<u>ON the EXPANSION of the UNIVERSE</u>

Albert Einstein (1879-1955) predicted a general expansion of the universe from his General Theory of Relativity. In fact, expansion or collapse (deflation) seemed an almost inevitable result of his original field equations. Einstein, however, did not believe that either of these results could be correct, since neither was supported by astronomical observations at the time that he had obtained them. The prevailing concept was one of a static island universe in which star motions were, to some extend, random, similar to the motions of the constituent particles in a gas.

Einstein, therefore, rewrote his field equations by adding another, wholly arbitrary, term – the so-called *cosmological term* – which enabled a static solution. This term was, by and large, a repulsion term. The effects of the new term could be directly predicted, and were found to be such as to be noticeable only over vast extents of time and space, vast even when compared with intergalactic distances; they were not felt at all locally. This characteristic was certainly a desirable one, since neither did any observational evidence exist to support its presence in the field equations.

Sometime later, when late astronomer, Sir Edwin Hubble (1889-1953), began to apply the spectroscope to distant galaxies, he observed a persistent red shift in the spectra of known elements. He also observed that the shift was greater the greater the distance of the galaxy from the Earth. Numerous astronomers offered explanations for the shift, including the reddening effects of interstellar or intergalactic dust, the possibility of a new and unknown mechanism operating at great distances from the Earth, and so on. But none of these interpretations rang true; they either failed to correctly interpret the data, or they failed on purely theoretical grounds, or both.

It was Hubble himself who correctly explained the red-shift as indicating that distant galaxies were radially moving away from the Earth. In every direction, these vast accumulations of stars and interstellar matter were moving outward at enormous speeds. He called this motion, *recession*. He showed that the velocity of recession was greater at greater distances. He also showed that this particular type of recession was consistent with the concept of a general *expansion* of the universe. We will examine this last point in more detail in just a little bit.

When Einstein heard of Hubble's results, he retracted his cosmological term, and declared it to have been one of his worst mistakes. Einstein had been forced to realize, after the fact, that he should have believed the results of his own theory rather than doubting them to the point of adding such an *ad hoc* term to the field equations. He had, in fact, predicted an, as yet, undiscovered cosmological effect of great importance. None-the-less, Einstein had also acted in good faith to the times when he added the term. A theory must be consistent with known facts after all.

After observing many galaxies, Hubble was able to quantify his results. His law of cosmic expansion states that an observer *at any point in the universe* will observe distant galaxies receding from him/her with *radial velocities proportional to their distance from the observer*. In mathematical notation, let **V** be the velocity of recession of a particular galaxy, and **S** be the distance of that galaxy from the observer who measures **V** (by measuring the red shift and doing some mathematics). Then **V** and **S** are related, to a first order, by a linear equation:

$$\mathbf{V = HS}$$

that is, the velocity of recession of a galaxy is *linearly proportional* to the distance of the galaxy from the observer through a constant **H**, called Hubble's constant. Hubble's constant is still being worked on to this day. At present, there is a range of measured values within which the actual constant is believed to lie.

Hubble's law is consistent with a general expansion of the space between galaxies (or galactic clusters), and is not a particular characteristic of the galaxies (clusters) themselves. This statement means that the galaxies themselves are not changing in any way; only the regions between them are expanding with time. If the expansion is run backward (as can be done with mathematics), then it would appear that, very long ago, all the matter of the universe was once compacted into a relatively small volume from which it was hurled outward by some titanic force. This idea is the basis for the *Big Bang*.

An interesting consequence of Hubble's law is that, if one looks out far enough, he/she will perceive the radial velocities of recession as approaching the speed of light. In other words, there is a finite value of **S** for which $\mathbf{V = c}$. This result in interesting because it implies that, for every observer in the universe, there is a unique, spherical horizon of which he/she is the center. The horizon is located at a distance $\mathbf{S = c/H}$. Since objects beyond the horizon would be receding from the observer at speeds greater than that of light, in violation of Special Relativity, there must exist a cosmic sensor that prevents the observer from ever seeing beyond his/her particular horizon.

The sensor is nothing more or less than the red shift itself. From the observer's point of view, the galactic red shift increases linearly with radial distance. Thus, the farther out an observer tries to look, the redder the universe will appear, until at immense distances it will no longer be visible at all, the light having all been red-shifted into the radio spectrum and beyond. Equivalently, the light frequency ν of photons from distant galaxies linearly *decreases* with increasing radial distance, becoming zero at $\mathbf{S = c/H}$. We know, from quantum mechanics, that the energy of a photon is $\mathbf{h\nu}$, where **h** is Planck's constant and is equal to $\mathbf{6.625 \times 10^{-27}}$ **erg-sec**. Thus, the energy received from more and more distant galaxies drops off with radial distance and becomes zero at $\mathbf{S = Hc}$.

Since we do not often allow for negative energy[1], we must conclude that there exists a natural horizon at $\mathbf{S = c/H}$ beyond which the observer may obtain no further information. But, what about an observer who is stationed right on *our* horizon? Well, he/she sees a locally normal universe that becomes progressively more and more red-shifted with distance. From this new observer's point of view, we are infinitely red-shifted and vanishing beyond his/her horizon. The new observer can see portions of the universe that we cannot; and we can see portions of the universe that he/she cannot. Such are the consequences of Hubble's law of cosmic expansion coupled with Einstein's Special Theory of Relativity!

[1] Binding energy is an exception. It is negative by convention, indicating the presence of an energy well.

It is not difficult to see how the interpretation of Hubble's law as cosmic expansion comes about. For a simple heuristic model, consider a hypothetical *two-dimensional universe* confined to the surface of a sphere[2] of radius **ρ**. The 2-dimensional universe is certainly curved – in fact, it is a *closed* universe. A line (great circle – beam of light) drawn in any direction will eventually return onto itself. But its center and radius are not contained within itself; rather, they exist in another higher-dimensional reality (our own 3-dimensional world), which is entirely inaccessible to the sphere's 2-dimensional inhabitants.

Now, place a 2-dimensional observer at any point **P** on the sphere, and place a 2-dimensional galaxy at a second point **P*** not identical with **P**. The *distance* **S** between **P** and **P***, *measured along the surface of the sphere,* may also be measured by the angle **θ**, subtended by **P** and **P***, at the center of the sphere in our 3-dimensional world. To the inhabitants of the hypothetical universe, the number represented by **θ** for any particular galaxy is obtainable from astronomical measurements, but is not necessarily treated in terms of a higher dimensional geometry[3]. It is treated, rather, as an abstract numerical measure related to the distance – a type of metric – between **P** and **P***. **S** and **θ**, in turn, are related by another "constant of proportionality," **ρ**.

Each galaxy will have its own specific metric **θ** related to its distance **S** through the constant **ρ**. A 2-dimensional mathematical physicist would use formal geometry in his/her 2-dimensional universe to write, for any galactic distance **S**:

$$\mathbf{S} = \boldsymbol{\rho\theta}$$

If they are anything like us, our 2-dimensional observers might measure the distance **S** in km. Let us say, for argument's sake, that they also measure the "constant" **ρ** in km. They would do so for purposes of convenience, perhaps. Finally, they would consider the metric **θ** to be an abstract, dimensionless quantity.

Now, let's complicate their world a little. Let's suppose that their 2-dimensional universe were not static, but uniformly expanding, like a balloon *inflating* in our 3-dimensional world. Then **ρ** would become a function of time, **ρ** = **ρ(t)**, increasing in value with increasing time, so that $\mathbf{D_t}\boldsymbol{\rho}$ (the time derivative of **ρ**) would have a positive value.

For OUR convenience, let us further suppose that **θ** is a *fixed* value for any given observer **P** and any given galaxy **P***. Thus, the 2-dimensional galaxies themselves are not actually moving about on the surface of the sphere or changing in any other way; only the space between them is increasing. From the point of view of an observer in the 2-dimensional universe, the distance **S** between **P** and **P*** will be seen as *increasing*:

$$\mathbf{D_t S} = \boldsymbol{\theta}\ (\mathbf{D_t}\boldsymbol{\rho}) > 0$$

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[2] Many authors suggest a balloon to represent cosmic space, with tiny spots painted on it to represent galaxies. In this case, the 2-dimensional universe is the *skin* of the balloon.

[3] Of course, the 2-dimensional mathematicians are aware of the existence of higher dimensional geometries as theoretical entities, but they know of no physical model that can be built of one. The 2-dimensional physicists, except for a few called *Relativists*, prefer to leave these abstract ideas alone and to work along more mundane lines!

In fact, for every **P** and **P***, this increase of **S** will be seen by **P** as a velocity **V** of *recession*. No matter where in the universe a 2-dimensional observer might be, he/she would see all the distant galaxies moving away from him/her.

Next, let us suppose that any variation in $\mathbf{D_t\rho}$ is negligible over a suitably long period of observational history[4]. This assumption is consistent with what we know of our OWN universe. Then, to a first approximation, $\mathbf{D_t\rho \sim k}$ (a constant), and we may write:

$$\mathbf{V = k\theta},$$

that is, the velocity of recession **V** is proportional to the metric $\boldsymbol{\theta}$. Finally, we take $\boldsymbol{\theta} = \mathbf{S}/\boldsymbol{\rho}$, and again assume that the variation in $\boldsymbol{\rho}$ is negligible over a suitably long period of observational history[5]. We may then write a version of Hubble's law for the 2-dimensional universe:

$$\mathbf{V = HS}$$

This law expresses the notion that the velocity of recession of any distant galaxy in the 2-dimensional universe, relative to some observer, is directly proportional to the galaxy's distance from an observer, to a first approximation. This is the same Hubble's law given earlier for *our* universe. The term **H** ($\mathbf{= k/\rho}$ in our 2-dimensional analog) represents Hubble's constant.

Hubble's constant has actually been estimated from observations of distant galaxies in the 20th century. The currently accepted value is in the range, **50 – 100 km/(sec Mpc)**. In this form, the distance **S** is measured in megaparsecs (**1 Mpc = 3.26 × $\mathbf{10^6}$ light years**), and the recessional velocity **V** is measured in kilometers per second. Whether this value represents a genuine universal constant or a number that varies slowly with time (and location?) is currently unknown.

There are several interesting consequences derivable from Hubble's constant. First, let us rewrite it with a change in units from Mpc to km. We obtain: $\mathbf{H = 1.6 \times 10^{-18}}$ **/sec – 3.2 ×** $\mathbf{10^{-18}}$ **/sec.** If we invert these values, we obtain $\mathbf{H^{-1} = 6.3 \times 10^{17}}$ **sec – 3.1 ×** $\mathbf{10^{17}}$ **sec ⇒ 10B – 20B years.** Some theorists have taken a number in this range to represent the age of the universe. Most books give a nominal value of **15B years** for the age of the universe.

We may also calculate the distance to our observational horizon. Let us use an average value for **H: H = 75 km/(sec Mpc)**. We know that

$$\mathbf{S_{Horizon} = c/H = (3 \times 10^5\ km/sec)/(75\ km/(sec\ Mpc)}$$

$$\mathbf{S_{Horizon} = 4{,}000\ Mpc = 1.3 \times 10^{10}\ light\ years = 13B\ light\ years!}$$

With NASA's Hubble Telescope, we can now see out to almost ***10 billion light years***. Thus, we can see about 75% of the way out to our own observational horizon. Using the Hubble Telescope, we estimate that there are about **5 ×** $\mathbf{10^{21}}$ **stars** in the observable

[4] Suppose that in some interval of time Δt (the period of observational history mentioned in the text), $D_t\rho$ increases or decreases by an amount $\Delta(D_t\rho)$. We are simply assuming that $\Delta(D_t\rho)/(D_t\rho) << 1$.
[5] Suppose that in the same interval of time Δt used in the previous note, ρ increased by $\Delta\rho = (D_t\rho)\Delta t$. Again, we are simply assuming that $\Delta\rho/\rho << 1$. These assumptions are consistent with the results of modern observational astronomy and modern astrophysics and cosmology.

universe! Let us calculate an average stellar number density (number of stars per unit volume of space). This number is only a rough estimate since the stars are not uniformly distributed throughout space but are clustered into galaxies. None-the-less, the result will be interesting and informative.

The volume of a sphere with radius **r** equal to $\mathbf{10 \times 10^9}$ **light years** is:

$$\mathbf{(4/3)\pi r^3 = 4 \times 10^{30}\ \text{cubic light years}}$$

Thus, we may expect an average stellar density of

$$\mathbf{(5 \times 10^{21}\ \text{stars})/(4 \times 10^{30}\ \text{cubic light years}) = 10^{-9}\ \text{stars per cubic light year}}$$

Remember, if you did this same calculation inside of a single galaxy, the value would be *considerably* higher.[6]

Now, let us estimate the number of stars in the *observable* universe. The volume of the observable universe is

$$\mathbf{(4/3)\pi(S_{Horizon})^3 = 9 \times 10^{30}\ \text{cubic light years}}$$

This value, combined with the stellar density, gives a rough numerical estimate of the number of stars in our observable universe:

$$\mathbf{(10^{-9}\ \text{stars per cubic light year}) \times (9 \times 10^{30}\ \text{cubic light years}) = 9 \times 10^{21}\ \text{stars}}$$

$$\mathbf{\sim 10^{22}\ \text{stars}}$$

We can go one more step, and estimate the *mass* of the observable universe. We will take the solar mass, $\mathbf{3 \times 10^{30}}$ **kg**, to be an estimate of the average mass of a star. There are stars more massive, to be sure, but there is also an immense population of dwarf stars. We are probably not off by more than an order of magnitude or so in this particular estimate. Using this value, we estimate the total mass of star-matter in the observable universe to be

$$\mathbf{(\sim 10^{22}\ \text{stars}) \times (3 \times 10^{30}\ \text{kg/star}) = 3 \times 10^{52}\ \text{kg}}$$

What can we say of the future history of our universe? Well, we can only speak in the broadest, most general terms at this stage of our understanding. Numerous models exist, based on General Relativity. But none of them have yet been verified by real data.

The first question we might ask is whether the universe will continue to expand. Essentially, this question boils down to asking whether there is sufficient mass in the universe to slow the expansion or even to reverse it through the action of gravitation. Whether the universe (i.) continues to expand, (ii.) expands to a certain limit then stops, or (iii.) expands to a certain limit then stops then begins to contract upon itself again, in the main, depends upon two factors:

a.) The cosmological model you are dealing with (i.e., whose theory do you favor?), and...

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[6] Our galaxy is roughly a disc with a radius of about 50,000 light years and a thickness of about 2,000 light years. Its volume is, therefore, approximately 2×10^{13} cubic light years. Our galaxy is estimated to have about 400,000,000,000 (400B) stars. The stellar density within our galaxy is, therefore, of the order 0.02 stars per cubic light year.

b.) The average mass density taken over the entire [observable] universe (a quantity sought by observational astronomers today).

The average mass density relates to whether there is sufficient total binding energy associated with gravity to keep the universe together: i.e., is this binding energy negative, zero, or positive? The case of negative binding energy favors expansion followed by collapse. The case of zero binding energy favors expansion with an ultimate "stop." The case of positive binding energy favors expansion that continues throughout infinite time.