Inversion of Multi-Angle Radiation Measurement

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Motivation

Observations of the polarization and reflectance in bands at 865 nm (negligible liquid water absorption) and 2250 nm (strong liquid water absorption) cannot be fitted by a simple atmospheric model consisting of a homogeneous cloud with a single particle size.

If we use the polarization (polarized reflectance) measurements and the reflectance measurements to retrieve cloud particle size independently, we find that it is frequently (although not always) the case that the size retrieved with the polarization measurements is larger than the size retrieved using the reflectance measurements (Figure 1).

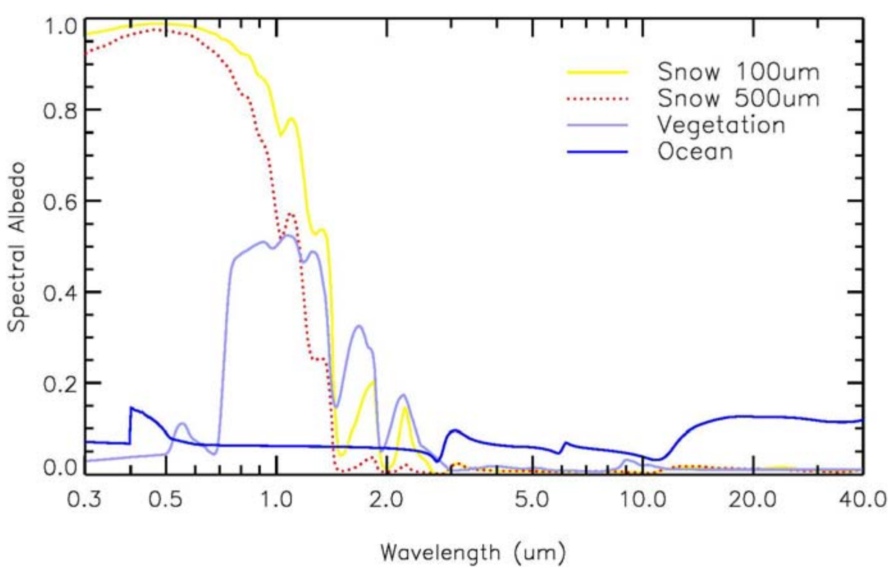

Figure 1. Effective radius retrievals using polarized reflectance (blue) and reflectance with effective variances of 0.05, 0.1, 0.15, and 0.2 (mauve, turquoise, green, red). Polarized reflectance provides effective variance retrieval also. Retrievals use data taken on 25 July 2003 during the CSTRIPE field experiment and use research scanning polarimeter (RSP) data taken at 410, 865, and 2250 nm over stratocumulus clouds near Monterey. Retrievals were validated against in situ measurements and agree to within ±0.5 μm when an appropriate level in the cloud is used for comparison.

When we look at the in situ observations of particle size within the cloud on days when the polarization and reflectance size retrievals differ, we find that there is a strong vertical gradient in particle size. If we then take a simple-minded look at the optical depths that contribute to the observed (polarized) reflectance we find that the polarized reflectance measurements respond to an optical depth of less than 3 while the reflectance measurements respond down to an optical depth of 20 (2250 nm) (Figure 2). The difference in sizes retrieved using polarization measurements and reflectance measurements can therefore be explained by the difference depths in the cloud that generate the two signals and the different particle sizes that are present at those depths.

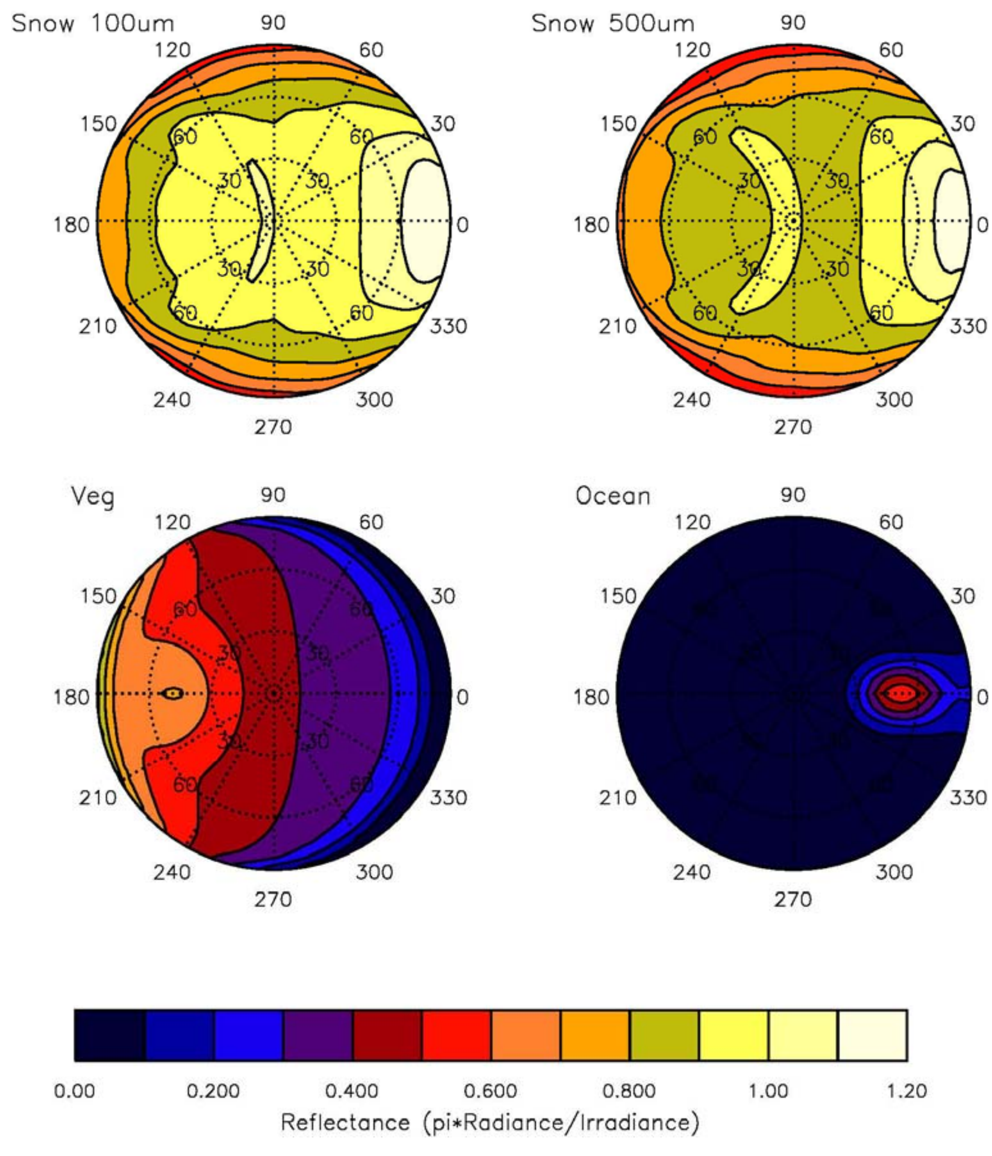

Figure 2. (a) Vertical variation of liquid water content (black line) and effective radius (red line) estimated for a transect through a cloud. Mean effective radius for straight and level legs are shown as red symbols. (b) (Polarized) reflectance ratio (reflectance value normalized by the value for a semi-infinite cloud) as a function of optical depth for a range of different viewing angles.

The question then is whether we can provide a practical method for retrieving the particle sizes in a vertically inhomogeneous cloud that is consistent with all available multi-angle, multi-spectral polarization and reflectance measurements. That is, can we fit the data using a simple model that is consistent with the typically observed vertical variation in cloud droplet size distributions.

Perturbation Theory

The equation of transfer for polarized light can be written in operator form as $\hat{\mathbf{L}}\mathbf{I} = \mathbf{S}$ where the transport operator is

$$\hat{\mathbf{L}} = \mathbf{s}_1.\nabla_1 + \sigma_{ext}(\mathbf{r}_1)N(\mathbf{r}_1)\int_{4\pi} \delta(\mathbf{s}_1, \mathbf{s}_1') - \varpi\frac{\mathbf{P}(\mathbf{s}_1, \mathbf{s}_1')}{4\pi} \bullet d\Omega_1'$$

I is the Stokes vector and S is the source term. The equation for the Green's function of this operator can formally be expressed as $\hat{\mathbf{L}}\hat{\mathbf{G}} = \delta(1,2)$ where δ (1,2) is a Dirac delta function in both space and angle variables. If we can determine the Green's function then the Stokes vector can be evaluated from the expression $\mathbf{I} = \hat{\mathbf{G}}\mathbf{S}$. If we now perturb the transport operator by altering the single-scattering properties of the particles present, or the number density of scatterers and/or absorbers then the effects of this perturbation on the Stokes vector can be expressed as $\left(\hat{\mathbf{L}} + \Delta\hat{\mathbf{L}}\right)\mathbf{I}' = \mathbf{S}$. The perturbed Stokes vector can be expressed as the sum of its unperturbed part I, and a perturbation ΔI. If we now neglect terms that are second order in the perturbation (i.e. the term $\Delta\hat{\mathbf{L}}\Delta\mathbf{I}$), we obtain the final result for the effects of perturbing the radiative transfer equation on the observed Stokes vector

$$\mathbf{I}' = \mathbf{I} - \int_0^{\tau_c} \hat{\mathbf{G}}(0,\tau)\Delta\hat{\mathbf{L}}(\tau)\hat{\mathbf{G}}(\tau,0)\mathbf{S}\,d\tau + O(\Delta^2).$$

where the optical depth variables indicate that this expression is specific to reflection and the two Green's functions are different. One ($\hat{\mathbf{G}}(\tau,0)$) allows the diffuse (and direct) radiation within the layer to be evaluated while the other ($\hat{\mathbf{G}}(0,\tau)$) is the (Mueller matrix generalization of the) "escape" function introduced by Twomey (1979).

Calculating Green's Functions

So, what are the Green's functions in terms of the more usual quantities that are calculated by radiative transfer codes?

$$\mathbf{G}(\tau,0;\mu,\mu';\varphi-\varphi') = \begin{cases} \mathbf{U}(\tau;\mu,\mu';\varphi-\varphi');\mu > 0,\mu' < 0 \\ \mathbf{D}(\tau;\mu,\mu';\varphi-\varphi') + \dfrac{\pi\exp(-\tau/|\mu|)}{|\mu|}\delta(\mu-\mu')\delta(\varphi-\varphi');\mu < 0,\mu' < 0) \end{cases}$$

$$\mathbf{G}(0,\tau;\mu,\mu';\varphi-\varphi') = \begin{cases} \mathbf{U}^{+}(\tau;\mu,\mu';\varphi-\varphi');\mu>0,\mu'<0 \\ \mathbf{D}^{+}(\tau;\mu,\mu';\varphi-\varphi') + \dfrac{\pi\exp(-\tau/|\mu|)}{|\mu|}\delta(\mu-\mu')\delta(\varphi-\varphi');\mu>0,\mu'>0) \end{cases}$$

In these equations, D and U are the usual downwelling and upwelling matrices at a depth τ below cloud top and D^+ and U^+ are the (adjoint) downwelling and upwelling radiation fields that would result if a source was placed in the direction of observation. The adjoint fields have simple relationships to the usual downwelling and upwelling fields that are given by the expressions

$$\mathbf{D}^{+}(\mu,\mu',\varphi-\varphi') = \mathbf{q}_4\mathbf{D}^{T}(\mu',\mu,\varphi-\varphi')\mathbf{q}_4$$
$$\mathbf{U}^{+}(\mu,\mu',\varphi-\varphi') = \mathbf{q}_4\mathbf{U}^{T}(\mu',\mu,\varphi-\varphi')\mathbf{q}_4$$

where the $_{q4}$ matrix is defined to be

$$\mathbf{q}_4 = \begin{Bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{Bmatrix}.$$

Practical Implementation

Discrete ordinates methods provide internal fields but for multi-angle observations calculations must be made for pseudo sources at all the observation angles to calculate the effect of perturbations. A method for the direct calculation of scalar (intensity only) Green's functions has been developed (Benedetti et al. 2002), but its generalization to realistic vertically inhomogeneous atmospheres and the inclusion of polarization (vector radiative transfer) is not trivial. However, doubling/adding calculations provide all the required angular informations and fast and accurate vector adding/doubling codes are readily available. So, if doubling/adding can be used to rapidly calculate the internal radiation field, this approach will allow us to calculate the required Green's functions. In fact, the downwelling and upwelling fields are required in the calculation of reflection and transmission matrices in the doubling/adding method and it is sufficient to save the last N downwelling and upwelling radiation fields from the doubling calculation of a layer in order to be able to calculate the internal field at 2^N-1 levels within the layer. Only simple multiplications of pairs of matrices are required in this calculation. The calculation of internal downwelling and upwelling fields for an inhomogeneous atmosphere with multiple layers is then quite straightforward (de Haan et al. 1987).

Weighting and Contribution Functions

The Green's functions can be used to calculate the effects of perturbations on the observed radiation field and also to examine the vertical weighting within a scattering medium that contributes to the observed field. The contribution function $C(\tau)= G(0,\tau)S(\tau)$ has been introduced by Benedetti et al. (2002). This function has the useful property that it satisfies the equation $\mathbf{R} = \int \mathbf{C}(\tau)d\tau$, which provides a useful check on the accuracy of the Green's function calculation. A natural definition of a weighting function is therefore $W(\tau)=C(\tau)/R$, which was introduced by Benedetti et al. (2002). This definition is however a little misleading. It tells you what fraction of light that experienced its first scattering at an optical depth of τ returned to the surface. For strongly forward-scattering media like clouds this is not the same as the contribution that light that has reached an optical depth τ makes.

A more informative weighting function can be obtained by using the perturbation approach outlined. The relative perturbation in reflectance caused by perturbations to the effective radius of the particle size distribution can be written as

$$\left.\frac{\Delta\mathbf{R}}{\mathbf{R}}\right|_{reff} = \int\left[\mathbf{W}_{Phase}(\tau) + \frac{\partial \ln \varpi(\tau)}{\partial r_{eff}}\mathbf{W}_{\varpi}(\tau) - \frac{\partial \ln \sigma_{ext}(\tau)}{\partial r_{eff}}\mathbf{W}_{ext}(\tau)\right]\Delta r_{eff}(\tau)d\tau$$

where $W_{\omega}(\tau)=G(0,\tau)P(\tau)G(\tau,0)/R$, $W_{ext}(\tau)=G(0,\tau)[I-P(\tau)]G(\tau,0)/R$ and $W_{Phase}(\tau)=G(0, \tau)[\delta P(\tau)/\delta r_{eff}]G(\tau,0)/R$ are the new weighting functions. One of the weighting functions (W_{ext}) gives the vertical weighting of perturbations associate with changes in the extinction cross section and/or number concentration of particles, while W_{ω} gives the vertical weighting of perturbations associated with changes in the single-scattering albedo (Platnick 2001) and W_{phase} provides the vertical weighting of contributions to the observed reflectance matrix from perturbations in the phase matrix cause by changes in particle size. The perturbation to the reflectance caused by changing the number concentration of particles can also be calculated and only depends on the weighting function W_{ext},

$$\left.\frac{\Delta\mathbf{R}}{\mathbf{R}}\right|_{N} = \int \mathbf{W}_{ext}(\tau)\Delta \ln N(\tau)d\tau.$$

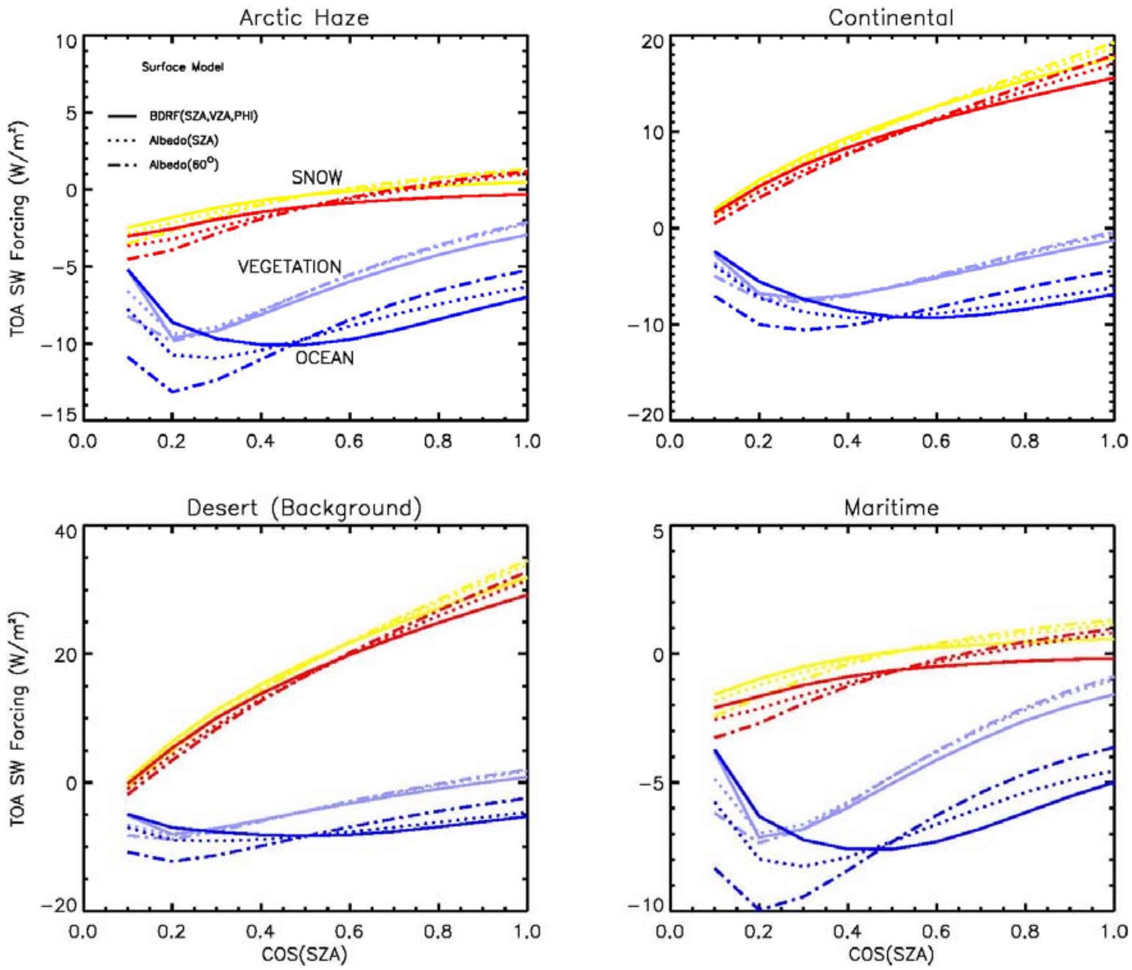

Figure 3. (a) Vertical distribution of relative contributions to reflectance from variations in extinction (solid lines) and single-scattering albedo together with phase matrix (dashed lines) caused by variations in effective radius from its base value of 10 µm. (b) Similar to a) but for relative contributions to polarized reflectance. Height of lines defines optical depth for calculation (4, 8, and, 16 @ 550 nm) and color indicates spectral band (red is 865 nm and black is 2250 nm).

In Figure 3, we show the contributions to perturbations in the reflectance and polarized reflectance from different depths in the cloud that are caused by varying (the vertical profile of) particle size. It is clear that the polarized reflectance is defined by the particle size near cloud top (optical depth less than three) in both spectral bands. The reflectance at 2250 nm is sensitive to particle size primarily because of the effect on single-scattering albedo for clouds with an optical depth of eight and larger, while perturbations to the reflectance at 865 nm by particle size are primarily the result of variations in extinction.

Thus, to fit multi-spectral, multi-angle polarimetric data we first fit the polarized reflectance by varying the particle size distribution. This defines the particle size at cloud top. We then perturb the particle size deeper into the cloud, either by varying the vertical profile of size, or using a two vertical layer

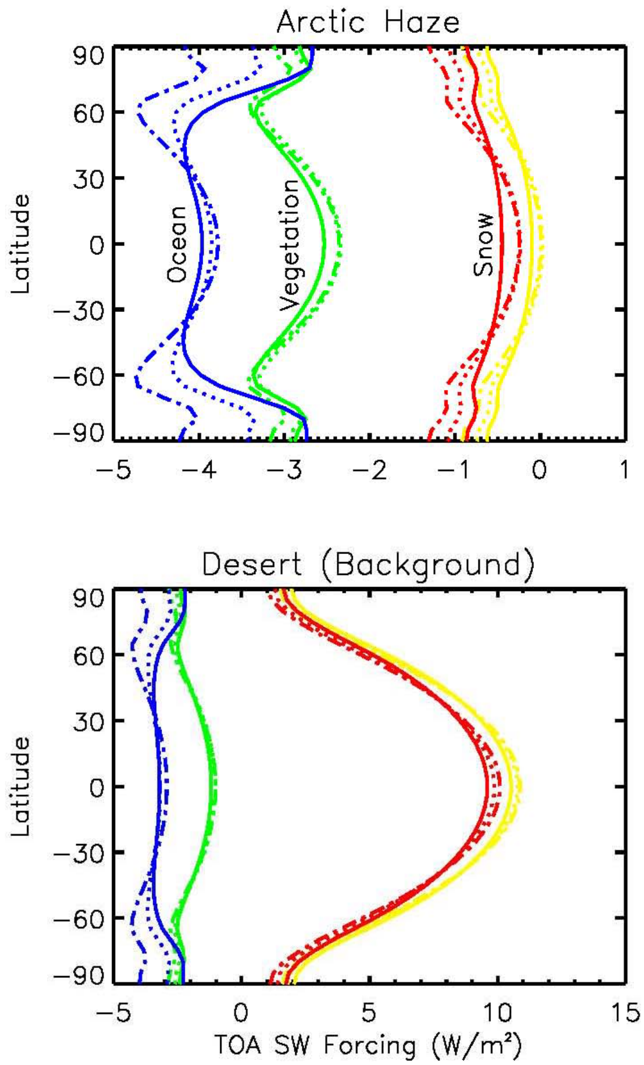

Figure 4. Comparison of data, show as circles (error bars show standard deviations for 1 km sample) with (a) single-layer cloud model, (b) two-layer cloud model.

model of the cloud, to reconcile the measured reflectance at 2250 nm. To fit the reflectance at both 865 nm and 2250 nm, it is also necessary to vary the particle number concentration to compensate for the effects of particle size on extinction at 865 nm. We assume that the particle size concentration is fixed with height.

Figure 4 shows an example of the differences between the fits to a dataset using a single-layer cloud model and a two-layer cloud model. As the degree of linear polarization is a ratio of polarized reflectance to reflectance it is sensitive to the cloud drople size both at the top and deeper into the cloud. It is only by allowing a two-layer model of cloud particle size that we obtain a reasonable fit (fitting errors comparable to expected measurement errors) between model and measurements.

Conclusions

Our need to reconcile models and measurements in an efficient manner that allows for the operational retrieval of particle sizes for a two-layer cloud led us to develop a new method for calculating the Green's functions for radiative transfer. The method uses the fact that doubling/adding codes can be easily used to calculate internal radiation fields at arbitrarily high resolution. We also have determined that the adjoint downwelling and upwelling vector radiation fields are simply related to the usual downwelling and upwelling vector radiation fields so that the entire Green's function can be determined from a single calculation. The Green's functions have then been used to calculate the particle sizes in a two-layer cloud that are consistent with both the reflectance and polarization measurements. This approach may be of use in other applications where adjoint calculations are used, particularly if multi-angle measurements are being analyzed.

References

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