Generalized Expressions for Effective Radius, Cloud Radiative Properties, and Their Application to Studies of the First Indirect Aerosol Effect

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Abstract

Radiative properties of clouds are often expressed as a function of effective radius r_e defined as the ratio of the third to the second moment of the cloud droplet size distribution, and the value of r_e in turn is parameterized as a "1/3" power-law: r_e = a(L/N)^1/3 where L is the cloud liquid water content, N is the cloud droplet number concentration and a is an increasing function of the relative dispersion of the cloud droplet size distribution. We have recently shown that the relative dispersion of the cloud droplet size distribution increases concurrently with the droplet concentration leading to a dispersion effect that significantly diminishes the Twomey effect. However, evaluation of the relationship between the relative dispersion and droplet concentration from ambient data is highly uncertain because differences in cloud dynamics from cloud to cloud, as well as from point to point in a single cloud, cause variations in L as well as N. Here we seek to reduce the uncertainties in the parameterization of dispersion effect caused by cloud dynamics by showing that the parameter is well represented by another power-law: a = b(L/N)^-c. A new power-law that has an exponent (1/3-c) < 1/3 due to the dispersion effect is then obtained for the effective radius. This new power-law, which has (L/N) as the independent variable, is then used to generalize various expressions for studying cloud radiative properties and the first indirect aerosol effect. These new expressions are used to estimate the effect of relative dispersion on the magnitude of the first indirect aerosol effect.

Introduction

Effective radius r_e (defined as the ratio of the third to the second moment of a droplet size distribution) is one of the key variables that are used for calculation of the radiative properties of liquid water clouds. The inclusion and parameterization of r_e in climate models has proven to be critical for assessing global climate change (Slingo 1990; Dandin et al. 1997). It has been demonstrated empirically (Bower and Choularton 1992; Bower et al. 1994; Liu and Hallett 1997; Liu and Daum 2000a), as well theoretically (Liu and Hallett 1997; Liu and Daum 2000b), that r_e can be expressed as a "1/3" power law of the ratio of the cloud liquid water content to the droplet concentration,

$$r_e = \beta\left(\frac{3}{4\pi\rho_w}\right)\left(\frac{L}{N}\right)^{1/3} \quad (1)$$

where r_e is the effective radius, L is the liquid water content, N is the droplet concentration, ρ_w is the water density, and the effective radius ratio β is a dimensionless parameter that depends on the spectral shape of the cloud droplet size distribution. The only difference among different parameterizations lies in the specification of the effective radius ratio β. Improving specification of β and its effect on evaluating indirect aerosol effects is the focus of this study.

Brief Review of Previous Studies

Specification of Effective Radius Ratio β

For clouds with a monodisperse droplet size distribution as described by a delta function $n(r) = N\delta(r-r_e)$, $\beta = 1$. This value of β was used by Bower and Choularton (1992) and Bower et al. (1994) to estimate the r_e of layer clouds and small cumuli. Martin et al. (1994) derived estimates of β of 1.08 for maritime, and 1.14 for continental stratocumulus clouds based upon analysis of in situ microphysical data. These expressions with fixed values of β totally ignore the dependence of β on the spectral shape. Pontikis and Hicks (1992) analytically derived an expression that relates the effective radius ratio β to the relative dispersion. Liu and Hallett (1997) derived another expression for β from the Weibull droplet size distribution which was itself obtained from the recently developed systems theory (e.g., Liu et al. 1995, Liu and Hallett 1998; Liu et al. 2002). Expressions can also be derived from the gamma and lognormal distributions that have been widely used to describe droplet size distributions. Liu and Daum (2000a, b) compared all the existing expressions to observations, and found that the expression corresponding to the Weibull or gamma distributions best describes the dependence of the effective radius ratio β on the relative dispersion, and the two expressions perform almost equally well. The expression corresponding to the gamma droplet size distribution is

$$\beta = \frac{\left(1+2\varepsilon^2\right)^{2/3}}{\left(1+\varepsilon^2\right)^{1/3}} \tag{2}$$

where ε is the relative dispersion.

Thus, the key to further improving the parameterization of r_e is to specify the relative dispersion. It would also be desirable to formulate the parameterization of the relative dispersion in terms of the liquid water content and/or the droplet concentration because these two variables are often predicted/diagnosed in state-of-the art climate models (Rotstayn 1997).

Dependence of Relative Dispersion on Pre-Cloud Aerosols

Recently, we (Liu and Daum 2002) have shown that the addition of anthropogenic aerosols to a marine air mass enhances not only the droplet concentration, but also the relative dispersion. Figure 1 shows the dependence of the relative dispersion on the droplet concentration. The points connected by lines represent cases identified by different investigators (see Liu and Daum 2002 for details) as evidence for the indirect aerosol effect. In each case, the points with lower droplet concentration were characterized as clean clouds and the higher points were characterized as similar clouds that were polluted by anthropogenic aerosols. The increased relative dispersion acts to offset the cooling of the first indirect aerosol effect by as much as 10% – 80% (Liu and Daum 2002), depending on the relationship between the relative dispersion and the droplet concentration. More evidence for the effect of the enhanced dispersion on indirect aerosol forcing has been later reported (Peng and Lohmann 2003).

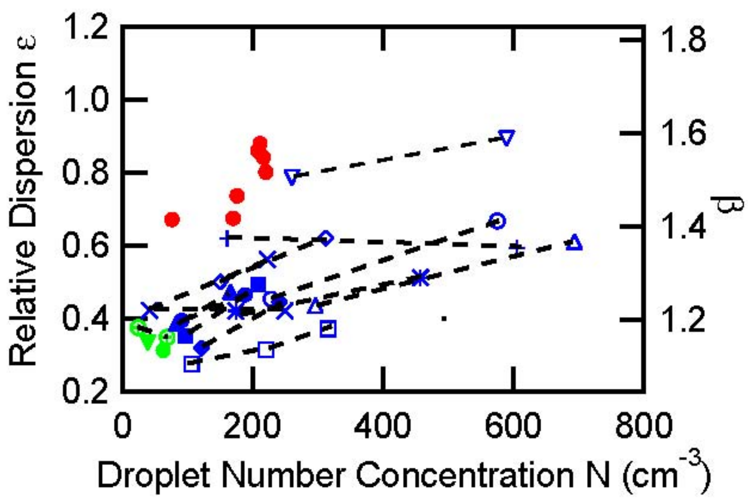

Figure 1. Relationship between the relative dispersion and the droplet concentration. See Liu and Daum (2002) for details about the data.

A New Power Law for Effective Radius Ratio β

Although Figure 1 clearly exhibits a substantial increase in the relative dispersion as the droplet concentration increases, the relationship is noisy. The "noise" likely arises from differences in cloud dynamics such as updraft velocity and turbulence. Cloud-to-cloud differences in liquid water content are also a potential reason for the scatter. It is desirable from Eq. (1) to parameterize the relative dispersion (or β) in terms of the ratio of the liquid water content to the droplet concentration, or specific water content γ. It is noteworthy that the specific water content for a cloud is the equivalent of the temperature for an ideal gas (Liu et al. 1995; Liu et al. 2002). Physically, uniform/regular adiabatic growth tends to cause narrowing of the droplet size distribution as droplets grow, producing a decrease in the relative dispersion with increasing specific water content (i.e., narrowing toward larger sizes). Furthermore, presenting data this way also minimizes the influence of different liquid water content and relaxes the assumption of a constant liquid water content (updraft velocity) used in previous studies of the dispersion effect. In fact, Wood et al. (2000) found that there is a negative correlation between the effective radius ratio β and the volume-mean radius, and that this correlation is better than that between the effective radius ratio β and the droplet concentration alone, suggesting that specifying the relative dispersion or β as a function of the specific water content will improve the parameterization of the relative dispersion.

Figure 2 shows the effective radius ratio β as a function of the specific water content calculated from cloud droplet size distributions measured with a forward scattering spectrometer probe (FSSP). The data come from several projects (North Atlantic Regional Experiment [NARE], ARM 1997 Spring IOP, ARM 1997 Fall IOP, ARM 1998 Spring IOP, ARM 2000 IOP, and FIRE-ACE). Each point in this figure represents a flight average. The result from Wood et al. (2000) (tilted dash) is also shown as a comparison. It is evident from this figure that the relative dispersion generally decreases when the specific water content increases as expected from the uniform adiabatic growth theory. The scatter in this figure is likely caused by differences in turbulent entrainment and mixing processes. Two points can be inferred from this figure. First, similar to the relative dispersion, β generally decreases when the specific water content increases. Second, as a first order approximation, the dependence of β on the specific water content can be approximated by a power-law

$$\beta = a_\beta \left(\frac{L}{N} \right)^{-b_\beta} \tag{3}$$

where the parameter $a_\beta = 0.13$ and the exponent $b_\beta = 0.11$.

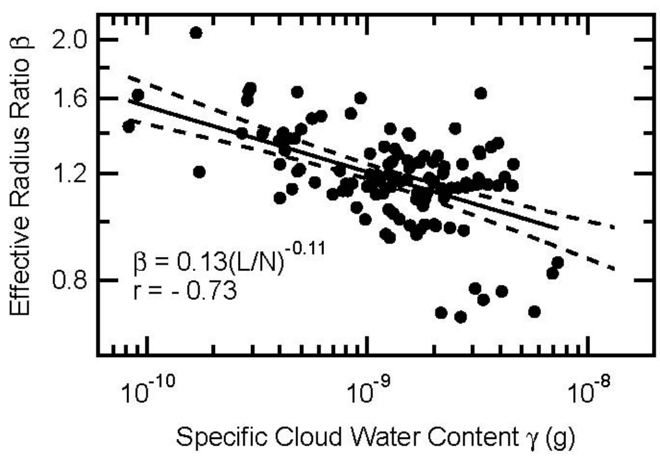

Figure 2. The relationship between the effective radius ratio β and the specific water content. The data are from several projects (ARM 1997 Spring IOP, ARM 1997 Fall IOP, ARM 1998 Spring IOP, ARM 2000 Spring IOP, NARE, and FIRE-ACE). The solid line represents the power-law fit to the data, and the dashed lines bound the 95% confidence interval.

Generalized Power-Law for Effective Radius and Evaluation of Indirect Aerosol Effect

Substitution of Eq. (3) into Eq. (1) yields a generalized power law expression for r_e that accounts for the dispersion effect

$$r_e = a_e \left(\frac{L}{N} \right)^{b_e} \tag{4a}$$

$$a_e = a_\beta \left(\frac{3}{4\pi\rho_w} \right)^{1/3} \tag{4b}$$

$$b_e = \frac{1}{3} - b_\beta \tag{4c}$$

A value of $b_e < 1/3$ due to $b_\beta > 0$ indicates that the dispersion effect results in a weaker dependence of r_e on γ, which in turn leads to a smaller indirect aerosol effect.

This point becomes more evident from the relative measure of the indirect aerosol effect that has been widely used in remote sensing studies of the first indirect aerosol effect,

$$I = -\frac{d \ln r_e}{d \ln N_a}, \tag{5}$$

where N_a is the number concentration of pre-cloud aerosols. It is often assumed that N is related to N_a by

$$N = a_a N_a^{b_a}, \tag{6}$$

where the exponent b_a is a measure of the Twomey effect. A combination of Eqs. (3), (5), and (6) yields

$$I = b_e b_a = \left(\frac{1 - 3b_\beta}{3} \right) b_a = I_0 + I_\varepsilon \tag{7}$$

where $I_0 = 1/3 b_a$ and $I_\varepsilon = -b_\beta b_a$ represent the Twomey effect and the dispersion effect, respectively. The above analysis suggests the first indirect aerosol effect is a sum of the Twomey effect and the dispersion effect, and the magnitude of the dispersion effect is proportional to that of the Twomey effect.

Conclusions

It is demonstrated that specification of the relative dispersion is the key to further improving the parameterization of the effective radius. It is argued from uniform adiabatic growth theory that the relative dispersion and the effective radius ratio should be represented in terms of the specific water content instead of the droplet concentration alone. Microphysical measurements from several projects conducted over continental and maritime air masses are analyzed. The results show that the relative dispersion decreases when the specific water content increases as expected from the uniform adiabatic theory. Furthermore, it is empirically demonstrated that the dependence of β on the specific water content can be approximately described by a power law. A power-law relationship between β and the specific water content leads to a generalized power law effective radius with an exponent less than 1/3 due to the dispersion effect. It is further shown from generalized power-law expression the effective radius that the first indirect aerosol effect is a sum of the competing effects: the Twomey effect and the dispersion effect.

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