

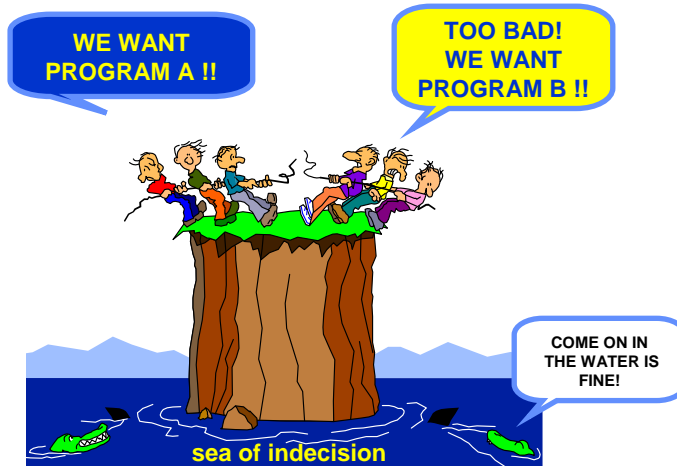
# An Illustrated Guide to the *ANALYTIC HIERARCHY PROCESS*

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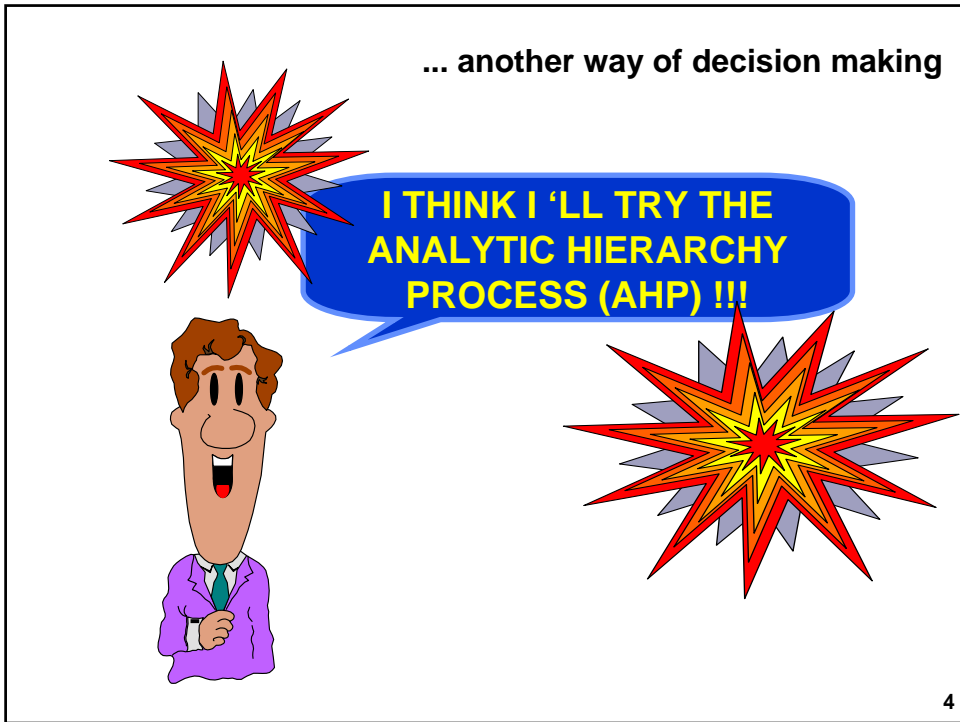
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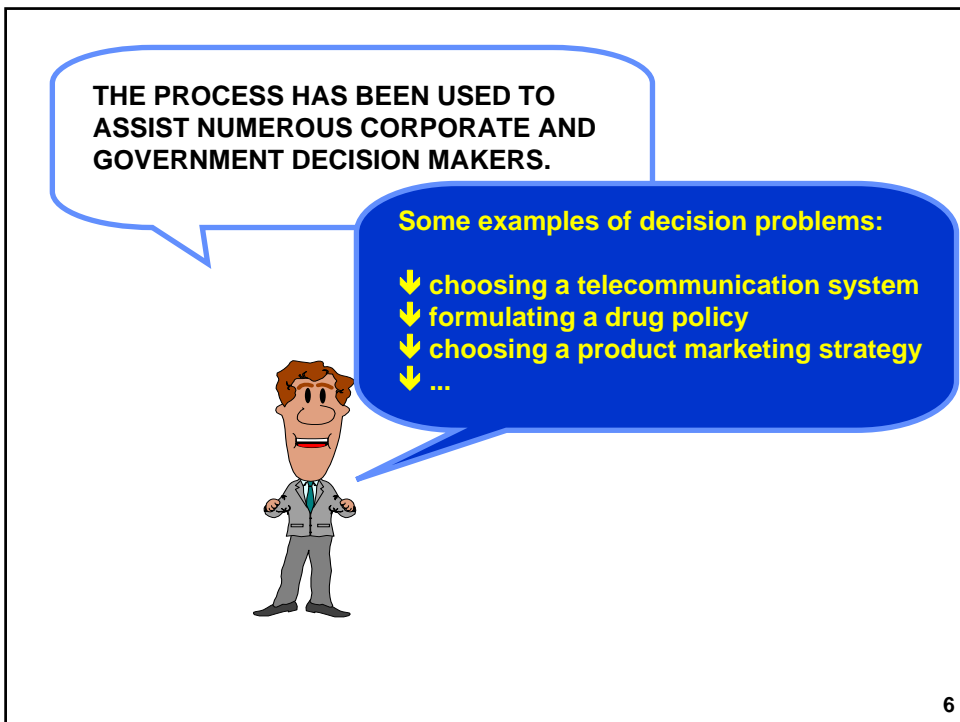
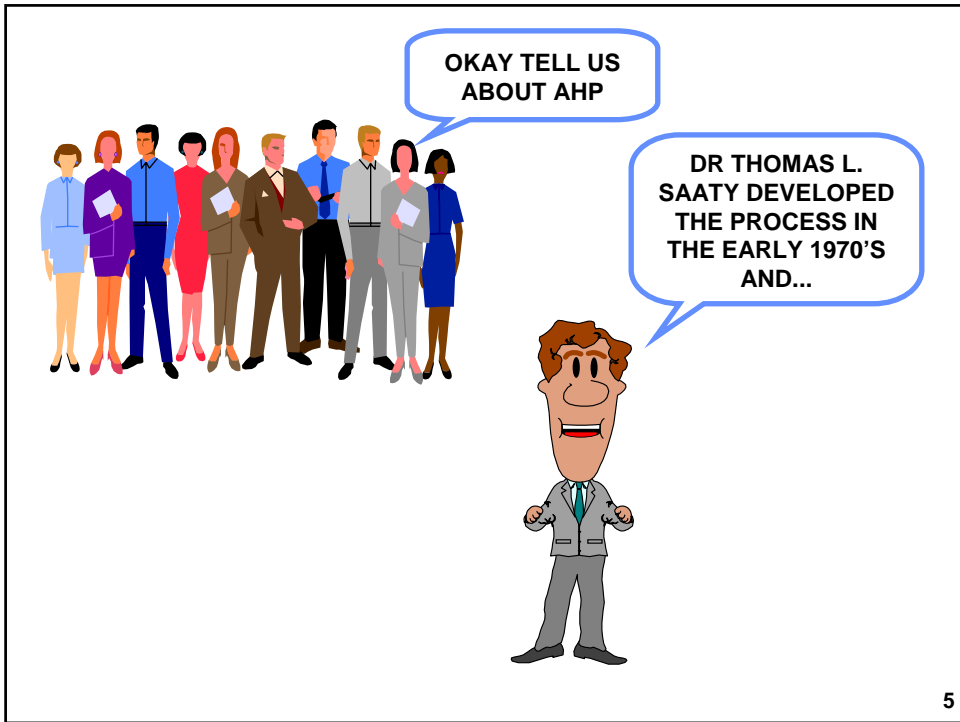
Do your decision conferences turn out like this?

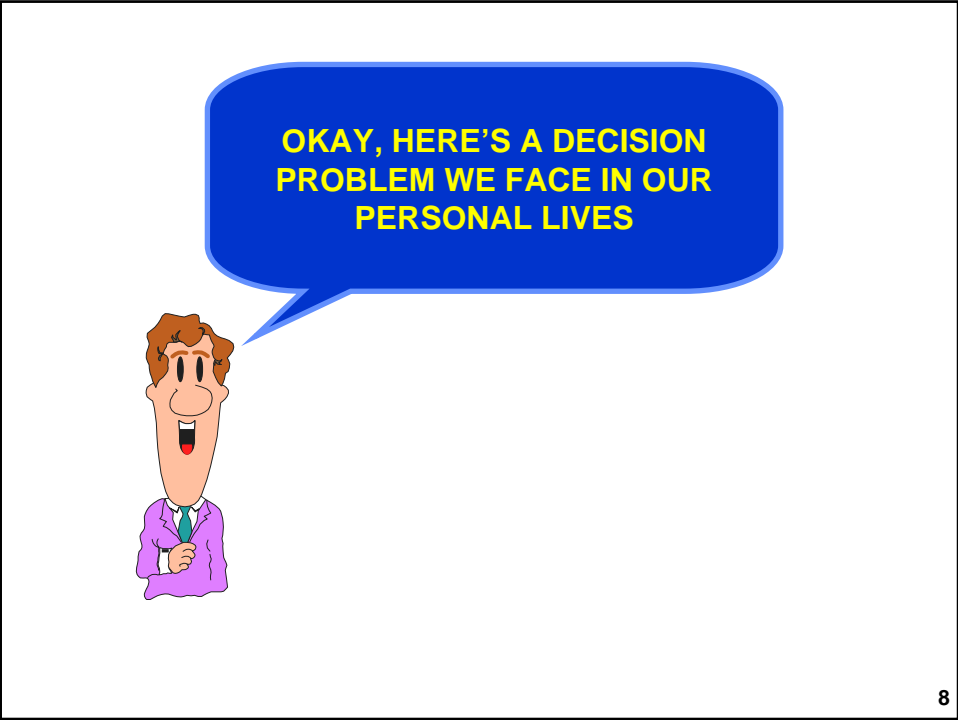
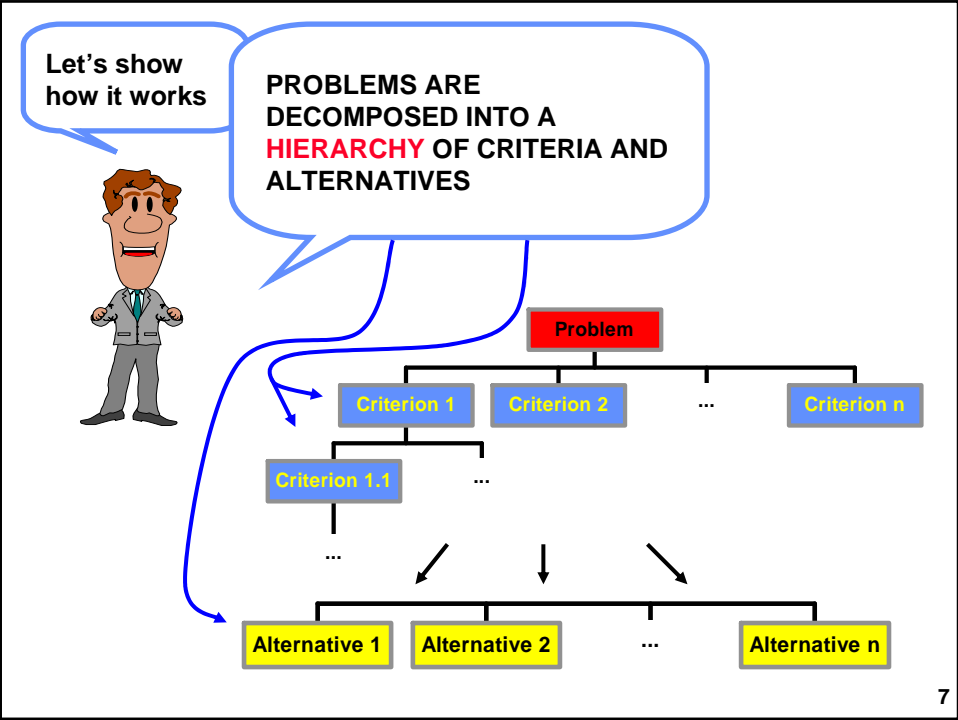


or does this happen?

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AN IMPORTANT PART OF THE PROCESS IS TO ACCOMPLISH THESE THREE STEPS



- **STATE THE OBJECTIVE:**
  - SELECT A NEW CAR
- **DEFINE THE CRITERIA:**
  - STYLE, RELIABILITY, FUEL ECONOMY
- **PICK THE ALTERNATIVES:**
  - CIVIC COUPE, SATURN COUPE, FORD ESCORT, RENAULT CLIO

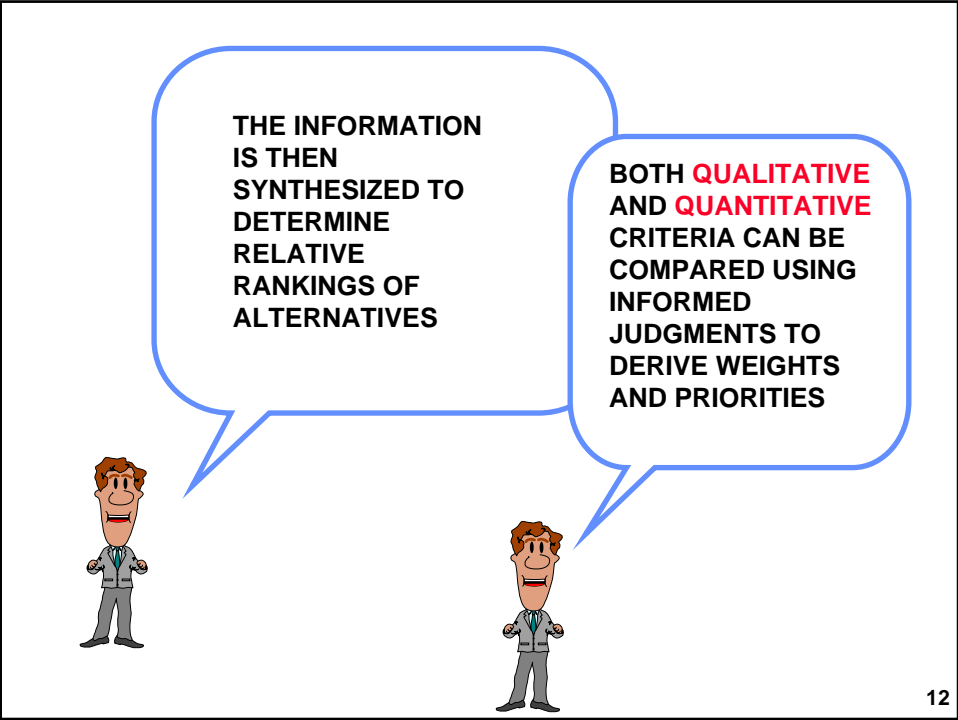
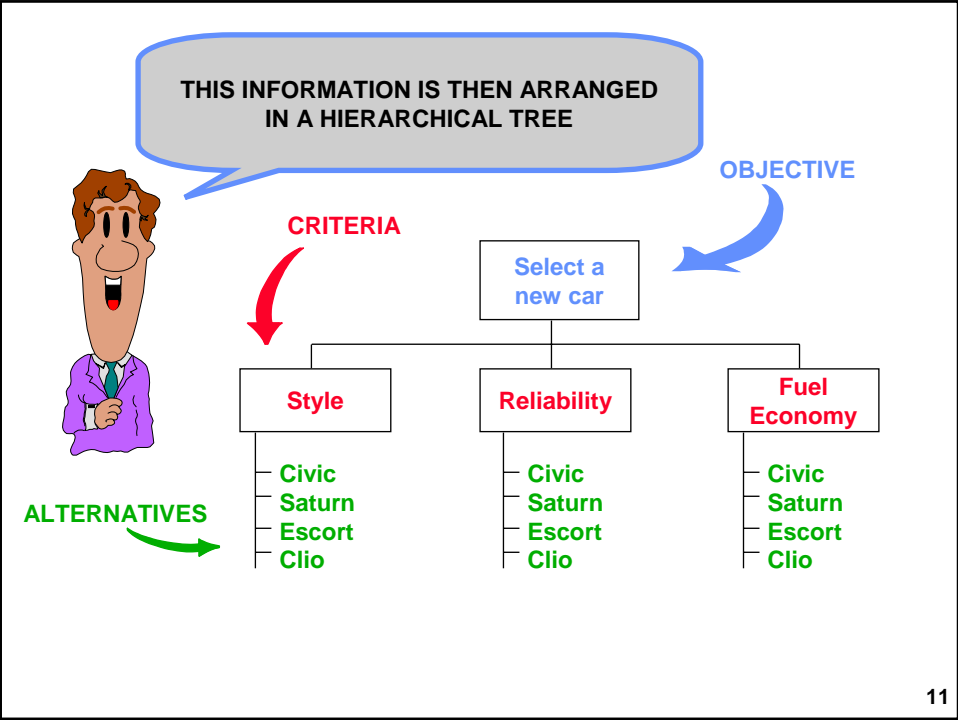
WHAT ABOUT COST?



(BE QUIET, WE'LL TALK ABOUT THAT LATER)

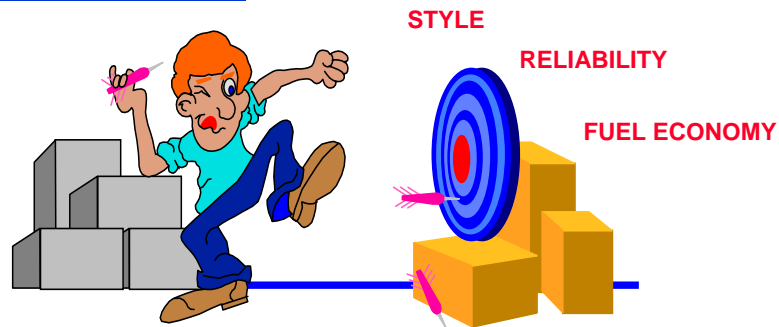
SKEPTIC-GATOR

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## HOW DO YOU DETERMINE THE RELATIVE IMPORTANCE OF THE CRITERIA?

Here's one way !

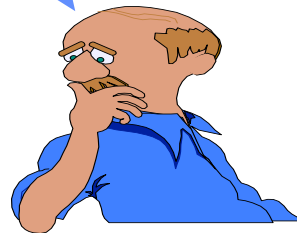


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HERE'S ANOTHER WAY

Hmm, I think reliability is the most important followed by style and fuel economy is least important so I will make the following judgements ....

USING **JUDGMENTS** TO DETERMINE THE RANKING OF THE CRITERIA



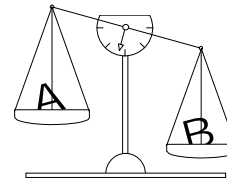
1. **RELIABILITY IS 2 TIMES AS IMPORTANT AS STYLE**
2. **STYLE IS 3 TIMES AS IMPORTANT AS FUEL ECONOMY**
3. **RELIABILITY IS 4 TIMES AS IMPORTANT AS FUEL ECONOMY**

he's not very consistent here ... that's o.k.

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## Pairwise Comparisons

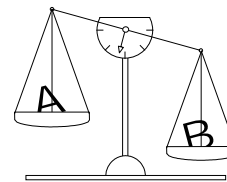


USING PAIRWISE COMPARISONS, THE RELATIVE IMPORTANCE OF ONE CRITERION OVER ANOTHER CAN BE EXPRESSED

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## Pairwise Comparisons



USING PAIRWISE COMPARISONS, THE RELATIVE IMPORTANCE OF ONE CRITERION OVER ANOTHER CAN BE EXPRESSED

1 equal 3 moderate 5 strong 7 very strong 9 extreme

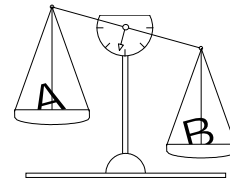
	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY		1/1	4/1
FUEL ECONOMY			1/1

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## Pairwise Comparisons



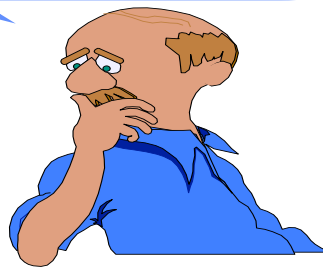
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1 equal 3 moderate 5 strong 7 very strong 9 extreme

	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY	2/1	1/1	4/1
FUEL ECONOMY	1/3	1/4	1/1

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How do you turn this MATRIX into ranking of criteria?



	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY	2/1	1/1	4/1
FUEL ECONOMY	1/3	1/4	1/1

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**HOW DO YOU GET A RANKING OF PRIORITIES FROM A PAIRWISE MATRIX?**

AND THE SURVEY SAYS



**ACTUALLY...**

**DR THOMAS L. SAATY, CURRENTLY WITH THE UNIVERSITY OF PITTSBURGH, DEMONSTRATED MATHEMATICALLY THAT THE EIGENVECTOR SOLUTION WAS THE BEST APPROACH.**

**REFERENCE : THE ANALYTIC HIERARCHY PROCESS, 1990, THOMAS L. SAATY**

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### **HERE'S HOW TO SOLVE FOR THE EIGENVECTOR:**

- 1. A SHORT COMPUTATIONAL WAY TO OBTAIN THIS RANKING IS TO RAISE THE PAIRWISE MATRIX TO POWERS THAT ARE SUCCESSIVELY SQUARED EACH TIME.**
- 2. THE ROW SUMS ARE THEN CALCULATED AND NORMALIZED.**
- 3. THE COMPUTER IS INSTRUCTED TO STOP WHEN THE DIFFERENCE BETWEEN THESE SUMS IN TWO CONSECUTIVE CALCULATIONS IS SMALLER THAN A PRESCRIBED VALUE.**

SAY WHAT!

SHOW ME AN EXAMPLE



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IT'S MATRIX ALGEBRA TIME !!!



	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY	2/1	1/1	4/1
FUEL ECONOMY	1/3	1/4	1/1

FOR NOW, LET'S REMOVE THE NAMES AND CONVERT THE FRACTIONS TO DECIMALS :

1.0000	0.5000	3.0000
2.0000	1.0000	4.0000
0.3333	0.2500	1.0000

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STEP 1: SQUARING THE MATRIX

THIS TIMES

1.0000	0.5000	3.0000
2.0000	1.0000	4.0000
0.3333	0.2500	1.0000

THIS

1.0000	0.5000	3.0000
2.0000	1.0000	4.0000
0.3333	0.2500	1.0000

I.E.  $(1.0000 * 1.0000) + (0.5000 * 2.0000) + (3.0000 * 0.3333) = 3.0000$

RESULTS IN THIS

3.0000	1.7500	8.0000
5.3332	3.0000	14.0000
1.1666	0.6667	3.0000

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**STEP 2 : NOW, LET'S COMPUTE OUR FIRST EIGENVECTOR  
(TO FOUR DECIMAL PLACES)**

FIRST, WE SUM THE ROWS

$$\begin{array}{r}
 \boxed{\begin{array}{r} 3.0000 + 1.7500 + 8.0000 \\ 5.3332 + 3.0000 + 14.0000 \\ 1.1666 + 0.6667 + 3.0000 \end{array}} = \begin{array}{r} 12.7500 \\ 22.3332 \\ 4.8333 \end{array} \quad \begin{array}{r} 0.3194 \\ 0.5595 \\ 0.1211 \end{array}
 \end{array}$$

SECOND, WE SUM THE ROW TOTALS  $\rightarrow$   $\begin{array}{r} 39.9165 \\ 1.0000 \end{array}$

FINALLY, WE **NORMALIZE** BY DIVIDING THE ROW SUM BY THE ROW TOTALS (I.E. 12.7500 DIVIDED BY 39.9165 EQUALS 0.3194)

THE RESULT IS OUR EIGENVECTOR  
( A LATER SLIDE WILL EXPLAIN THE MEANING IN TERMS OF OUR EXAMPLE)

$$\boxed{\begin{array}{r} 0.3194 \\ 0.5595 \\ 0.1211 \end{array}}$$

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**THIS PROCESS MUST BE ITERATED UNTIL THE EIGENVECTOR SOLUTION DOES NOT CHANGE FROM THE PREVIOUS ITERATION (REMEMBER TO FOUR DECIMAL PLACES IN OUR EXAMPLE)**

CONTINUING OUR EXAMPLE,  
AGAIN, STEP 1: WE SQUARE THIS MATRIX

$$\boxed{\begin{array}{r} 3.0000 \quad 1.7500 \quad 8.0000 \\ 5.3332 \quad 3.0000 \quad 14.0000 \\ 1.1666 \quad 0.6667 \quad 3.0000 \end{array}}$$

WITH THIS RESULT  $\rightarrow$

$$\boxed{\begin{array}{r} 27.6653 \quad 15.8330 \quad 72.4984 \\ 48.3311 \quad 27.6662 \quad 126.6642 \\ 10.5547 \quad 6.0414 \quad 27.6653 \end{array}}$$

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**AGAIN STEP 2 : COMPUTE THE EIGENVECTOR (TO FOUR DECIMAL PLACES)**

27.6653	+	15.8330	+	72.4984	=	115.9967	0.3196
48.3311	+	27.6662	+	126.6642	=	202.6615	0.5584
10.5547	+	6.0414	+	27.6653	=	44.2614	0.1220
						<b>TOTALS</b>	362.9196    1.0000

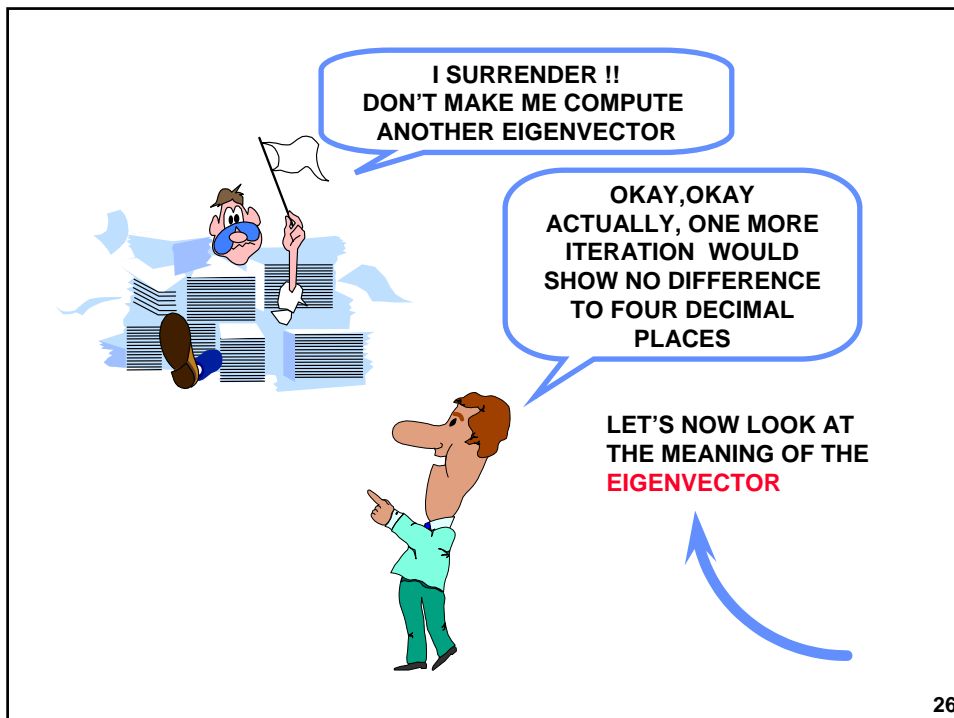
**COMPUTE THE DIFFERENCE OF THE PREVIOUS COMPUTED EIGENVECTOR TO THIS ONE:**

0.3194	-	0.3196	=	- 0.0002
0.5595	-	0.5584	=	0.0011
0.1211	-	0.1220	=	- 0.0009

**TO FOUR DECIMAL PLACES THERE'S NOT MUCH DIFFERENCE HOW ABOUT ONE MORE ITERATION?**




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HERE'S OUR PAIRWISE MATRIX WITH THE NAMES



	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY	2/1	1/1	4/1
FUEL ECONOMY	1/3	1/4	1/1


AND THE COMPUTED EIGENVECTOR GIVES US THE RELATIVE RANKING OF OUR CRITERIA

STYLE	0.3196	← THE SECOND MOST IMPORTANT CRITERION
RELIABILITY	0.5584	← THE MOST IMPORTANT CRITERION
FUEL ECONOMY	0.1220	← THE LEAST IMPORTANT CRITERION

NOW BACK TO THE HIEARCHICAL TREE...

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HERE'S THE TREE WITH THE CRITERIA WEIGHTS



OBJECTIVE

CRITERIA


Select a new car 1.00

- Style .3196
  - Civic
  - Saturn
  - Escort
  - Clio
- Reliability .5584
  - Civic
  - Saturn
  - Escort
  - Clio
- Fuel Economy .1220
  - Civic
  - Saturn
  - Escort
  - Clio

ALTERNATIVES

WHAT ABOUT THE ALTERNATIVES?

I'M GLAD YOU ASKED...



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IN TERMS OF STYLE, PAIRWISE COMPARISONS DETERMINES THE PREFERENCE OF EACH ALTERNATIVE OVER ANOTHER



**STYLE**

	CIVIC	SATURN	ESCORT	CLIO
CIVIC	1/1	1/4	4/1	1/6
SATURN	4/1	1/1	4/1	1/4
ESCORT	1/4	1/4	1/1	1/5
CLIO	6/1	4/1	5/1	1/1

AND...

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IN TERMS OF RELIABILITY, PAIRWISE COMPARISONS DETERMINES THE PREFERENCE OF EACH ALTERNATIVE OVER ANOTHER



**RELIABILITY**

	CIVIC	SATURN	ESCORT	CLIO
CIVIC	1/1	2/1	5/1	1/1
SATURN	1/2	1/1	3/1	2/1
ESCORT	1/5	1/3	1/1	1/4
CLIO	1/1	1/2	4/1	1/1

ITS MATRIX ALGEBRA TIME!!!

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COMPUTING THE EIGENVECTOR  
DETERMINES THE RELATIVE  
RANKING OF ALTERNATIVES  
UNDER EACH CRITERION



RANKING	STYLE		RANKING	RELIABILITY	
3	CIVIC	.1160	1	CIVIC	.3790
2	SATURN	.2470	2	SATURN	.2900
4	ESCORT	.0600	4	ESCORT	.0740
1	CLIO	.5770	3	CLIO	.2570

WHAT ABOUT FUEL ECONOMY?



SKEPTIC-GATOR

ANOTHER GOOD QUESTION...

AS STATED EARLIER,  
AHP CAN COMBINE BOTH QUALITATIVE  
AND QUANTITATIVE INFORMATION



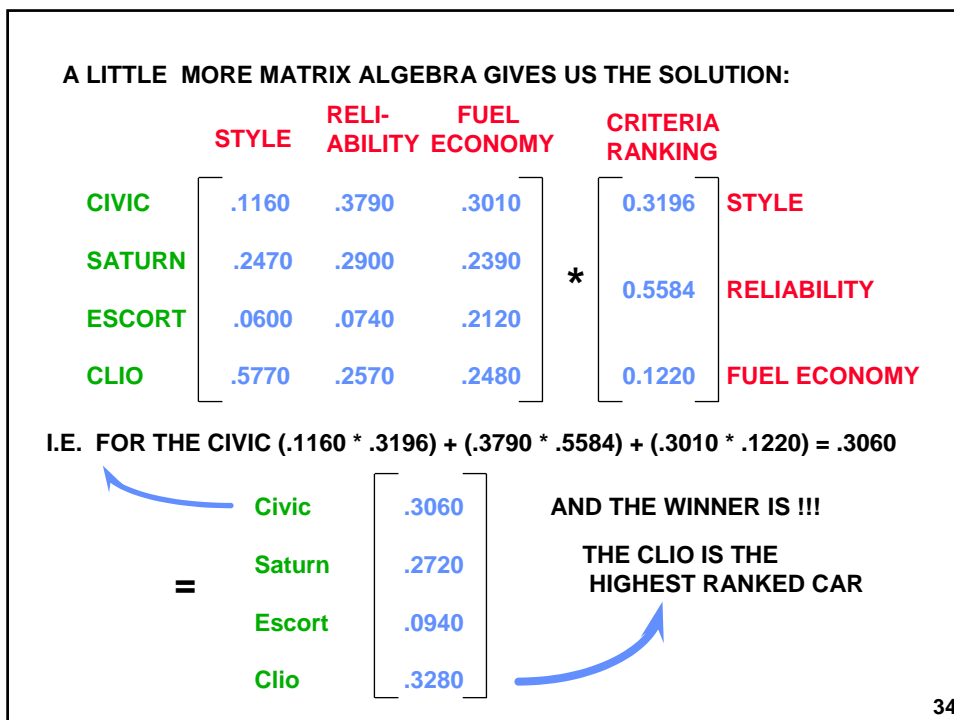
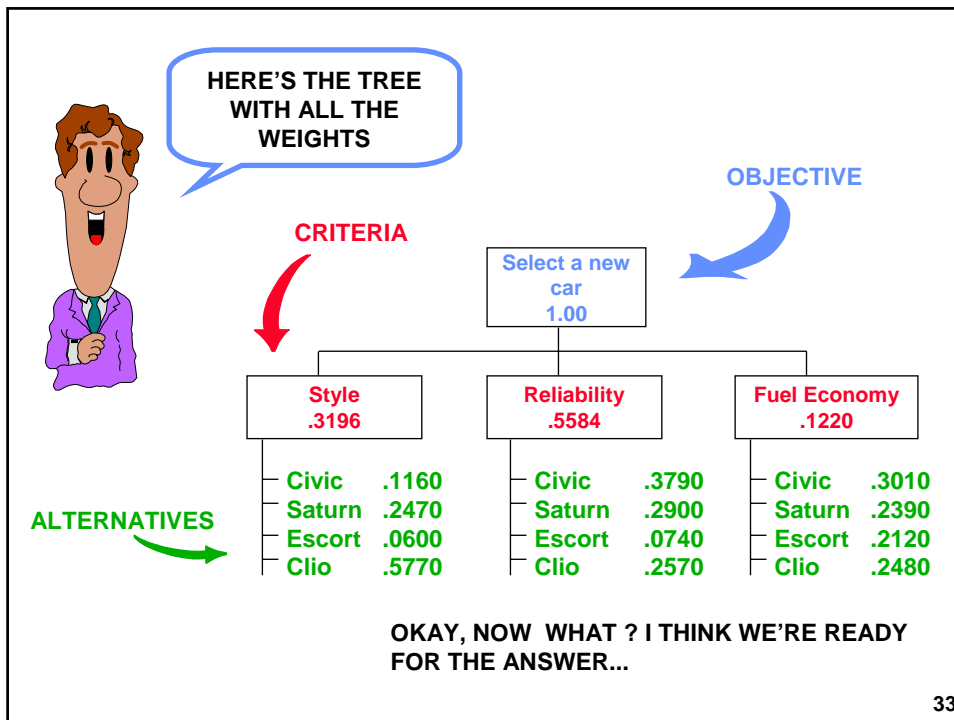
FUEL ECONOMY INFORMATION IS OBTAINED FOR EACH  
ALTERNATIVE:

	FUEL ECONOMY (MILES/GALLON)		
CIVIC	34	$34 / 113 =$	.3010
SATURN	27	$27 / 113 =$	.2390
ESCORT	24	$24 / 113 =$	.2120
CLIO	28	$28 / 113 =$	.2480
	<u>113</u>		<u>1.0000</u>

NORMALIZING THE FUEL ECONOMY INFO  
ALLOWS US TO USE IT WITH OTHER RANKINGS







IN SUMMARY, THE ANALYTIC HIERARCHY PROCESS PROVIDES A LOGICAL FRAMEWORK TO DETERMINE THE BENEFITS OF EACH ALTERNATIVE



- |           |       |
|-----------|-------|
| 1. Clio   | .3280 |
| 2. Civic  | .3060 |
| 3. Saturn | .2720 |
| 4. Escort | .0940 |

WHAT ABOUT COSTS?



SKEPTIC-GATOR

WELL, I'LL TELL YOU...

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ALTHOUGH COSTS COULD HAVE BEEN INCLUDED, IN MANY COMPLEX DECISIONS, **COSTS** SHOULD BE SET ASIDE UNTIL THE BENEFITS OF THE ALTERNATIVES ARE EVALUATED



OTHERWISE THIS COULD HAPPEN...

YOUR PROGRAM COST TOO MUCH I DON'T CARE ABOUT ITS BENEFITS



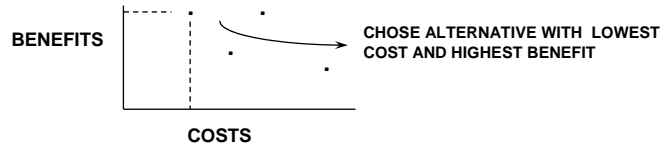
DISCUSSING COSTS TOGETHER WITH BENEFITS CAN SOMETIMES BRING FORTH MANY POLITICAL AND EMOTIONAL RESPONSES

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**WAYS TO HANDLE BENEFITS AND COSTS INCLUDE THE FOLLOWING:**



**1. GRAPHING BENEFITS AND COSTS OF EACH ALTERNATIVE**



**2. BENEFIT TO COST RATIOS**

**3. LINEAR PROGRAMMING**

**4. SEPARATE BENEFIT AND COST HIERARCHICAL TREES AND THEN COMBINE THE RESULTS**

**IN OUR EXAMPLE...**

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**LET'S USE BENEFIT TO COST RATIOS**



	<b>COST \$</b>	<b>NORMALIZED COSTS</b>	<b>BENEFIT - COST RATIOS</b>
<b>1. CLIO</b>	18,000	.3333	.3280 / .3333 = .9840
<b>2. CIVIC</b>	12,000	.2222	.3060 / .2222 = 1.3771
<b>3. SATURN</b>	15,000	.2778	.2720 / .2778 = .9791
<b>4. ESCORT</b>	9,000	.1667	.0940 / .1667 = .5639
	<u>54,000</u>	<u>1.0000</u>	


(REMEMBER THE BENEFITS WERE DERIVED EARLIER FROM THE AHP)

**AND...**

**THE CIVIC IS THE WINNER WITH THE HIGHEST BENEFIT TO COST RATIO**

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**AHP CAN BE USED FOR VERY COMPLEX DECISIONS**




**MANY LEVELS OF CRITERIA AND SUBCRITERIA CAN BE INCLUDED**

```
graph TD;
    GOAL[GOAL] --> C1[ ];
    GOAL --> C2[ ];
    C1 --> C1_1[ ];
    C1 --> C1_2[ ];
    C1 --> C1_3[ ];
    C2 --> C2_1[ ];
    C2 --> C2_2[ ];
    C2_1 --> C2_1_1[ ];
    C2_1 --> C2_1_2[ ];
    C2_1 --> C2_1_3[ ];
```

**HERE'S SOME EXAMPLES**

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**AHP CAN BE USED FOR A WIDE VARIETY OF APPLICATIONS**



- STRATEGIC PLANNING**
- RESOURCE ALLOCATION**
- SOURCE SELECTION**
- BUSINESS/PUBLIC POLICY**
- PROGAM SELECTION**
- AND MUCH MUCH MORE...**

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