterizations	Assumptions	Conversion Coefficient c	Threshold y_c
Kessler	Fixed collection kernel	$c_K = a_K \left(1 - \frac{L_c}{L}\right)$	L_c
Manton and Cotton	Fixed collection efficiency, monodisperse spectrum and fixed terminal velocity	$c_{MC} = \alpha_{MC} N^{-1/3} L^{4/3}$ $\alpha_{MC} = \pi\kappa_1 \left(\frac{3}{4\pi\rho_w}\right)^{4/3} E_{MC}$	R_{3c}
Baker	Fixed collection efficiency and monodisperse spectrum	$c_{Baker} = \alpha_{Baker} N^{-1/3} L^{4/3}$ $\alpha_{Baker} = \pi\kappa_1 \left(\frac{3}{4\pi\rho_w}\right)^{4/3} E_4$	R_{3c}
Boucher	Fixed collection efficiency and fixed, broader spectrum	$c_{Boucher} = \alpha_{Boucher} N^{-1/3} L^{4/3}$ $\alpha_{Boucher} = \pi\kappa_1 \left(\frac{3}{4\pi\rho_w}\right)^{4/3} E_4 (1.1)^4$	R_{4c}
Generalized R_4	Fixed collection efficiency	$c_4 = \alpha_4 N^{-1/3} L^{4/3}$ $\alpha_4 = \pi\kappa_1 \left(\frac{3}{4\pi\rho_w}\right)^{4/3} E_4 \beta_4^4$	R_{4c}

New R_6 Parameterization

Although the various R_4 parameterizations are significant improvements of the original Kessler parameterization, they still suffer from the implicit deficiency that the collection efficiency is treated as a constant. Here we develop a new parameterization that further removes this assumption. Long (1974) showed that the collection kernel can be well approximated by

$$K(R, r) = \kappa_2 R^6 , \tag{2}$$

where the empirical constant κ_2 = 1.9 x 1011 cm^{-3} s^{-1}. Substitution of (5) into (3) yields the new R_6 parameterization

$$P_6 = c_6 LH\left(R_6 - R_{6c}\right) = \alpha_6 N^{-1/3} L^{7/3} H\left(R_6 - R_{6c}\right), \quad (3a)$$

$$\alpha_6 = \left(\frac{3}{4\pi\rho_w}\right)^2 \kappa_2 \beta_6^6 \left(\frac{L}{N}\right)^{2/3} \quad (3b)$$

$$c_6 = \alpha_6 N^{-1/3} L^{4/3} \quad (3c)$$

The dependence of the dimensionless β_6 on the spectral dispersion ε is well described by

$$\beta_6 = \left[\frac{\left(1+3\varepsilon^2\right)\left(1+4\varepsilon^2\right)\left(1+5\varepsilon^2\right)}{\left(1+\varepsilon^2\right)\left(1+2\varepsilon^2\right)}\right]^{1/6} \quad (4)$$

Applications and Comparisons

Both the Kessler scheme and the various R_4 schemes include empirical coefficients that are tunable over a wide range of values (e.g., a coefficient in the Kessler scheme and α coefficient in the R_4 schemes). The new R_6 parameterization suggests that the wide range of the tunable coefficients is due to the variability of droplet concentration, liquid water content and spectral dispersion. This is demonstrated in Figure 1.

Furthermore, several empirical expressions that do not have adjustable parameters have been proposed by fitting results of numerical simulations (Berry 1968; Beheng 1994; Khairoutinov and Kogan 2000). Figure 2 shows the comparison of our new parameterization with these empirical parameterizations. Clearly, there are significant differences among the existing empirical parameterizations, and our simple parameterization well represents the average behavior of those existing parameterizations. Daum and Liu (2003) apply this new parameterization to investigate the effect of spectral dispersion on the autoconversion rate and the second indirect aerosol effect.

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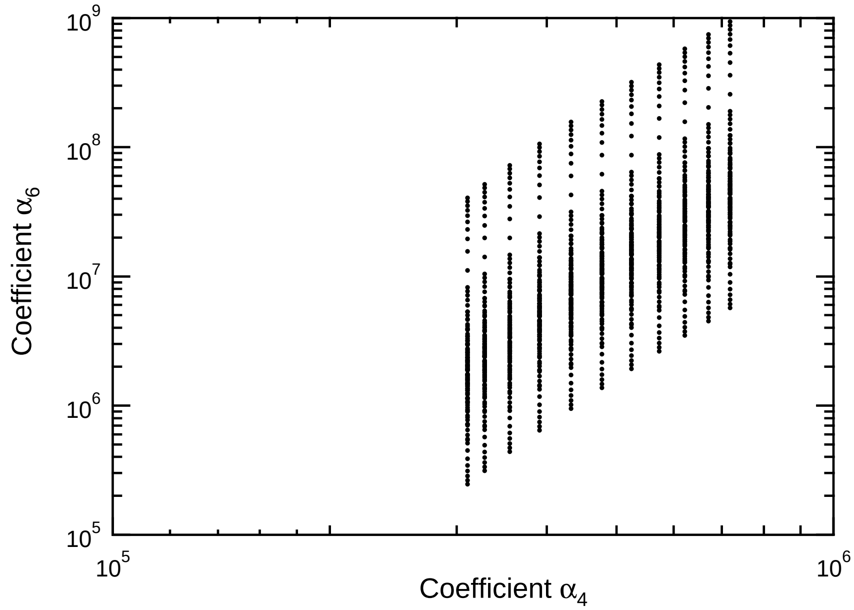

Figure 1. This figure shows α values calculated from the new scheme for a monodisperse cloud droplet size distribution. The contour value denotes the log(α). The α coefficient varies by three orders of magnitude depending on the combination of the droplet concentration and the LWC. The effect of the spectral dispersion is referred to Daum and Liu (2003).

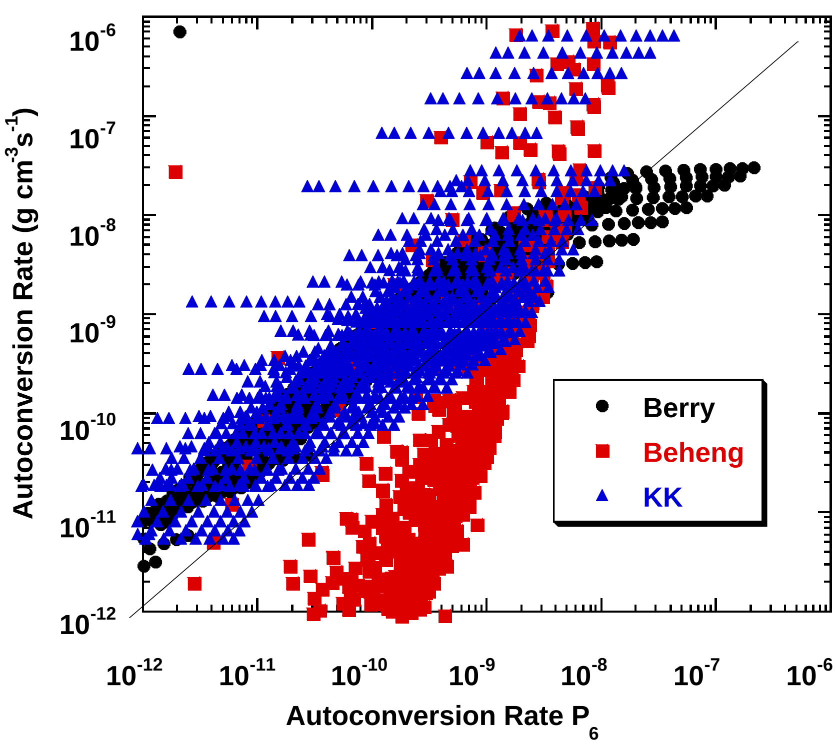

Figure 2. Shows the comparison of our new scheme (solid line) with previous parameterizations. The solid line represents our new R_6 parameterization. KK represents the scheme given by Khairoutdinov and Kogan (2000). The results support our new R_6 scheme.

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