A Note on the Centrifugal and Coriolis Accelerations as Pseudo Accelerations

Let's begin with a Cartesian coordinate system K at rest in Newton's absolute space. Let **v** be the velocity vector of a particle in K. We may represent **v** in the usual way:

$$\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$$

where **i, j, k,** are unit vectors along the x, y, and z axes, and v_x, v_y, and v_z are the x, y, and z components of the velocity, respectively.

Let us now assume that a force is acting and inquire into the acceleration **a** of the particle in K. The acceleration is merely the time derivative of the velocity:

$$\mathbf{a} = d\mathbf{v}/dt = (dv_x/dt)\mathbf{i} + (dv_y/dt)\mathbf{j} + (dv_z/dt)\mathbf{k}$$

Notice that the derivative operator d /dt "attaches" only to the x, y, and z components of the velocity but not to the unit vectors **i, j,** and **k**. Why is this? It is because K is stationary in absolute space. Since the unit vectors are attached to K, they are stationary also. The derivatives dv_x/dt, dv_y/dt, and dv_z/dt are the x, y, and z components of the acceleration due to the force. We may therefore write:

$$\mathbf{a} = d\mathbf{v}/dt = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$

where $a_x = dv_x/dt$, $a_y = dv_y/dt$, and $a_z = dv_z/dt$.

OK. Now let's introduce a second Cartesian system K*. We will let the origins of K and K* coincide, although this condition is merely a convenience. Assume that K* is rotating with respect to K (and, therefore, with respect to absolute space). There are unit vectors **i*, j*,** and **k*** in K* just as there were unit vectors **i, j,** and **k** in K. Since these unit vectors are attached to K*, they must also be rotating with K*.

Let us now write the velocity **v*** of a particle in K* (let it be the same particle that we used previously; then, we will be able to its compare motion as viewed from K and from K*):

$$\mathbf{v^*} = v_x^*\mathbf{i^*} + v_y^*\mathbf{j^*} + v_z^*\mathbf{k^*}$$

where the starred terms have similar definitions in K* as were given before in K.

If we now seek the acceleration **a*** of the particle in K*, we must again differentiate, but this time with a somewhat different outcome:

$$\mathbf{a^*} = d\mathbf{v^*}/dt = \mathbf{[}(dv_x^*/dt)\mathbf{i^*} + (dv_y^*/dt)\mathbf{j^*} + (dv_z^*/dt)\mathbf{k^*]}$$
$$+ \mathbf{[}v_x^*(d\mathbf{i^*}/dt) + v_y^*(d\mathbf{j^*}/dt) + v_z^*(d\mathbf{k^*}/dt)\mathbf{]}$$

Or:

$$\mathbf{a^*} = \mathbf{[}a_x^*\mathbf{i^*} + a_y^*\mathbf{j^*} + a_z^*\mathbf{k^*]} + \mathbf{[}v_x^*(d\mathbf{i^*}/dt) + v_y^*(d\mathbf{j^*}/dt) + v_z^*(d\mathbf{k^*}/dt)\mathbf{]}$$

Notice the additional set of terms on the right, involving the derivatives of the unit vectors **i*, j*,** and **k***. (I have enclosed two different groupings of terms each in square brackets **[]**.) These derivatives are non-zero because of the rotation.

The first set of terms is the acceleration due to the imposed force, and is the analogous to the acceleration that we found in K:

$$\mathbf{a_f*} = a_x*\mathbf{i*} + a_y*\mathbf{j*} + a_z*\mathbf{k*}$$

I have appended the subscript “f” merely to remind us that THIS acceleration is due to the imposed force.

The second set of terms has a different interpretation. Such terms do not occur in K. If they did, we might argue that they also must represented accelerations due to an imposed force. But we cannot. This second set of terms arises solely due to the rotation of K*.

The second set of terms, in fact, resolves into the centrifugal and Coriolis accelerations that are observed only in rotating frames of reference:

$$\mathbf{a*_{Cent.}} + \mathbf{a*_{Cor.}} = v_x*(d\mathbf{i*}/dt) + v_y*(d\mathbf{j*}/dt) + v_z*(d\mathbf{k*}/dt)$$

I will not include the details of the derivation at this time. I will, however, make note of some important facts implicit in the above expression:

- The combined centrifugal and Coriolis accelerations are dependent on the particle’s velocity in K*. Notice the presence of the terms v_x, v_y, and v_z. If these terms all vanished, the velocity in K* would be zero, and there would be no centrifugal and Coriolis accelerations regardless of the values of the derivatives. It is only when these velocities have real number values apart from zero that the centrifugal and Coriolis accelerations may arise. The larger the v’s, the larger the accelerations.
- The combined centrifugal and Coriolis accelerations are also dependent on the angular velocity of K*, as represented by the derivatives d**i***/dt, d**j***/dt, and d**k***/dt. If these terms all vanished, the rotation would cease, and there would be no centrifugal and Coriolis accelerations regardless of the particle’s velocity. It is only when these terms become non-zero that centrifugal and Coriolis accelerations may arise. The larger the derivatives, the larger the accelerations.

The centrifugal and Coriolis accelerations do not arise as a result of physically imposed forces, but as a result of a special property of the system K* - i.e., the property of rotation. Thus, these accelerations are called pseudo-accelerations, or “false” accelerations. They are merely an artifact of the rotation.