Radiative Transfer in 3D Clouds: A Perturbation Theoretical Approach

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Introduction

The Department of Energy Atmospheric Radiation Measurement (ARM) Program is committed to improving our understanding of cloud-radiation interactions, including the effects of three-dimensional (3D) spatial variability. It is generally assumed that to simulate accurately radiative transfer through a realistic cloudy layer one should use numerical approaches such as Monte Carlo (Marchuk et al. 1980) or Spherical Harmonic Discrete Ordinate Method (SHDOM) (Evans 1998). However, it usually requires too much computer time to make a simulation for a realistic model, which is inconvenient when just we need an answer on a simple question like how significant the 3D effects are for a given problem. The perturbation method is what comes to mind if we need to go further into modeling of the radiative transfer through cloud atmosphere starting from the simplest framework of one dimensional radiative transfer (Chandrasekhar 1950; Lenoble 1985).

Recently, two perturbation approaches have been used to address 3D radiative transfer problems. One is somewhat orthodox (Morse and Feshbach 1952) and based on assuming that some term in the radiative transfer equation is small enough to be considered as a small parameter to construct a perturbation series (Li et al. 1994 1995; Galinsky and Ramanathan 1998; Galinsky 2000; Lyapustin and Knyazikhin 2002). A different type of perturbation approach may be formulated on the basis of both the direct and corresponding adjoint problem (Marchuk 1964; Box et al. 1989; Ustinov 1991; Polonsky and Box 2002). This approach can be derived using a variational principle (Pomraning 1965), thus allowing one to obtain a required perturbation solution practically without effort.

The goal of this paper is to demonstrate how perturbation technique can be employed differently for the problem of cloud optics starting from the independent pixel approximation (IPA).

Definition of the Problem

Let us consider radiation propagation through an internally variable cloud, which has the outer shape of a plane-parallel slab, and is illuminated by a steady, uniform, and collimated beam (i.e., sunlight). We introduce a Cartesian coordinate system with the origin on the upper slab surface with the z-axis directed toward the inner normal. The direction is defined by the Euler polar, θ and azimuth φ angles. The radiance $I(\vec{r},\vec{n})$ at the point $\vec{r}$ in the direction $\vec{n}$ can be calculated using the framework of the 3D radiative transfer equation (RTE) (e.g., [Ishimaru 1978])

$$\vec{n}\cdot\vec{\nabla}I(\vec{r},\vec{n})+\sigma_e(\vec{r})I(\vec{r},\vec{n})=\frac{\sigma_s(\vec{r})}{4\pi}\int_{4\pi}P(\vec{r},\vec{n},\vec{n}')I(\vec{r},\vec{n}')d\vec{n}'+S(\vec{r},\vec{n}) \quad . \tag{1}$$

Here $\vec{r}$ and $\vec{n}$ are the vectors which define the direction and position, respectively, $\sigma_e(\vec{r})$ and $\sigma_s(\vec{r})$ are the extinction and scattering coefficients, respectively, and $P(\vec{r},\vec{n},\vec{n}')$ is the phase function normalized as

$$\frac{1}{4\pi}\int_{4\pi}P(\vec{r},\vec{n},\vec{n}')d\vec{n}'=1 \quad , \tag{2}$$

$S(\vec{r},\vec{n})$ is the source function which in the case of the Sun beam has the form

$$S(\vec{r},\vec{n})=F_0\delta(\vec{n}-\vec{n}_0)\delta(z) \quad .$$

Here F_0 is the Sun flux at the top of the cloud. Certainly, Eq. (1) has to be complimented with the boundary conditions:

$$\begin{aligned} I(\vec{r},\vec{n})|_{z=0} &= 0, && \mu>0, \\ I(\vec{r},\vec{n})|_{z=H} &= \frac{1}{\pi}\int_{\mu>0}A(\vec{r},\vec{n},n')I(\vec{r},\vec{n})_{z=H}\,d\vec{n}, && \mu<0. \end{aligned} \tag{3}$$

Here $A(\vec{r},\vec{n},\vec{n})$ describes the reflection properties of the underlying surface if one exists.

Perturbation Technique

We may safely assume that the most measurements (signals) in cloud optics can be describe by integration of the radiance with a receiver function $R(\vec{r},\vec{n})$

$$P = \int_{\Xi} R(\vec{r},\vec{n})\, I(\vec{r},\vec{n}) d\vec{r} d\vec{n} \quad , \qquad (4)$$

where Ξ denotes the region of interest in the position-direction space. This signal can also be obtained as a solution of the corresponding adjoint problem (Marchuk 1995)

$$P = \int_{\Xi} S(\vec{r},\vec{n})\, \tilde{I}(\vec{r},\vec{n}) d\vec{r} d\vec{n},$$
$$-\vec{n}\cdot\vec{\nabla}\tilde{I}(\vec{r},\vec{n}) + \sigma_e(\vec{r})\tilde{I}(\vec{r},\vec{n}) = \frac{\sigma_s(\vec{r})}{4\pi}\int_{4\pi} P(\vec{r},-\vec{n},-\vec{n}')\,\tilde{I}(\vec{r},\vec{n}')d\vec{n}' + R(\vec{r},\vec{n}). \qquad (5)$$

If one knows both the direct and adjoint solutions of the same problem at the same time then it is possible to estimate (Polonsky and Box 2002) how a small variation of the radiative transfer equation parameters affects the signal *P* in the first order of magnitude of those variation

$$\Delta P = -\int_{\Xi} \tilde{I}(\vec{r},\vec{n})\, \Delta\mathbf{L} I(\vec{r},\vec{n}) d\vec{r} d\vec{n} \quad . \qquad (6)$$

Here **ΔL** is the variation of the operator **L** of the RTE defined as

$$\mathbf{L} = \vec{n}\cdot\vec{\nabla} + \sigma_e(\vec{r}) - \frac{\sigma_s(\vec{r})}{4\pi}\int_{4\pi} d\vec{n}'\, P(\vec{r},\vec{n},\vec{n}') \otimes \quad . \qquad (7)$$

Note that the notation $\otimes$ is used to indicate that the final term is an integral operator, not merely a definite integral.

The main perturbation formulae have an elegant physical meaning. For the sake of demonstration let an extinction coefficient perturbation at point $\vec{r}_p$ be considered. We have that

$$\Delta P = -\int_{4\pi} \tilde{I}(\vec{r}_p,\vec{n})\, \Delta\sigma_e(\vec{r}_p) I(\vec{r}_p,\vec{n}) d\vec{n} \quad . \qquad (8)$$

Here, $\Delta\sigma_e(\vec{r}_p) I(\vec{r}_p,\vec{n})$ can be interpreted as an auxiliary source placed at the point $\vec{r}_p$ which generates changes in the radiance field; $\tilde{I}(\vec{r}_p,\vec{n})$ describes how the emission of this source affects the signal measured by our receiver.

Thus, a similar interpretation of Eq. (6) can be given by introduction of a set of auxiliary sources corresponding to the variation of the radiative transfer operator which perturb the radiance field. Then, we calculate how these auxiliary sources affect the receiver signal. Note that a similar but fully empirical technique called the quasi single scattering approximation was suggested by Dolin and Saveliev (Dolin and Saveliev 1971) and was used extensively especially in image transfer theory (Zege et al. 1991; Dolin and Levin 1995). The perturbation theory provides a more rigorous basis for this approximation.

Independent Column Approximation

We consider the propagation of the Sun's light through the slab which means that the source is independent of the horizontal coordinates. If the boundary condition formulated in form Eq. (6) is also horizontally uniform and, additionally,

$$\begin{aligned} \sigma_e(\vec{r}) &= \sigma_e(z), \\ \sigma_s(\vec{r}) &= \sigma_s(z), \\ P(\vec{r},\vec{n},\vec{n}') &= P(z,\vec{n},\vec{n}'), \\ A(\vec{r},\vec{n},\vec{n}') &= A(\vec{n},\vec{n}'), \end{aligned} \qquad (9)$$

then, Eq.(1) can be simplified

$$\mu \frac{d}{dz} I(\vec{r},\vec{n}) + \sigma_e(z) I(\vec{r},\vec{n}) = \frac{\sigma_s(z)}{4\pi} \int_{4\pi} P(z,\vec{n},\vec{n}') I(\vec{r},\vec{n}') d\vec{n}' + S(z,\vec{n}) . \qquad (10)$$

This is a well-known 1D radiative transfer equation which is generally used in the cloud remote sensing (e.g., see King et al. [King et al. 1997]). Moreover, this equation serves as the base of the independent column approximation (ICA) (Cahalan et al. 1994) with parameters, which depends on horizontal position:

$$\mu \frac{d}{dz} I(\vec{r},\vec{n}) + \sigma_e(\vec{r}) I(\vec{r},\vec{n}) = \frac{\sigma_s(\vec{r})}{4\pi} \int_{4\pi} P(\vec{r},\vec{n},\vec{n}') I(\vec{r},\vec{n}') d\vec{n}' + S(z,\vec{n}) , \qquad (11)$$

where, in $\vec{r} = (x, y, z)$, the first two coordinates are treated as parameters and only the third one is treated as independent variable in the RTE.

Comparing Eqs. (1) and (11), we find that the former can be considered as a perturbation of the later with

$$\Delta \mathbf{L}_{\text{IPA}} = \sin(\theta)\cos(\varphi)\frac{d}{dx} + \sin(\theta)\sin(\varphi)\frac{d}{dy} . \qquad (12)$$

Because of the form of Eq. (12) in the case of the normal illumination and the nadir observation ($S(z,\vec{n})$ and $R(z,\vec{n})$ are axially symmetric) we have

$$\Delta P = 0. \quad (13)$$

This feature has to be expected since the equation we used to obtain the non-perturbed solutions includes no information about the radiance spreading in the horizontal directions. So, the only correction to the IPA this technique can provide is to estimate the strength of the adjacency effect which takes place in the case of the oblique illumination and/or non-nadir observation of a given pixel, which originates from the fact, that to be reflected from a given column, the photons have to enter the cloud into a neighbor pixel. Seemingly, this effect becomes more significant the more oblique illumination is.

Correction to Heating Rates

To demonstrate possibilities provided by the perturbation technique, we shall estimate the error which the ICA makes because of horizontal inhmogeneity of the extinction coefficient. We shall employ two different approaches: the full 3D simulation with SHDOM (Evans 1998) and the perturbation approach discussed in the previous section. Moreover, we shall go further and use the diffusion approximation for the perturbation calculation which is mathematically equivalent to the 2-stream approximation extensively used in climate modeling with the global spectral model. For simplicity we consider only a x-dependent cloud model with the extinction coefficient shown on Figure 1 (it is adapted from the I3RC protocol [Cahalan et al. 2005]).

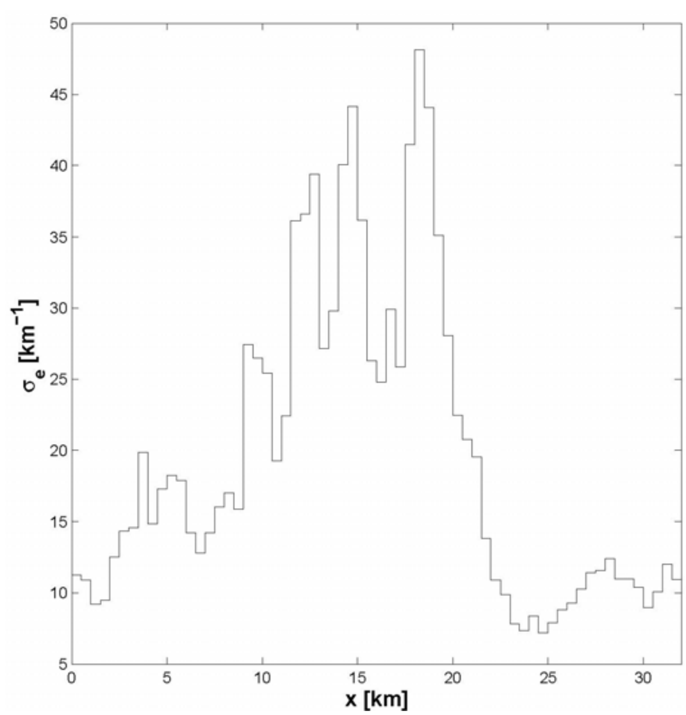

Figure 1.

The results of the both simulations are depicted on Figure 2 showing that our simulation quite accurately recognizes the region at ≈22 km where a substantial cooling can be observed (the absolute value of this effect can be as large as 40% of the whole heating rate estimated using the ICA). The figure shows that the correction estimated using the perturbation approach lacks smoothness which is characteristic to the SHDOM results, but this disagreement is understandable from the point of view how approximate the perturbation approach is, starting from a fully 1D solution. Additionally, the perturbation corrections are more significant near cloud top and tends to zero at the bottom. This is a consequence of the radiance distribution becoming more axially-symmetric the deeper the light penetrates into the cloud.

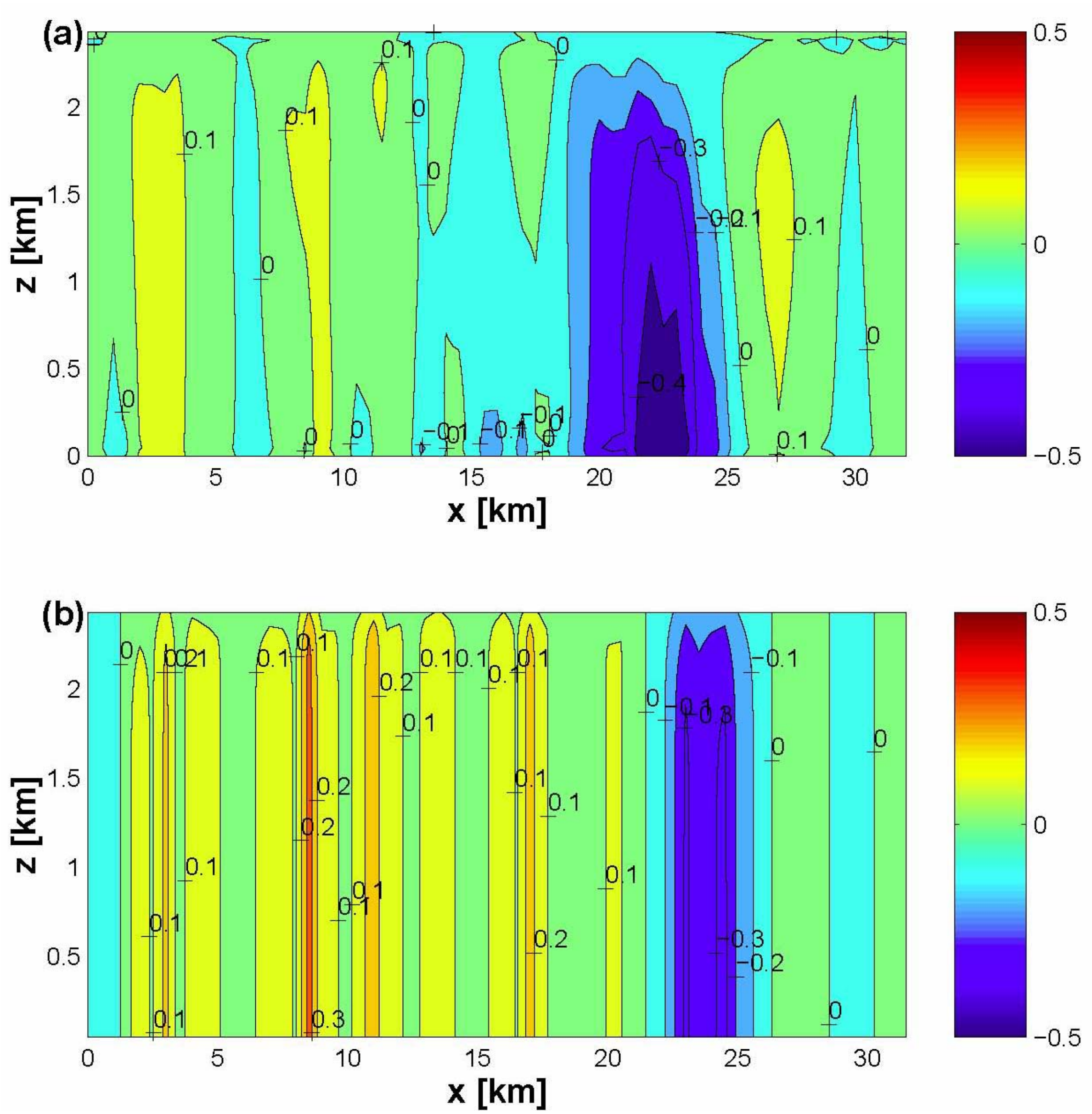

Figure 2. Correction to the IPA for the heating rates calculated by SHDOM (a) and employing the perturbation technique (b).

Conclusion

We have demonstrated how a relatively simple technique – that's extremely efficient computationally – allows one to estimate the strength of the 3D effect preserving the framework of the ICA. For the price of a few extra calculations (comparable to the 2-stream approximation complexity level) one can obtain an important test regarding whether the ICA framework is sufficiently accurate and correct it as needed. We have applied this perturbation technique to the estimation of 3D impact on in-cloud solar photon density (a surrogate for heating rate) which is of direct interest to cloud modelers (e.g., Gu and Liou 2001). Moreover, it can be done for purpose of either solving a direct problem of radiative transfer through the atmosphere or using the ICA framework for the cloud optical properties retrieval.

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