2-D PIC SIMULATIONS OF ELECTRON FLOW IN THE MAGNETICALLY INSULATED TRANSMISSION LINES OF Z AND ZR [*]

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Abstract

The ZR project [1] to refurbish the Z accelerator at Sandia National Laboratories [2] is scheduled for completion in FY06. An important factor for the success of ZR is limiting current losses in the vacuum section. On Z today, late-time losses of 5 – 10% are observed, which are known to occur in the post-hole convolute region. The most direct way to mitigate these losses is to limit the electron flow into the convolute from the magnetically insulated transmission lines (MITLs). The key design consideration is the radial profile of the MITL gap.

The MITL gap profile is a compromise between two competing constraints—limiting both the electron flow into the convolute and the MITL inductance. Larger gaps reduce the flow, while smaller gaps reduce the inductance. 2-D particle-in-cell (PIC) simulations suggest that the current Z A-level MITL profile is close to optimum, in the sense of minimizing the flow for its total inductance. For ZR, operating at ~40% higher voltage and ~30% higher current, the decision was made to limit the flow into the convolute to be no larger than on Z today. Time-accurate simulations, with a Z-pinch load, show that profiles based on Z, but with 20% larger gaps meet this condition. It does not appear to be possible to meet the flow limitation without this increase in inductance.

I. INTRODUCTION

The vacuum section of Z is shown in Fig. 1. Power is conducted radially inward from the insulator stack at r ~ 1.6 m to the load with four radial MITLs, which are coupled together in parallel with a double post-hole convolute at r ~ 10 cm. Electrons are emitted from each MITL cathode and drift inward towards the convolute and the load. It is currently not feasible to model the entire vacuum section with a full 3-D simulation. However, 3-D simulations of the convolute have been performed with restricted MITLs extending out to only 20 – 25 cm [3-5]. They show that most of the MITL electrons are lost to the anode in small areas around the magnetic null regions in the convolute. The intense local energy deposition heats the anode surface above the threshold for plasma formation long before the Z-pinch implodes. Dense anode plasma effects cannot be accurately modeled in these 3-D simulations, but are strongly suspected to be the cause of the late-time current losses in the convolute seen on Z.

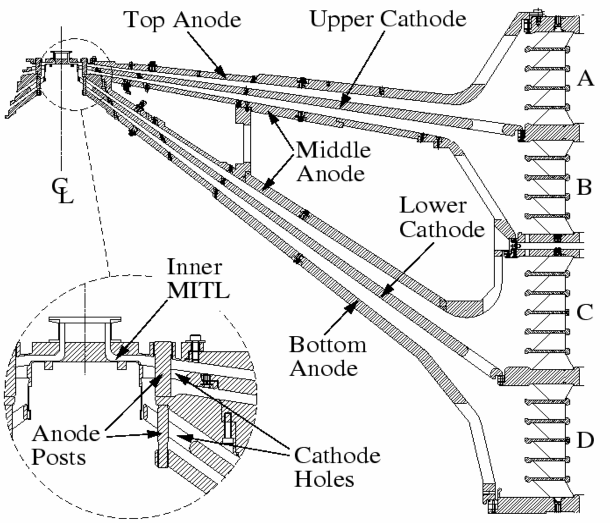

Figure 1. R-z geometry of the Z vacuum section, with inset showing the convolute and inner MITL. Power flows to the left from the stack at the far right (r ~ 1.6 m).

Although the convolute is intrinsically 3-D, two insights from the 3-D simulations indicate that the MITLs can be modeled economically with just 2-D r-z simulations, terminated with an inner radial boundary just upstream of the convolute. First, even slightly upstream of the convolute, the boundary and electron flow currents in the MITLs are essentially azimuthally symmetric. Second, all electrons lost in the convolute are emitted in the MITLs — there is no emission in the convolute itself, because of the excess space-charge of the MITL electrons flowing in. Provided that appropriate boundary conditions are applied, the electron flow through the inner radial boundary should accurately model the actual flow into the convolute in the full 3-D system.

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The simulations described here are done with Sandia's QUICKSILVER code [6], using an inner radial boundary model described in Ref. 7. A complementary effort to the work in Section III has also been done with the LSP code [8] by Mission Research Corporation [9].

II. THEORY

For an infinitely long MITL with uniform vacuum impedance Z_0, the line is insulated at a given voltage once the current exceeds a threshold value. In equilibrium, the electron flow current I_e, anode boundary current I_a, and cathode boundary current I_c satisy $I_a = I_c + I_e$. The voltage and currents are related to a good approximation by [10]

$$V = Z_0(I_a^2 - I_c^2) - \frac{mc^2}{2e}\left(\frac{I_a^2}{I_c^2} - 1\right). \qquad (1)$$

For a strongly undermatched load, $Z_L << Z_0$, the electron flow is much smaller than the boundary currents, and is confined to a thin sheath at the cathode. For a radial transmission line, the vacuum impedance is $Z_0 = 60d/r$, where d(r) is the gap. Applying the $Z_L << Z_0$ uniform impedance MITL expressions locally, we have

$$I_e \cong \frac{Z_L^2}{2Z_0^2} I \propto \frac{V^2}{I}\frac{r^2}{d^2}, \text{ and} \qquad (2)$$

$$g \cong \frac{mV}{eB^2 d} \propto \frac{V}{I^2}\frac{r^2}{d}, \qquad (3)$$

where $I = I_c \approx I_a$, and g is the sheath thickness.

III. RESISTIVE LOAD SIMULATIONS

An important first step for analysis of MITL performance is to model a single level, terminated with a resistive load, and driven with a simple voltage pulse that rises up to a fixed value. The simulation geometry is a radial MITL in cylindrical coordinates, extending out to r = 1.38 m, with electron emission for r < 1.3 m. The baseline case uses the gap profile from the Z A-level MITL – constant gap, d = 1 cm, from the convolute out to r = 20 cm, and then increasing to d = 4.2 cm at r = 1.3 m. This Z MITL is inclined at only 8° from horizontal, so these simulations closely model the actual geometry.

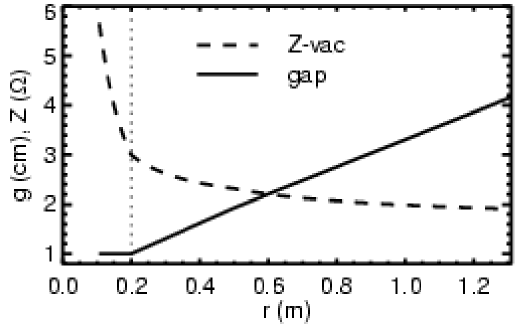

Figure 2. Gap and impedance profile of the Z A-level MITL. The gap is constant, d = 1 cm, for r < 20 cm.

The impedance profile is shown in Fig. 2. We use Z_L = 0.46 Ω; at late time this gives the operating point for Z at peak current, V = 2.3 MV, I = 5 MA. With this load, we are strongly undermatching the vacuum impedance of Z_0 = 5.66 Ω at the inner radius of the MITL at r = 10.6 cm. Eq. 3 predicts that the sheath thickness varies from 0.53 mm at r = 1.3 m down to only 15 μm at r = 10.6 cm.

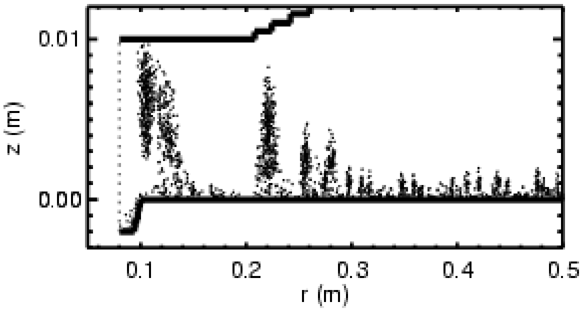

Figure 3. Particle plot for the inner 50 cm of the Z A-level MITL simulation at late time.

The most noticeable feature of these simulations is the unstable electron flow at small radius. The flow breaks up into large vortices extending across the gap for r < 20 cm, as illustrated in Fig. 3. The radial profile of the electron flow, averaged over a 20 ns time window, is shown in Fig. 4. It is seen that Eq. 2 is a very good fit to the flow profile, except at small radius. The same behavior is seen in Ref. 9. An important feature of Fig. 4 is that many of the electrons emitted at large radius are retrapped back to the cathode, rather than flowing into the convolute.

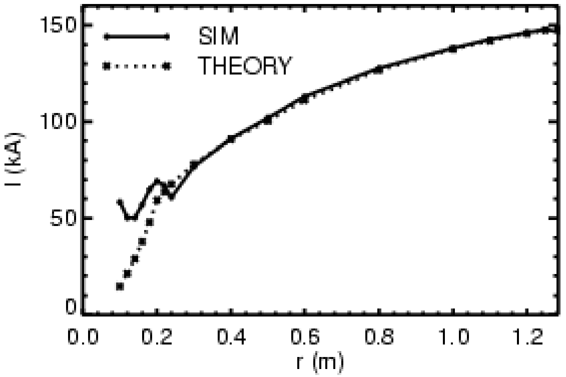

Figure 4. Radial profile of the electron flow, averaged over a 20 ns time window, at late time. Also shown is the profile predicted by Eq. 2.

It is instructive to compare these results with another simulation using a uniform impedance MITL — one in which d/r is constant. We chose Z_0 = 2.4 Ω, which gives a total MITL inductance of 9.3 nH, close to the baseline value of 9.1 nH. This MITL has inner and outer gaps of 4 mm and 5.2 cm respectively. Fig. 5 compares the time history of the electron flow into the convolute for these two simulations. The uniform-Z MITL does not have any electron vortices, and the fluctuation level of the flow is relatively small. On the other hand, we note the extreme enhancement to the average flow from the vortices for the baseline MITL. The baseline level is about 15 kA, consistent with Eq. 2, but the vortices increase the average net flow current by a factor of 3 – 4.

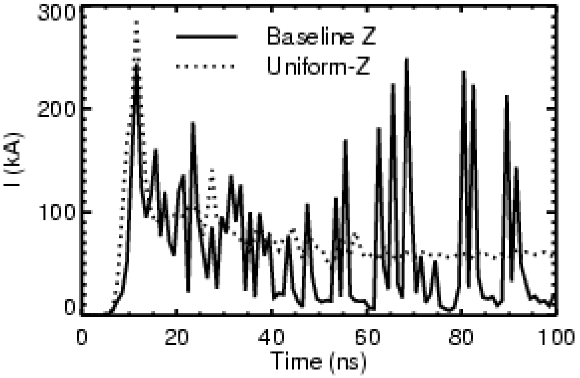

Figure 5. Time history of the electron flow into the convolute for the baseline and uniform-Z MITLs.

On Z, the very large ratio (~13) of the outer to inner radius makes a uniform impedance MITL impractical. For a reasonable total inductance, the inner gap is so small that the average net flow is no lower than Z, even without vortices. Gap closure is also a concern for small gaps, which is why the baseline setup uses gaps no smaller than 1 cm [11]. However, Fig. 5 shows that the net flow could be substantially reduced if the vortices could be removed.

Vortex formation appears to depend on the radial gradient of Z_0. In the baseline setup, $Y_0(r) \equiv -dZ_0(r)/dr$ increases monotonically moving inward from large radius. Denoting r_v as the radius at which vortices are launched into the gap (r_v ~ 30 cm from Fig. 3), $Y_0(r) < Y_v \equiv Y_0(r_v)$ for $r > r_v$. This suggests that vortex formation might be suppressed if a profile could be constructed in which Y_0 is always smaller than Y_v everywhere. This requires gaps smaller than 1 cm at inner radius (we used 6 – 8 mm), but larger than the uniform impedance MITL. We have empirically tested several different profiles, but with discouraging results. In no case was vortex formation suppressed. Although some profiles had slightly lower average flow into the convolute, it was not statistically significant, given that the vortices cause such huge perturbations in the flow.

IV. Z-PINCH LOAD SIMULATIONS

Ultimately, we are most interested in computing the electron flow for a Z-pinch load, driven with a time-accurate, forward-going wave. We set up a single MITL to be modeled with a 2-D PIC geometry, with the inner and outer radial boundaries attached to 1-D transmission lines (TLs) [7]. The other three levels are modeled entirely with 1-D TLs. The inner ends of the 1-D TLs for the four levels are coupled together in parallel with a transmission line model of the convolute and inner MITL. The inner line is terminated with a standard 1-D imploding Z-pinch load model [3]. This setup computes the flow into the convolute from one level, while accounting for the contribution to the load current from the other three levels. It is accurate provided that the flow currents and losses in the MITLs are small compared to the boundary currents, as is the case here, except for a short period of time before the MITLs are insulated.

In this study, it is important to do all four levels. We expect more flow from the lower levels because they operate at both higher voltage and lower current. For the Z A-level simulations, we could approximate the MITL as a purely radial line with <1% error. However, for the C and D levels, the MITL angles are ~35°, and we must model the actual biconic geometry. Because of the thin electron sheath, it is critical not to stair-step the cathode. To accomplish this, we do these simulations in spherical coordinates, with the MITL cathode located on a constant-θ conical surface.

For ZR, the MITLs come into the convolute at steeper angles, to give better diagnostic access to the Z-pinch load. The gap profiles are based on the Z profiles. They are constant 1 cm gap out to only 13.6 cm, and open up to 20% larger than the Z gaps for r > ~ 20 cm, to reduce the electron flow at higher voltage. Since the ZR MITLs are longer because of the steeper angles, and the gaps are also opened up, they have substantially higher inductance.

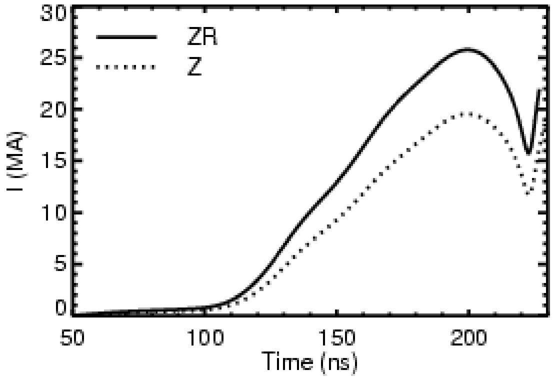

Figure 6. Time history of the load current for the ZR and Z A-level simulations.

The Z simulations use a Z-pinch load and forward-going wave from Z shot 540 [3]. The ZR simulations use the same waveshape, with $V_{oc,max}$ and the wire array mass scaled to give the same implosion time, but with a peak load current of ~26 MA. This allows us to compare with the Z results without the added complication of differences in the shape of the rising part of the pulse. The time history of the actual load current from the Z and ZR A-level simulations is shown in Fig. 6. The peak values are 25.8 and 19.6 MA respectively. The simulations for the B, C, and D levels agree with their corresponding A-level values to within 0.4%.

In the 2-D PIC MITL, the cells at the cathode start emitting electrons 8 ns after the normal E-field exceeds 100 kV/cm. Fig. 7 compares the flow current into the convolute on A-level. The initially larger flow for t < 140 ns occurs before the MITL insulates. The flow on ZR is lower because the relative increase in the boundary currents is higher than the voltage increase. After this early phase, the flow decreases, although electron vortices are clearly present. For t > 200 ns, the rising voltage as the load implodes causes the flow to increase once again.

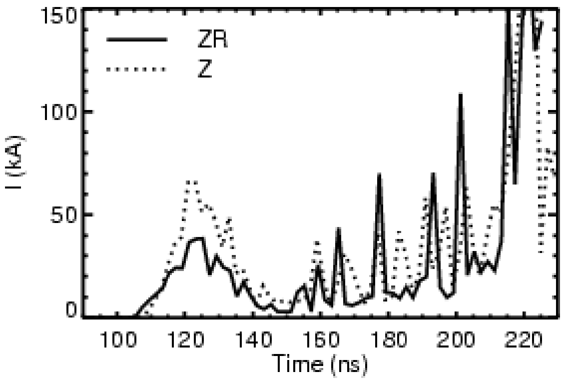

Figure 7. Time history of the electron flow current into the convolute for the ZR and Z A-level simulations.

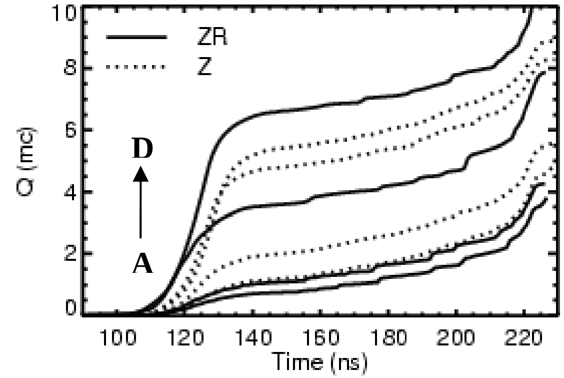

Figure 8. Net charge into the convolute on all four levels. For both Z and ZR, the charge increases monotonically from the lowest value on A-level to highest on D-level.

Fig. 8 shows the time integral of the flow current into the convolute. A-level has the lowest flow and net charge, followed, in ascending order, by B-, C-, and D-level. The net charge into the convolute for ZR is substantially smaller than Z for the A, B, and C levels. Only on D-level is the flow higher on ZR than Z. Summed over all four levels, the total charge flowing into the convolute for ZR is about 10% lower than on Z.

V. SUMMARY

We have performed a series of time-accurate, 2-D PIC simulations of all four MITLs on Z and ZR with a Z-pinch load. The new ZR MITL profiles limit the flow current into the convolute to slightly less than it is on Z today, while delivering 30% more current to the load. To accomplish this, the MITL gaps have been opened up by 20% over those on Z today. Although the increase in inductance is undesirable, another series of 2-D simulations with a resistive load suggest that there is no substantially better MITL profile that can reduce the flow current with lower inductance.

VI. REFERENCES

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