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A General Method for Fatigue Analysis of Vertical Axis Wind Turbine Blades*

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Abstract

The fatigue life of wind turbine blades that are exposed to the random loading environment of atmospheric winds is described with random data analysis procedures. The incident wind speed and the stresses caused by these winds are expressed in terms of probability density functions, while the fatigue life vs stress level relationship is treated deterministically. This approach uses a "damage density function" to express fatigue damage as a function of wind speed. By examining the constraints on the variables in the damage density expression, some generalizations of the wind turbine fatigue problem are obtained. The area under the damage density function is inversely related to total fatigue life. Therefore, an increase in fatigue life caused by restricted operation in certain wind regimes is readily visualized. An "on parameter", which is the percentage of total time at each wind speed that the turbine actually operates, is introduced for this purpose. An example calculation is presented using data acquired from the DOE 100-kW turbine program.

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Acknowledgments

The calculations required to produce the figures in this paper would not have been possible without the work of Jerry McNerney and Tim Leonard in developing the AUTOSYM computer simulation, The efforts of Nolan Clark and Ron Davis in collecting data for the DOE 100-kW turbine at the USDA station in Bushland, TX are greatly appreciated.

A General Method for Fatigue Analysis of Vertical Axis Wind Turbine Blades

Introduction

The fatigue life of wind turbine blades is a necessary ingredient in the estimate of the cost effectiveness of a wind turbine system. Ideally, rotor control parameters (such as the cutin and cutout wind speeds) should he selected to maximize both turbine life and rate of energy capture. However, energy capture (on a yearly basis) and blade fatigue life are conflicting quantities that require trade offs to determine the most suitable operating mode for a given wind system. The goal of this fatigue analysis is to define techniques that help to clarify the balance between rotor life expectancy and the rate of energy production.

Because the turbine operates in a random-loading environment, a statistical approach to defining the operating stresses is used. Reference 1 describes the method used to produce an estimate of blade life based upon descriptions of the cyclic stresses and wind speeds in terms of probability density functions (pdf). By using Miner's cumulative damage rule, the damage produced by the stress cycles at each stress amplitude within the pdf are integrated to estimate the blade life. However, Reference 1 does not include the effects of the control system. Since evaluating the control system that turns the turbine on and off is a goal of the fatigue analysis, it is important to know which wind speeds cause the bulk of the fatigue damage. A damage density function (ddf) that plots damage as a function of wind speed is introduced for this purpose. In a similar way, the amount of energy available at each wind speed is expressed in terms of an energy density function (edf). Both the maximum energy and maximum damage are accumulated if the wind turbine is operated (on) at all times. The life expectancy of the rotor is extended by incorporating a high-wind cutout in the turbine control algorithm. The effect of this cutout algorithm is expressed by plotting an "on parameter" that shows the fraction of the total available time at each wind speed actually operated by the turbine. In general, the on parameter is less than the unity between cutin and cutout, and above zero elsewhere because of the wind speed averaging times associated with the control algorithms. The operating edf and ddf are found by multiplying the maximum edf and ddf by the on parameter. The area under the edf is related to the annual energy production and the area under the ddf is related to the inverse of the blade fatigue life

This approach allows the analyst to visualize the effect of changing algorithm parameters on the fatigue life and energy production rate of the turbine. The ideal algorithm is one that removes as much area as possible from the ddf, thus increasing rotor life expectancy without taking area from the edf, thus maximizing the annual energy production. It is important to remember that the fatigue life and energy capture of a turbine are inseparable components of a cost of energy estimate.

The damage accumulated while the turbine is parked and during start-stop cycles is not included in this analysis procedure. Data collected during testing of several Darrieus Vertical Axis Wind Turbine (VAWT) systems support the assumption that these events have negligible effect on blade fatigue life. This is not necessarily a universal truth, but should be checked for each turbine individually.

Damage Density and Energy Density

In order to examine the effects of the operating parameters on the rotor fatigue life, it is convenient to know which wind speeds are responsible for the most damage. This can be visualized by plotting the damage as a function of wind speed in terms of a ddf. The ddf is derived in Appendix A. Although the stress levels (and hence the damage rate) continue to rise as wind speed increases, the amount of wind available at higher wind speeds decreases. The net result is that the ddf goes to zero as wind speeds continue to rise. The area under the ddf is the inverse of the blade life expectancy. The ddf for the blade to tower joint of the DOE 100-kW turbine at the Bushland, TX site is shown in Figure 1. This ddf is calculated as if the turbine operates at all times. The calculation of this ddf and the data used to produce it are included in Appendix B.

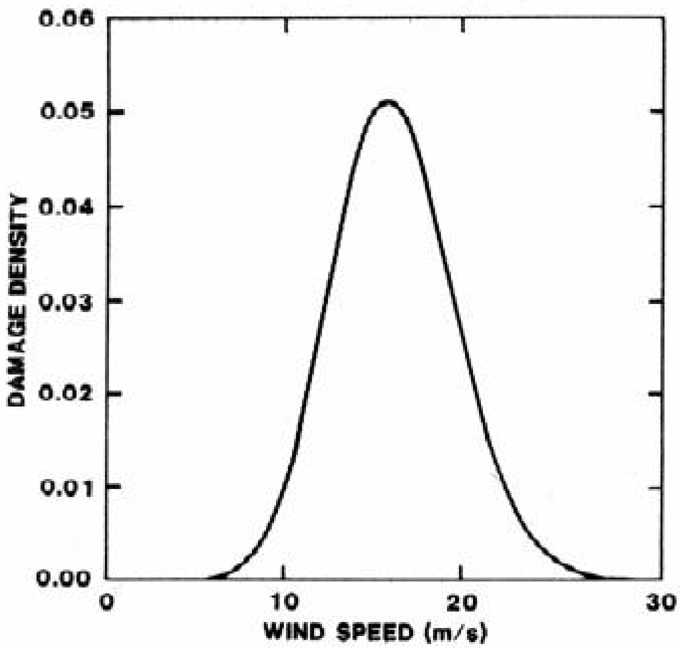

Figure 1. Damage Density Function (ddf) for the Welded Blade Joint of the DOE 100-kW Turbine at Bushland, TX

To summarize the results of Appendices A and B, if the pdf of wind is Rayleigh,

$$P_v(V) = \frac{\pi V}{2\bar{V}^2} \exp\left(\frac{-\pi V^2}{4\bar{V}^2}\right) \quad (1)$$

the standard deviation of the stress varies linearly with wind speed,

$$\sigma(V) = mV \quad (2)$$

and the number of cycles to failure can be described by a power function of the standard deviation of the stresses,

$$N(\sigma) = K\sigma^b \quad (3)$$

then the maximum ddf, assuming that the turbine is always operating, will be as shown in Appendix B, Eq (B2)

$$d_v(V) = \frac{\pi m^{-b}}{2\bar{V}^2 K} V^{(1-b)} \exp\left(\frac{-\pi V^2}{4\bar{V}^2}\right) \quad (4)$$

The relationships described by Eqs (1), (2), and (3) have been found to be very accurate for the DOE 100-kW machines and are useful approximations.

The general shape of the maximum ddf is as shown in Figure 1. The peak of the ddf is at a wind speed of

$$V_{peak} = \bar{V}\sqrt{\frac{2}{\pi}(1-b)} \quad (5)$$

Observe that the peak of the maximum ddf is a function only of the average wind speed (V) and the fatigue life exponent (b). (The fatigue life exponent is the slope of the RMS stress vs cycles to failure curve when plotted log-log.) At the high-cycle end of the fatigue life curve, the slope is, in general, quite flat; typical values of the exponent (b) are between -5 and -12. This puts the peak of the ddf between 2 and 3 times the annual average wind speed. As a result of the flat nature of the fatigue-life curve at high cycles, small changes in the operating stress level or in the fatigue-life curve produce large changes in the life estimate. Therefore, a reliable fatigue-life estimate requires an accurate description of the operating stresses and a statistical description of the fatigue-life characteristics. A family of fatigue-life curves for various confidence levels should be produced. Then a fatigue-life estimate can be described based upon a specified confidence level. Unfortunately, defining the statistics of the fatigue-life curve requires a large number of tests of the blade components.

The energy-capture characteristics of a turbine in a particular site are often described using an edf analogous to the ddf. The maximum edf is the product of the wind speed pdf and the power curve. A convenient way to visualize the trade off between fatigue life and energy capture is to plot both the edf and the ddf on the same wind speed axis. Figure 2 is a plot that includes the edf and two extreme cases of the ddf; the first with a high slope on the fatigue-life curve (b = -5) that results in a peak at $2\bar{V}$, and the second with a low slope on the fatigue curve (b = -12) that produces a peak at $3\bar{V}$. The peak of the maximum edf will typically be at about 1.5 times the average wind speed. Note that the bulk of the damage is usually accumulated at higher wind speeds than the bulk of the available energy. This makes it possible to obtain a significant extension in the blade fatigue life without sacrificing a great deal of the available energy by shutting down the turbine at the appropriate time.

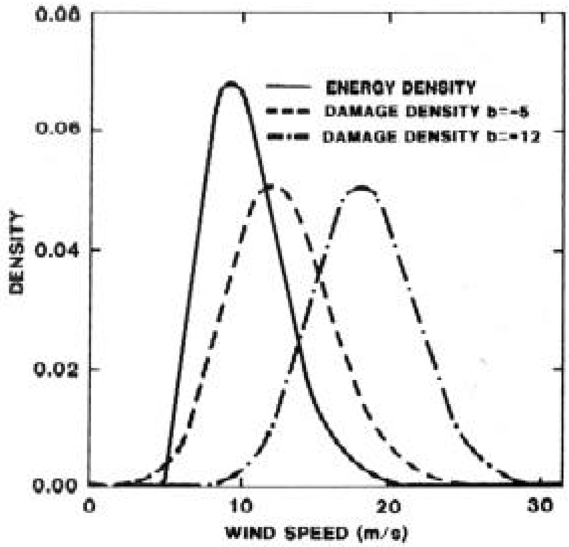

Figure 2. Energy Density Function (edf) With Two Limiting Cases of the Damage Density Function (ddf)

Control Algorithm Effects

The purpose of the cutout algorithm is therefore to balance the extension of fatigue life with the reduction in the annual energy capture, both of which result from shutting down the turbine in high winds. To visualize the effect of the control algorithm on the operation of the turbine, an "on parameter" is defined which shows the fraction of the total available time at each wind speed that a turbine operates. The on parameter for a given wind speed is zero if the turbine never operates at that wind speed and 1 if the turbine always operates when the winds are at the given wind speed. Figure 3 plots the on parameter for several cutout wind speeds. This on parameter was calculated by a Sandia computer simulation called AUTOSYM[2] using wind data collected at Bushland, TX. The algorithm shuts the turbine off when a 2-s average of the wind speed exceeds the cutout and will not restart until 15 min after any cutout wind speed excellence. This is by no means an optimum algorithm but is included as an example. The dashed line would be the on parameter for a turbine that is always on. The on parameter never reaches 1 because the winds may be inside the operating range while the turbine is in the middle of a start-up averaging period or in a 15-min shutdown after a high wind cutout. As expected, operation in winds above cutout is practically eliminated. But a by-product of this algorithm (or any algorithm) is a considerable reduction in operating time in winds well below cutout.

The actual, or operating, edf and ddf are obtained by multiplying the maximum edf and pdf by the on parameter. The ddf's in Figure 4 are calculated by combining the on parameters in Figure 3 with the damage density in Figure 1. Since the area under the ddf is the inverse of the blade fatigue life, the reduced areas under the ddf's associated with lower cutout wind speeds indicate increased blade life. Similarly, the family of edf's for different cutout wind speeds plotted in Figure 5 show how the annual energy capture is affected by the cutout algorithm. Note that, with a 20.1 m/s (45 mph) cutout, almost all of the available energy is captured while a substantial fraction of the fatigue damage is still eliminated. Table 1 lists the blade fatigue life expectancy and annual energy capture associated with edf's and ddf 's for the blade-to-tower joints on the DOE 100-kW turbine at Bushland, TX, shown in Figures 5 and 4, respectively.

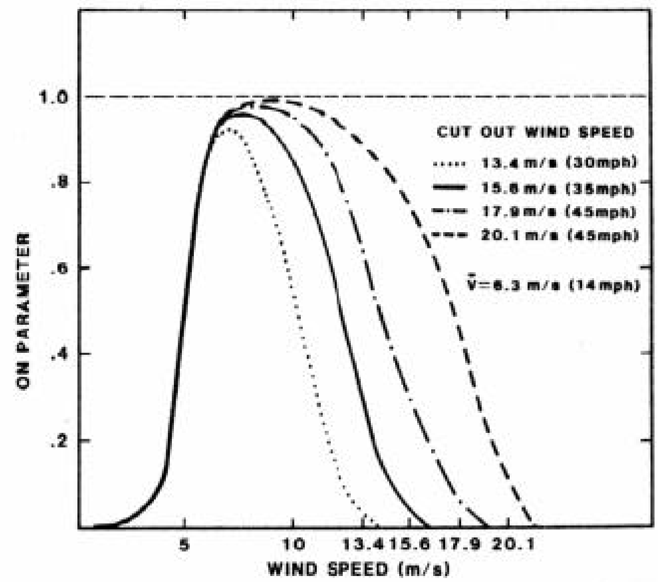

Figure 3. On Parameters for Several Values of Cutout Wind Speed for the DOE 100-kW Turbine at Bushland, TX

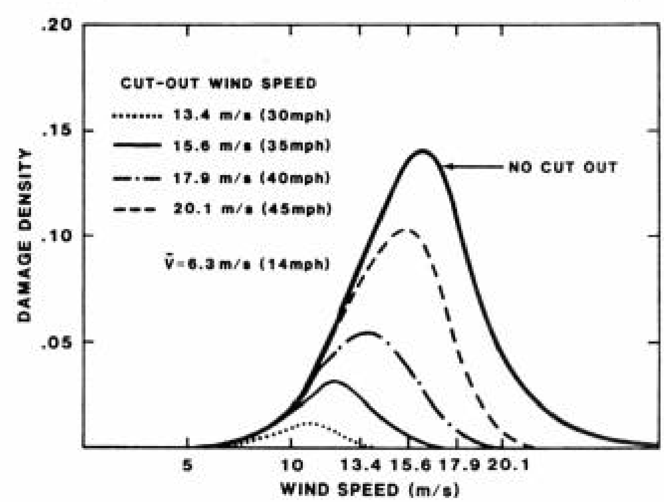

Figure 4. Damage Density Functions for Several Values of Cutout Wind Speed

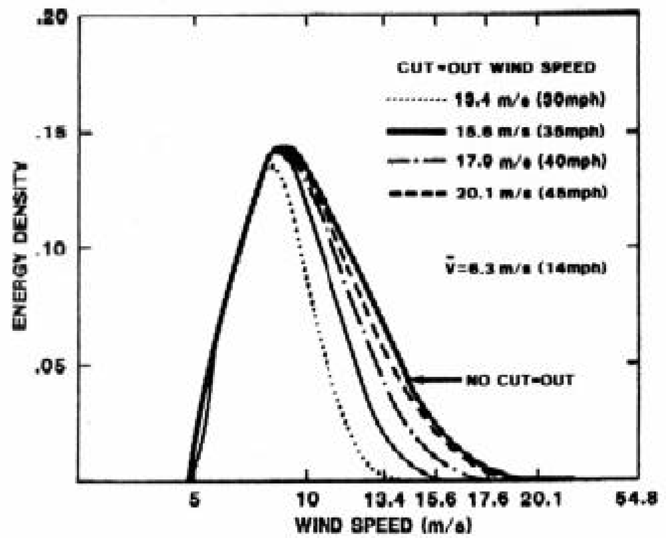

Figure 5. Energy Density Functions for Several Values of Cutout Wind Speed

Table 1. Effects of Cutout Wind Speed

Cutout Wind Speed (m/s)	(mph)	Annual Energy (fraction of maximum)*	Expected Joint Life (multiple of minimum)*
134	30	0.49	22.0
15.6	35	0.69	7.0
17.9	40	0.811	1.1
20.1	45	0.92	1.6
22.4	50	0.95	1.2

*Maximum energy and minimum joint life are calculated based upon a turbine that never shuts down in high winds. For the DOE 100-kW turbine at Bushland ($\overline{V}$ = 6.3 m/s; 14 mph) the maximum energy is 200 000 kWh and the minimum joint life is 2.8 yrs.

Statistical Considerations

The data presented here is used to produce an estimate of the mean component fatigue life. There are two very important points that must be made: first, this is the mean (or 50%) confidence level of the fatigue life; second, it is the fatigue life of a single component (such as one joint) and not the entire rotor. If there is a P percent confidence that a component will not fail in T years and there are n components in the rotor, then the confidence that the rotor will not fail in T yr is

$$P_{rotor} = P^n \tag{6}$$

For example, if there is a 50%, confidence that one joint on the Bushland turbine will not fail for 5 yr and there are four similar joints on the rotor experiencing the same stress levels, there is only a 6.25% confidence that the rotor will not fail in 5 yr. In order to obtain a 50% confidence or mean rotor life, the fatigue life of each joint would have to he known at an 84% confidence level. Producing an RMS-N curve at a given confidence level requires repeated tests at each stress level. Reference 3 outlines a procedure for evaluating the fatigue life curve at a given confidence level.

Summary

By creating maximum damage and energy density functions, the lower hound on fatigue life and the upper bound on annual energy production are defined.

Figure 2 indicates that the bulk of the energy and the bulk of the damage will usually lie in different wind speed regimes. Therefore, a good cutout algorithm can reduce the damage significantly without crippling the system by overly restricting the annual energy production. The cutout algorithm is characterized by the on parameter that shows the fraction of the total available time at each wind speed the turbine actually operates. The actual ddf and edf are the product of the on parameter and the maximum ddf and edf. The expected time to failure is the inverse of the area under the actual ddf and the annual energy production is the area under the actual edf. The use of these density functions should aid the analyst in visualizing the fatigue life/energy capture trade-off.

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APPENDIX A

Derivation of the Damage Density Function

Even though wind turbine blades have the cyclic loads caused by rotating through upwind and downwind orientations producing what could be expected to be deterministic loads, the turbulence in the wind causes the blade stresses to be stochastic rather then deterministic. There is evidence[1] that the distribution of cyclic stress amplitudes at any given wind speed follows the distribution of peak values of a Gaussian narrow band random process: namely, a Rayleigh distribution. This conditional probability density function (pdf) is

$$P_{s|v}(S,V) = \frac{S}{\sigma(V)^2} \exp\left(\frac{-S^2}{2\sigma(V)^2}\right) \quad \text{(A1)}$$

where

S = stress amplitude
V = wind speed
$\sigma(V)$ = standard deviation of the stress signal (written here as a function of wind speed)

In many cases, $\sigma(V)$ may be a linear function of wind speed as it is for the DOE 100-kW rotor (Appendix B).

$$\sigma(V) = mV \quad \text{(A2)}$$

The pdf for the stress amplitudes can be found from the joint pdf of stress amplitude and wind speed. The joint pdf is the product of the conditional pdf and the wind speed pdf. If the wind pdf is Rayleigh,

$$P_{s|v}(S,V) = \frac{S}{\sigma(V)^2} \exp\left(\frac{-S^2}{2\sigma(V)^2}\right) \quad \text{(A3)}$$

the joint pdf of stress and wind speed will be

$$P_{s,v}(S,V) = \frac{S}{\sigma^2} \exp\left(\frac{-S^2}{2\sigma^2}\right) \frac{\pi V}{2\overline{V}^2} \exp\left(\frac{-\pi V^2}{4\overline{V}^2}\right) \quad \text{(A4)}$$

The pdf of stress is the integral over all wind speeds of the joint pdf.

$$P_s(S) = \int_0^\infty P_{s,v}(S,V)dV \quad \text{(A5)}$$

Substituting (A4) into (A5) and evaluating the integral yields

$$P_s(S) = CK_0(k\sqrt{R}) \quad \text{(A6)}$$

where

$C = \pi S/2\overline{V}^2 m^2$

$k^2 = 2S^2/m^2$

$R = \pi/4\overline{V}^2$

$\overline{V}$ = annual average wind speed

$K_o(\,)$ = modified Hankel function of order 0

Plots of the stress pdf of the DOE 100-kW rotor for three annual average wind speeds are shown in Figure A1.

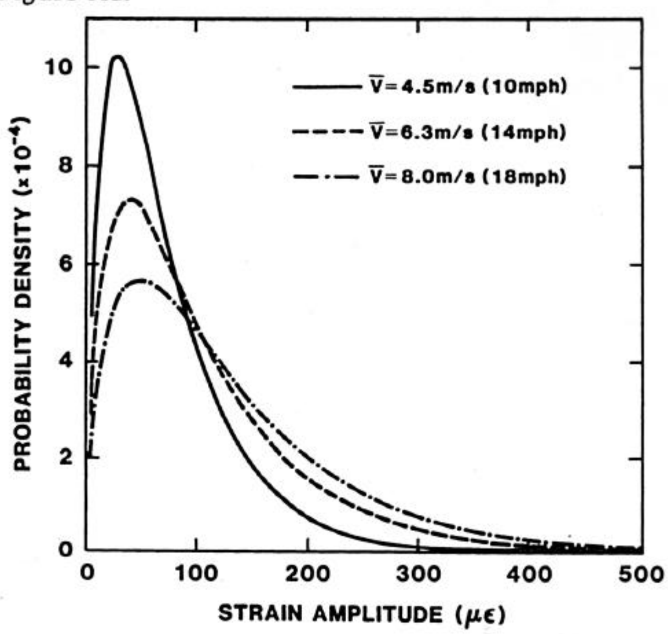

Figure A1. Probability Density Functions of the Vibratory Strain Amplitudes for the Welded Joint of the DOE 100-kW VAWT for Three Different Annual Average Wind Speeds

The total fatigue damage is the sum of the damage lone by each stress amplitude, The damage is calcu-ated by dividing the number of cycles at each tress level (described by $P_S(S)$ times the total number of ycles) by the number of cycles to failure at each stress evel. When the sum is equal to one, the total number of ycles equals the number of cycles to failure.

$$1 = \int_0^\infty \frac{n_f P_s(S)}{N(S)}\, dS \qquad \textbf{(A7)}$$

where

n_f = total cycles to failure (including all levels)
$N(S)$ = number of cycles to failure at the stress amplitude S

The time to failure is the number of cycles to failure divided by the cyclic frequency. The estimated time to failure, assuming the turbine is always oper-ating, is given by

$$\frac{1}{T} = f\int_0^\infty \frac{P_s(S)}{N(S)}\, ds \qquad \textbf{(A8)}$$

where

T = estimated time to failure
f = mean crossing rate

If, however, the fatigue life of the component is nown as a function of the standard deviation of the ress rather than the stress amplitude, solving for the stimated time to failure is much simpler and voids some of the inaccuracies of using Miner's Rule. nce the standard deviation of stress can be written as a function of wind speed (as in Eq (A2)), and the number of cycles to failure is a function of the stress standard deviation, the number of cycles to failure can be written directly as a function of wind speed.

$$N(\sigma) = N(\sigma(V)) = N(V) \qquad \textbf{(A9)}$$

The time to failure can be solved for by integrating all possible wind speeds, again assuming that the turbine is always operating.

$$\frac{1}{T} = \int_0^\infty f\, \frac{P_v(V)}{N(V)}\, dV \qquad \textbf{(A10)}$$

The integrand of this expression is the damage density function (ddf).

$$d_v(V) = f\, \frac{P_v(V)}{N(V)} \qquad \textbf{(A11)}$$

This is actually the maximum ddf because it represents the relative amounts of damage accumulated at each wind speed for a turbine that is always operating. Whenever the turbine operates less than full time, the magnitude of the ddf is reduced and the area under the ddf is also reduced. Eq (A10) shows that the time to failure is inversely related to the area under the ddf. Therefore, the increase in life expectancy caused by restricted operation can be visualized by examining the effect on the ddf. For example, if the turbine blade characterized by the ddf in Figure 1 were always operated in winds below 16 m/s (36 mph) and never above, the expected time to failure of the blade would be about twice that of the blade on a turbine that always operates.

APPENDIX B

Example Damage Density Function Calculation

The DOE 100-kW Vertical Axis Wind Turbines (VAWTS) have been operating in Rocky Flats, CO, Bushland, TX, and Martha's Vineyard, MA. As a concrete example of the fatigue analysis method described above, the field data collected at Bushland and the testing done on the DOE 100-kW system are presented here. The welded joint at the blade-to-tower connection was selected for this fatigue life estimate. To produce a maximum ddf for the turbine always operating, the component fatigue life, operating stress levels, and cyclic stress frequency must be characterized. The wind distribution used for this example is for the Bushland site which closely matches a Rayleigh distribution with a 6.3 m/s (14 mph) mean.

Fatigue Life Data

The testing was accomplished by mounting a specially designed test specimen on an electromagnetic shaker. The specimen was shaken at its first resonant frequency with a random narrow band input. The resulting cyclic stress amplitude distribution matches the operating conditional pdf of cyclic stress amplitude. This distribution is characterized by the RMS of the stress signal about the mean. The system was designed such that the mean stress at the joints during operation is near zero. The fatigue-life curve plots RMS stress level at zero mean against a number of cycles to failure (failure is defined as a crack detectable by visual inspection). Figure B1 shows the results of the test with an estimated mean time to failure curve expressed as a bilinear log-log fit. The slope of the curve at the high cycle end corresponds to b = -9.5 in the equation

$$N(\sigma) = K\sigma^{b} \quad \text{(B1)}$$

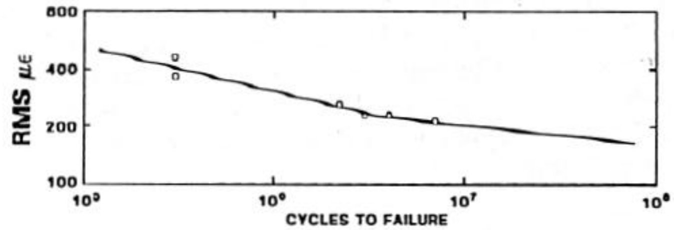

Figure B1. RMS Strain vs Number of Cycles to Failure With a Narrow Band Random Loading for the DOE 100-kW Welded Blade Joint

Operating Stress Levels

The turbine at Bushland has strain gages mounted at each joint in the same locations as the strains were monitored in the fatigue tests. The standard deviation of the stress signal during operation was calculated, using a bins type approach as outlined in References 1, 4. Figure B2 is a plot of the measured stress standard deviation as a function of wind speed. One feature of the short averaging time associated with the bins-measuring procedure is that the highest wind-bins estimates are too low and the lowest wind estimates are too high.[5] With this in mind, a linear fit to the data with a slope of 128 μ∈/ (m/s) (57 μ∈/ mph) is reasonable.

The Maximum Damage Density

By combining the fatigue life curve with the operating stress data, the number of cycles to failure is written directly as a function of wind speed as in Eq (A8). The average cyclic stress frequency can be estimated by taking the ratio of the second moment to the zero moment of the stress frequency spectrum. A typical blade stress spectrum from the DOE 100-kW rotor is shown in Figure B3. The mean crossing rate calculated from this spectrum is 3.5 H2, which is higher han might be expected by a visual inspection of the pectrum. As a check on the accuracy of this estimate or use in a fatigue calculation, the number of occurrences of each stress amplitude were counted by the method outlined in Reference 1 using several threshold values or minimum cyclic stress amplitude, The results are hown in, Figure B4. As the threshold level is ncreased, the cyclic stress amplitudes approach the xpected Rayleigh distribution. The solid line in Figure 34 is the Rayleigh distribution with a frequency of 3.5 Iz. It is important to note that the mean crossing rate is ignificantly affected by the high frequency content of he spectrum, even though the high frequency content nay appear to be insignificant.

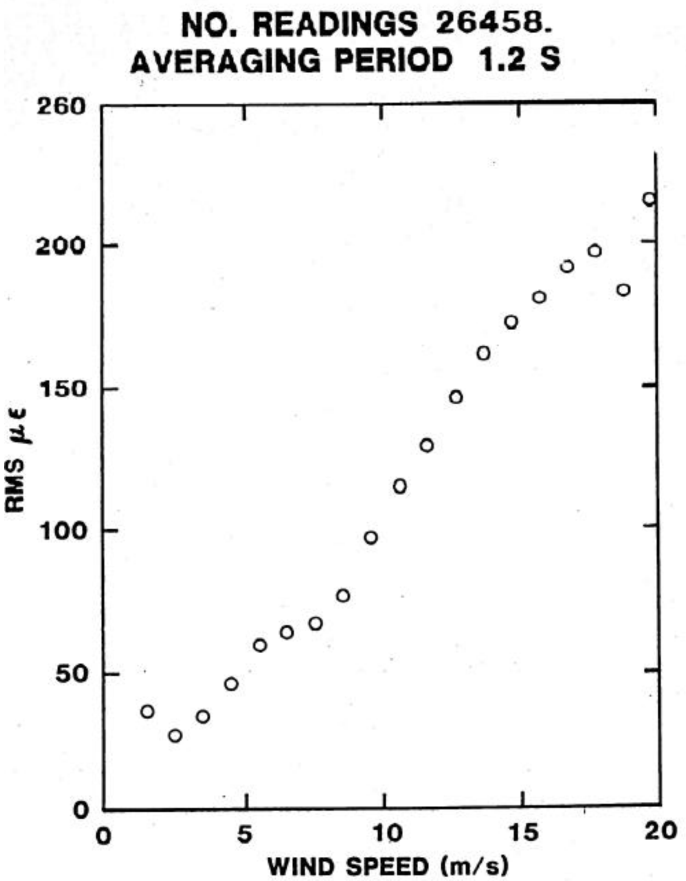

Figure B2. Measured RMS Strain vs Wind Speed for the Welded Blade Joint at Bushland, TX

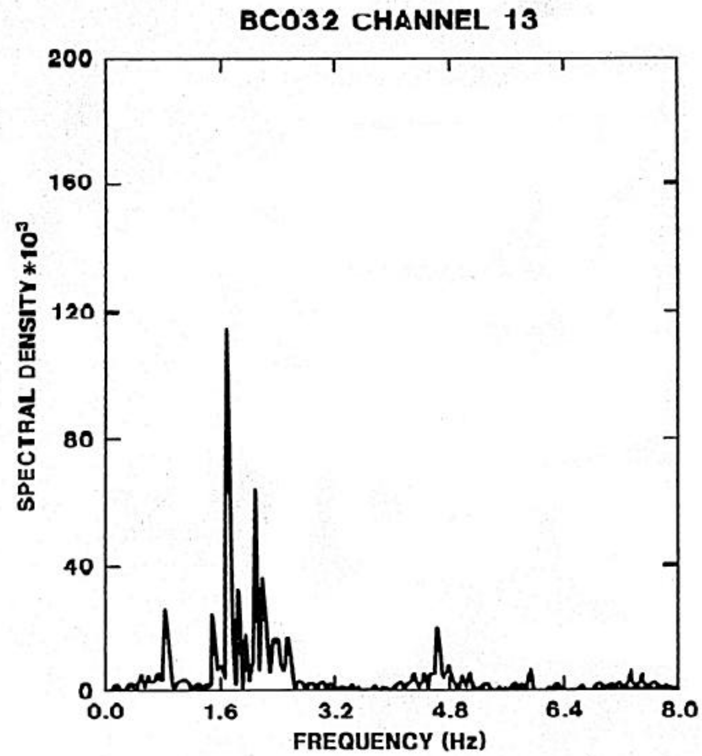

Figure B3. DOE 100-kW VAWT Blade Stress Power Spectral Density

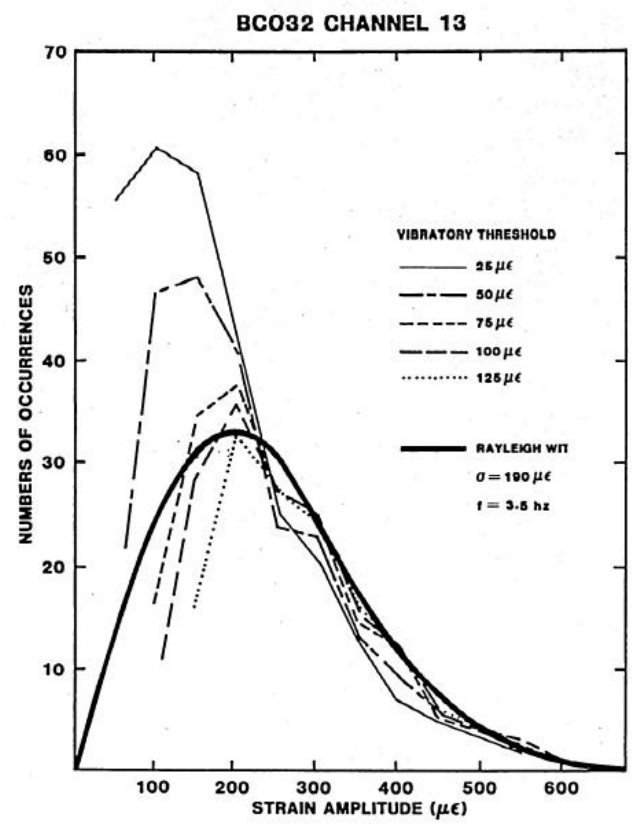

Figure B4. Number of Occurrences of each Vibratory Strain Amplitude Level for Several Values of Threshold Strain Amplitude. The heavy line is the theoretical Rayleigh distribution for a record of the same length with identical variance and a frequency of 3.5 Hz.

A11 the necessary ingredients to evaluate Eq (A11) for the maximum ddf are supplied above. The Rayleigh wind speed distribution from Eq (A3), the number of cycles to failure as a function of wind speed as produced by Eq (A9), and the average cyclic stress frequency are substituted into Eq (A11). This results in a maximum ddf with the functional form

$$d_v(V) = \frac{\pi m^{-b}}{2\bar{V}^2 K} V^{(1-b)} \exp\left(\frac{-\pi V^2}{4\bar{V}^2}\right) \quad \textbf{(B2)}$$

The resulting maximum ddf for the welded joint on the Bushland l00-kW turbine is shown in Figure B5. The time to failure of this component, if the turbine operates at all times, is the inverse of the integral of the maximum ddf: 2.8 yr. The actual life of the joint on a turbine with an active control system can be considerably longer. The increases in joint life expectancy using various cutout wind speeds are included in Table l.

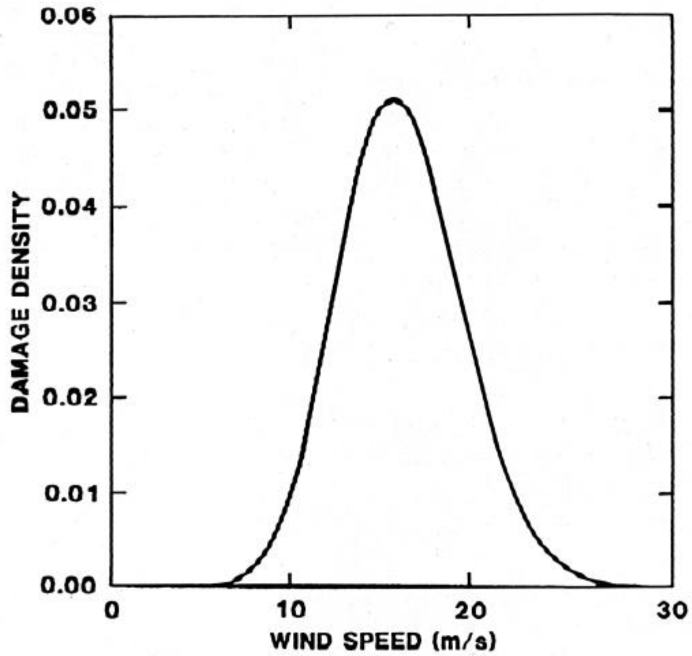

Figure B5. Damage Density Function (ddf) for the Welded Blade Joint of the DOE 100-kW Turbine at Bushland, TX