Low mass recyclable transmission lines for Z-pinch driven inertial fusion

S. A. Slutz and C. L. Olson
Sandia National Laboratories, Albuquerque, New Mexico 87185-1186

Per Peterson
University of California, Berkeley, California 94720-1730

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Recyclable transmission lines (RTLs) are being studied as a means to repetitively drive Z pinches. Minimizing the mass of the RTL should also minimize the reprocessing costs. Low mass RTLs could also help reduce the cost of a single shot facility such as the proposed X-1 accelerator and make Z-pinch driven nuclear space propulsion feasible. Calculations are presented to determine the minimum electrode mass to provide sufficient inertia against the magnetic pressure produced by the large currents needed to drive the Z pinches. The results indicate an electrode thickness which is much smaller than the initial resistive skin depth. This suggests that the minimum electrode thickness may be not be solely determined by inertial effects, but also by the ability of the electrode to efficiently carry the current. A series of experiments have been performed to determine the ability of the electrodes to carry current as a function of the electrode thickness. The results indicate that electrodes much thinner than the initial resistive skin depth can efficiently carry large currents presumably due to the formation of a highly conducting plasma. This result implies that a transmission line with only a few tens of kilograms of material can carry the large Z-pinch currents needed for inertial fusion. [DOI: 10.1063/1.1533789]

I. INTRODUCTION

Z-pinch physics has developed rapidly in the last few years. The use of wire arrays has resulted in the efficient conversion of pulsed power generated electrical current into thermal x rays. Nearly 2 MJ of thermal x rays have been generated by this approach[1] with an overall efficiency greater than 15%, and much higher efficiencies should be possible with pulse power machines optimized for efficiency. Z-pinch generated thermal x rays have been used to drive hohlraums[2] to temperatures greater than 145 eV, which is high enough to be of interest for driving inertial fusion capsules. One inertial fusion scenario[3] is to use two Z pinches to drive a central hohlraum containing a fusion capsule. Since Z-pinch implosions are subject to the Rayleigh–Taylor instability, this approach has the advantage of separating the nonuniformly emitting Z-pinch implosion from the inertial fusion capsule, but at the price of relatively low efficiency. Calculations[3] indicate that high yields (~0.4–1.2 GJ) could be obtained with 16 MJ of x-ray energy provided by two pinches driven with approximately 60 MA of current each. An alternate scheme[4] could provide much higher efficiency and thus lower the driver energy. In this "dynamic hohlraum" approach, a Z-pinch plasma is imploded onto a "convertor," which surrounds the capsule. Numerical simulations[5] indicate that a single pinch with 12 MJ of kinetic energy (55 MA) could drive a 0.5 GJ yield capsule, using this approach. Recent experiments indicate that the radiation generated within this convertor is relatively unaffected by the Rayleigh–Taylor instability[6] indicating that this higher efficiency approach may indeed be feasible.

Pulse power machines are robust and inexpensive when compared to other approaches for generating high energy densities, such as lasers or heavy ion beams, and the capability to operate reliably at high repetition rates has been demonstrated at small scale.[7] Thus pulsed power driven Z pinch could be an attractive approach to inertial fusion energy. However, a Z-pinch driven fusion explosion will destroy a portion of the transmission line that delivers the electrical power to the Z pinch. On the present Z machine, these electrodes are constructed from five tons of stainless steel. The cost of repairing the transmission line would outweigh the value of the energy created by the fusion explosion. Thus, up until recently, it has been assumed that this technology is limited to single-shot experiments.

Various means of providing standoff for Z pinch have been suggested. One possible approach[8] is to use a high velocity projectile to compress a seed magnetic field. The compression of the field can generate the large current required to drive a Z pinch. This approach has difficulty generating short current pulses and will require either a large area projectile, an opening switch or a very large $B \gg 1$ T seed field. The seed field could possibly be generated by an electron beam, but this results in a fairly complicated and probably expensive system. Another approach is to use an ion beam to deliver power to an inverse diode[9] as proposed by one of the authors (S.A.S.). The inverse diode is a magnetically insulated gap, which also serves as a transmission line to deliver current to the Z pinch. The cathode side would be constructed from a thin foil that allows the ions to pass through, delivering their current to the anode. This current then flows through the Z-pinch wire assembly and back to the cathode foil. A potential problem in the inverse diode is that the large space-charge of the beam current is sufficient to generate a virtual anode that could reflect the ion beam, unless electrons can effectively neutralize this space-charge. The effectiveness of the electron neutralization depends on electromagnetic fluctuations that allow them to cross magnetic field lines. Analytic theory[10] and numerical simulations[11] suggest this process is very effective. However, this process would have to be studied experimentally. Another potential problem is that the anode of the inverse diode will become a plasma and thus a source of ions. This will result in a loss of efficiency. The problem may be reduced by using a high-Z material for the anode, but surface contamination by hydrocarbons could still be a problem. This approach has complex physics issues that must be resolved. Also the inverse diode will be destroyed on each shot. Since it is a moderately complicated piece of equipment this approach may not be cost effective.

The most promising concept is the recyclable transmission line (RTL), which emerged at a workshop[9] at Sandia National Laboratories and was developed further at the Snowmass[12] workshop on fusion energy. This concept is much simpler than the two we have discussed previously. The idea is to construct the final portion of the transmission lines which delivers current to the Z pinch out of material that can be recycled inexpensively.

These RTLs could be formed inexpensively by casting of appropriate materials such as the reactor coolant flibe (fluorine, lithium, beryllium). Since flibe is an insulator, a conducting coating would be required. Preliminary experiments on the Saturn facility indicate that either lead or aluminum could be used.[13] Recent analysis suggest that an alloy of iron/carbon/tungsten would be a better choice for the conducting material. The components of this alloy are inert and immiscible in flibe, and will form a solid precipitate that can be recovered mechanically from the molten flibe by filtering and centrifugation processes. Activation products from these materials have relatively short half lives, so no long-lived radioactive waste would be generated. By maintaining carbon and tungsten concentrations around 1%–2% by weight, the iron maintains the properties of steel and can be formed using the same processes used for fabricating sheet-metal components of automobiles, although remote fabrication will be required due to the activity from short-lived activation products.

In this paper, we investigate the option of using low mass RTLs as a means of reducing the cost of reprocessing. Low mass RTLs could be used for inertial fusion energy, or to help reduce the cost of a single shot facility high yield facility such as the proposed X-1 accelerator. An additional application is pulsed nuclear space propulsion. Low mass RTLs are critical to this application because a portion of the RTL will become part of the rocket propellant. The portion of the RTL that impacts the craft could be captured and recycled. Since the portion of the RTL that is not captured is proportional to the total mass of the RTL, high specific impulses require low mass RTLs. Details of this application will be presented in a future publication. The present work will focus on the Z-pinch inertial fusion energy application.

Large currents are needed to drive Z-pinch inertial fusion. Numerical simulations[5] indicate that a current in excess of 55 MA will be required to drive a capsule with a fusion yield of 500 MJ. Numerical simulations, results of which will be published in a future article, indicate that substantially higher currents (~100 MA) will be required to drive capsules with a yield of several giga joules. Note that there are several high gain scenarios such as the fast ignitor concept[14] that could substantially reduce the required current. However, we shall assume that roughly 100 MA is required throughout the rest of this paper to maintain a conservative stance. High drive current generates very large magnetic pressure, which tends to push the electrodes apart. This motion is opposed by the inertial mass of the electrodes. The requirement that the gap between the electrodes does not change excessively during the current pulse places a minimum inertial mass for the electrodes. This is calculated in Sec. II.

It is found that the required areal density of the electrodes decreases strongly with radius and the outer portion of the RTL could be very thin. This calculated thickness is much smaller than the resistive skin depth of the cold electrode material. This suggests that there might be excessive resistive losses if very thin electrodes are used. It is difficult to calculate the magnitude of this effect, due to the complicated nature of surface breakdown phenomenon which can lead to highly conducting plasmas. Therefore experiments were performed to investigate the resistive effects of very thin electrodes. These experiments indicate that 20 μm of mylar is sufficient to carry the current with acceptable resistive losses. This result indicates that a transmission line with a mass as little as 2 kg could be used for energy applications. Note that we have assumed that the electrodes are thin sheets of material, possibly in the form of ribbons. An alternative idea,[9] suggested by Hammer, is to use an array of wires. This might allow even smaller transmission wire masses, but we were concerned that the explosion of the wires might cause unacceptable power flow losses so we pursued the more conservative scenario. The experiments on thin sheet electrodes are described in Sec. III. Some issues associated with the magnetic insulation of the transmission line are included in Sec. IV. A discussion of the results is provided in Sec. V.

II. CALCULATIONS OF MINIMUM RTL INERTIAL MASS

The RTL option for standoff is to construct a portion of the MITL (magnetically insulated transmission line) out of material that can be recycled. We wish to minimize the amount of material that is recycled each shot. The magnetic field generated by the current within the transmission line produces a pressure which pushes the electrodes apart. This pressure is typically much higher than the material strength of the electrode materials so the electrodes will accelerate away from each other during the current pulse. This outward motion of the electrodes increases the inductance of the transmission line making it more difficult to deliver the high currents required by the Z pinch. Assuming thin electrodes, the movement is determined by Newton's equation

$$F=\frac{B^2}{2\mu_0}A=M\frac{d^2x}{dt^2}, \tag{1}$$

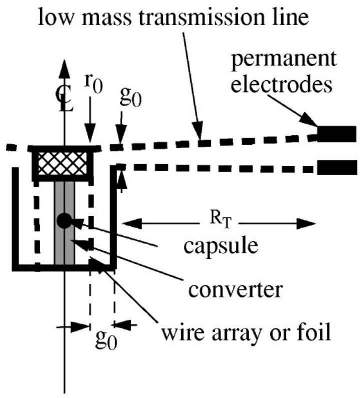

FIG. 1. A schematic of a low mass transmission line.

where x is the displacement of the electrodes and the magnetic field is determined by the relation $B=\mu_0 I/2\pi r$. The current profile produced by pulsed power accelerators such as Z can be approximated by

$$I=I_p\left(\frac{3\sqrt{3}}{2}\right)^{1/2}\tau\sqrt{1-\tau^4}, \tag{2}$$

where I_p is the peak current, $\tau=t/t_p$ and t_p is total current pulse length. This form admits an analytic solution to the Z-pinch implosion,[15] which we shall find useful. Equations (1) and (2) yield the result

$$x(t_p)=\frac{11\sqrt{3}}{224\pi\Gamma(r)}\left(\frac{\mu_0}{4\pi}\right)\left(\frac{I_p t_p}{r}\right)^2, \tag{3}$$

where $\Gamma(\mathbf{r})$ is the areal density (kg/m^2) of the electrodes. As can be seen from Eq. (3), the electrode motion is limited by the areal density (thickness) of the electrodes. Thus the acceptable amount of electrode motion determines the minimum areal density of the electrodes and hence the total mass of the transmission lines.

The electrode motion needs to be limited to a small fraction, Δg, of the transmission line gap, g. The outer portion of a low mass electrodes will have little shear strength. This limits this portion of the transmission line to sections of conics which can be supported by tensile strength alone. An example of this geometry is shown in Fig. 1. Note that the central portion of the RTL needs to be much thicker since the magnetic field pressure scales as r^{-2}. Therefore the central section can have a more complicated structure such as that depicted in Fig. 1. Power flow experiments indicate that the gap must remain finite near the pinch. Therefore we assume a gap given by

$$g=g_0+\Delta\theta(r-r_0). \tag{4}$$

Using Eqs. (3) and (4) we can solve for the areal density of the electrodes

$$\Gamma(r)=\frac{11\sqrt{3}}{112\pi}\left(\frac{\mu_0}{4\pi}\right)\frac{t_p^2 I_p^2}{\Delta g r^2[g_0+\Delta\theta(r-r_0)]}, \tag{5}$$

which scales roughly as r^{-3} for $r\gg r_0$. At some radius, r_x, the areal density of the electrode as calculated by Eq. (5) could be smaller than a minimum, Γ_n, required for structural strength or to conduct the current with acceptable resistive losses. We shall assume that $\Gamma=\Gamma_n$ for $r>r_x$. The total transmission line mass, assuming two electrodes, is then found from the integral

$$M_{\mathrm{tot}}=2\int_{r_0}^{R_T}2\pi r\Gamma(r)dr \tag{6}$$

with the result

$$M_{\mathrm{tot}}=\frac{11\sqrt{3}}{28\Delta g}\left(\frac{\mu_0}{4\pi}\right)\frac{t_p^2 I_p^2}{[g_0-\Delta\theta r_0]}\ln\left\{\frac{g_0 r_x}{r_0[g_0+\Delta\theta(r-r_0)]}\right\}+2\pi\Gamma_n(R_T^2-r_x^2). \tag{7}$$

Since this function decreases monotonically with $\Delta\theta$, it is instructive to consider the limit $\Delta\theta=\Gamma_n=0$, which yields

$$M_{\min}=\frac{11\sqrt{3}}{28\Delta g}\left(\frac{\mu_0}{4\pi}\right)\frac{t_p^2 I_p^2}{g_0}\ln\left(\frac{R_T}{r_0}\right). \tag{8}$$

Using values appropriate for a fusion reactor, i.e., $I_p=100$ MA, $r_0=3$ cm, $R_T=4$ m, $t_p=150$ ns, and $\Delta g=0.1$, we obtain the surprisingly small result that $M_{\min}=0.37$ kg. This should be compared to the mass of the transmission line in the Z accelerator, which weighs approximately 5 tons. Note that four transmission lines are used in the present Z accelerator with a current adding convolute just before the Z pinch. Using only two electrodes increases the voltage requirement, as calculated in Sec. IV. The higher voltage requirement should be attainable with a properly modified accelerator using present technology. Using these same parameters, Eq. (5) yields

$$\Gamma(r)=\frac{11\sqrt{3}}{112\pi}\left(\frac{\mu_0}{4\pi}\right)\frac{t_p^2 I_p^2}{\Delta g r^2 g_0}=\frac{6.1\times10^{-3}}{r^2}\ \mathrm{kg/m^2}, \tag{9}$$

where r is in meters. At a 1 m radius, this corresponds to an electrode thickness of 0.75 μm for steel or 6 μm for plastic. If such extremely thin electrodes cannot be constructed which can carry the large currents needed to drive the Z pinch, the mass of the transmission line will be dominated by the second term in Eq. (7) and to a good approximation the transmission line mass needed to drive a fusion capsule is given by the simple expression

$$M_{\mathrm{tot}}=2\pi\Gamma_n R_T^2. \tag{10}$$

In Sec. III we present the results of experiments on the Saturn facility to determine the appropriate value of Γ_n.

III. LOW MASS TRANSMISSION LINE EXPERIMENTS

In Sec. II it was found that, at the outer portion of the low mass RTL, very thin electrodes have sufficient inertia to resist the current generated magnetic pressure. This electrode thickness is much smaller than the resistive skin depth of typical cold electrode materials (~50 μm for aluminum). This suggests that there might be excessive resistive losses if very thin electrodes are used. However, it is difficult to calculate the magnitude of this effect, due to the complicated nature of surface breakdown phenomenon, which can lead to

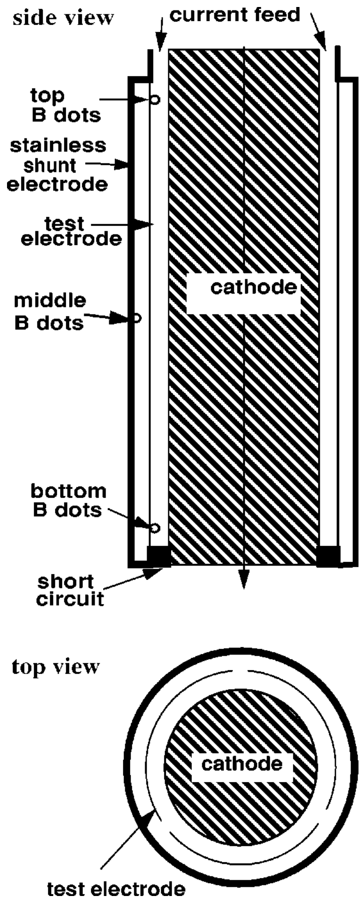

FIG. 2. A schematic of the experimental design.

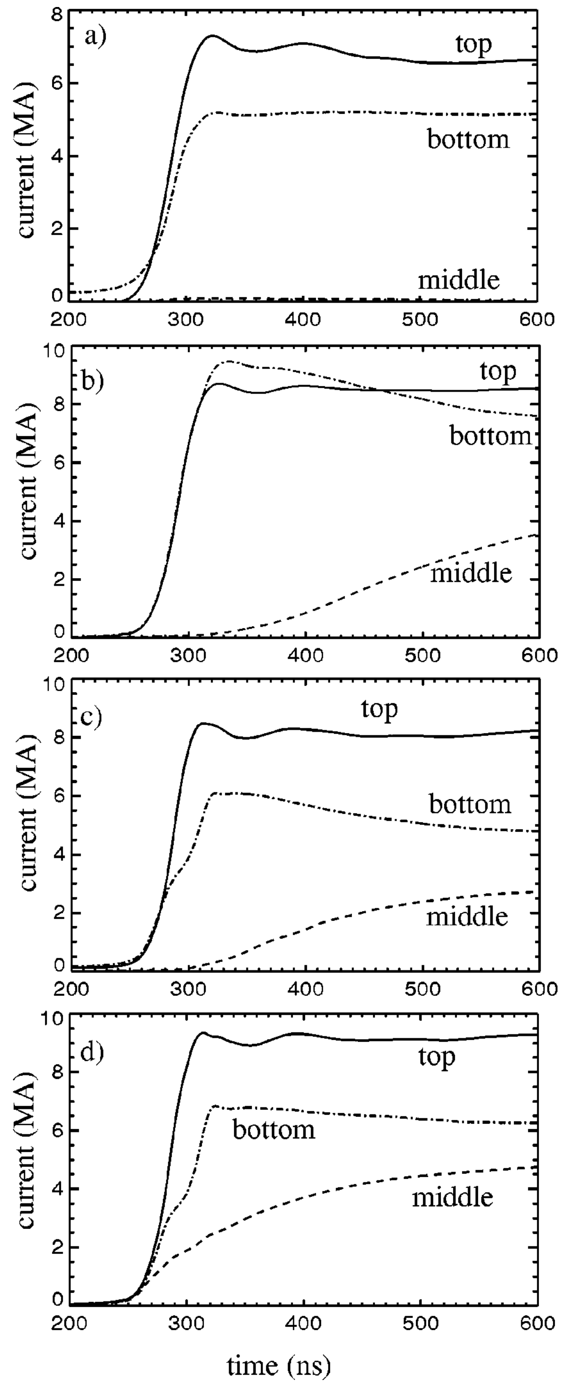

FIG. 3. The currents plotted as a function of time for (a) 250 μm carbon steel, (b) 100 μm carbon steel, (c) 50 μm carbon steel, and (d) 20 μm of mylar.

highly conducting plasmas. Therefore we designed an experiment to investigate the resistive effects of very thin electrodes. A schematic of the experimental setup is depicted in Fig. 2. Current is fed from the top by the Saturn accelerator with a nominal maximum short circuit current of about 10 MA. The test hardware is a coaxial transmission line with a short circuit at the end furthest from the accelerator (bottom). Since the transmission line is only 30 cm long it acts as a lumped inductance of approximately 4 nH. This brings the peak expected current down to about 9 MA. The hardware is divided into three azimuthal sections of 120° each, which are held together on the top and bottom by rings. Current monitors (Bdots) are placed in each of the azimuthal segments at the positions labeled in Fig. 2. The top and bottom Bdots determine the degree of magnetic insulation obtained within the transmission line. The top and bottom Bdots should have the same current profiles if the transmission line is 100% insulated. If the test electrode has a significant resistance, current will flow through the shunt electrode. This current is monitored by the middle Bdots.

Three carbon steel test electrodes of thicknesses 250, 100, and 50 μm were tried. Carbon steel has been identified as an excellent electrode material for Z-pinch driven fusion energy, due to its low activation and good separability from the reactor coolant material flibe. We also tried a 20 μm mylar test electrode as a possible candidate material for pulsed nuclear space propulsion.

The measured currents for these four shots are plotted in Fig. 3. A significant loss of current just below the top Bdot was indicated by post-shot inspection of the hardware for all of the shots except the 100 μm steel (case b). This is consistent with the measured currents at the top being larger than the bottom, except for case b, where the bottom current is slightly higher than at the top for a period of time. Since this is not actually possible, the difference gives some measure of the accuracy of the measurements. We speculate that the current loss was caused by the glue which was used to attach the test electrode just below the top Bdot monitor. We will use

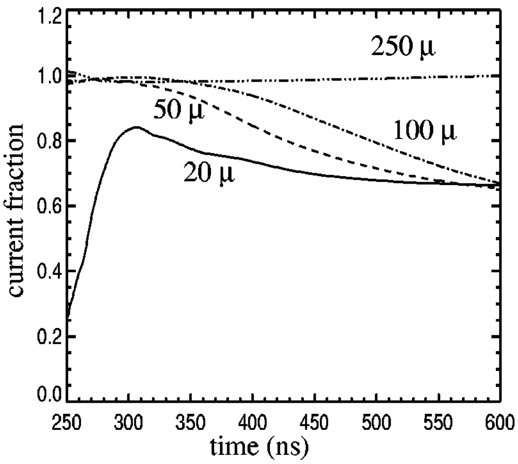

FIG. 4. The fraction of the current carried by the test electrode as a function of time.

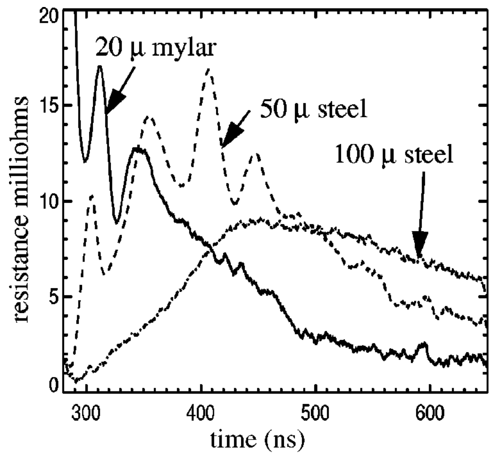

FIG. 5. The effective resistance of the test electrodes is plotted as a function of time.

the bottom Bdot current as a measure of the current that was carried by the test electrodes.

The current measured by the middle Bdots is considerably lower than either the top or the bottom for all of the shots. This is the current carried by the shunt electrode. As can be seen this current is negligible for the 250 μm case and increases as the test electrode thickness is decreased. Thus most of the current is being carried by the test electrode during the time of a typical Z-pinch implosion ($\sim$1.2–1.3 times the current rise time or about 100 ns for the Saturn accelerator). The top and bottom current monitors remain near the peak current for 400 ns after the initial rise. This behavior has often been seen with Bdot monitors. It is often assumed that the Bdots ''flash'' (short out possibly due to plasma formation in the loop) and do not register the negative voltage required to bring the integrated current back down. However, the rising current measured by the middle Bdots in this experiment implies that there is at least 4 MA of current carried by the cathode 400 ns after the initial current rise. This suggests that the top and bottom Bdots have not flashed and are actually measuring current which takes a long time to be resistively damped.

The fraction of the current carried by the test electrode is plotted for each shot in Fig. 4. The current carried by the test electrode is the difference between the bottom current and the middle current. This is then divided by total current, which is given by the bottom monitor. Note that the fraction of the current carried by the test electrode is high for all the carbon steel shots and that the fraction increases with thickness as we would expect. The 20 μm mylar electrode appears to start off as an insulator and then breaks down to carry most of the current. It should be noted that the fraction of the current carried by the test electrode would increase if the shunt electrode was moved further away due to increasing inductance. In a reactor scenario the first wall would play the role of the shunt electrode and due to the very large inductance would carry an insignificant current. Thus the fraction of the current carried by the test electrode is only a qualitative measure of performance.

The data can be used to obtain an effective resistance of the test electrode. The current carried by the test electrode is the difference between the bottom and the middle currents. The voltage across the test electrode is approximately LdI/dt, where $L \sim 4$ nH is the inductance between the test and shunt electrodes (gap$=$3 mm) and dI/dt is given by the unintegrated middle Bdots. Dividing the voltage across the test electrode by the current it carries yields a measure of the resistance of the test electrode. This effective resistance is plotted for each of the shots (except 250 μm steel) in Fig. 5. The 250 μm steel test electrode showed negligible effective resistance. This is presumably because the electrode was too thick for the magnetic field to diffuse through during the pulse. The rising portion of the resistance for the other steel test electrodes is probably due to the finite time it takes for magnetic diffusion. The falling portion of the curve should be more representative of the true resistance of the electrode. Note that the 20 μm mylar test electrode starts with a high effective resistance. This is because the mylar is initially an insulator and the magnetic field diffuses quickly through it. As the mylar breaks down the effective resistance drops below the thicker steel electrodes, probably because a relatively low atomic number plasma is formed at the surface.

We estimate the energy lost due to joule heating from the integral $E_{\mathrm{joule}}=\int_0^t R_{\mathrm{eff}}(t)I(t)^2dt$, where $I(t)$ is the full current entering from the test electrodes. The maximum energy that Saturn could deliver to a Z-pinch load ($E_{\max}\sim 140$ kJ). We plot the ratio of $E_{\mathrm{joule}}/E_{\max}$ as a function of time in Fig. 6. An optimal Z-pinch implosion occurs a little after peak current ($\sim$350 ns) in this experiment. At that time the resistive losses are approximately 7%, 32%, and 56% for 100 μm steel, 50 μm steel, and 20 μm mylar, respectively. The progression of increased losses with decreasing electrode thickness is clearly indicated. The Z-pinch energy and the resistive loss both scale as I^2, but the resistance also scales as L_T/r_T, where L_T is the length of the transmission line and r_T is the radius. If we scale our experimental data up to a transmission line carrying 100 MA of current we must increase the transmission line radius by roughly a factor of 10 ($r_T=40$ cm) to maintain the same current density. Thus the fractional loss will be the same for a transmission line length

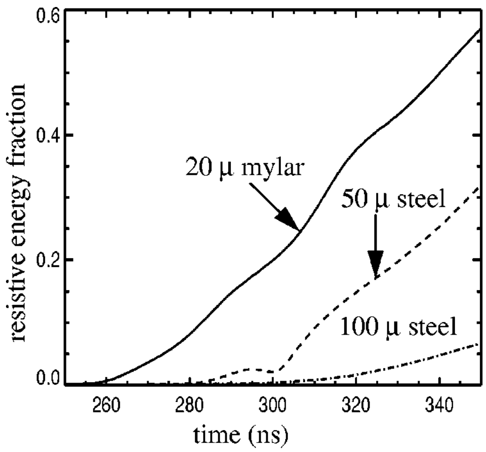

FIG. 6. The calculated resistive loss in the test electrode is plotted as a function of time.

of 3 m. The required standoff distance is not well known. Calculations indicate[16] that 4 m should be adequate even for yields of several giga joules. Assuming $R_T=4$ m, we have estimated the resistive losses for each test electrode material. We also use Eq. (10) to estimate the total mass of the low mass RTL assuming the minimum areal density is the same as each of the test electrodes. The results are summarized in Table I.

The resistive losses should be considered only a rough estimate, since that actual geometry will not be the same as the experiment (conics rather than coaxial). These results indicate that the low mass RTL mass can be quite modest (~80 kg) with a very small resistive loss (~7%). Substantially lower masses could be used with an acceptable increase in resistive losses. Pulsed power accelerators have been designed with efficiencies approaching 50%. In Sec. IV we show that roughly 1/2 of the forward going power delivered by the accelerator can be converted to kinetic energy of a Z pinch. Thus the overall efficiency of a system using 20 μm mylar electrodes would be approximately 10%. Since the conversion of Z-pinch kinetic energy into radiation is very efficient,[1] we can expect an overall efficiency of about 10%. This is competitive with the most efficient proposed laser drivers. Increasing the electrode mass to tens of kilograms would double the efficiency. Such low masses should reduce the cost of recycling transmission lines in an inertial fusion energy application. However, Eq. (10) is not an adequate approximation for applications such as microfission or magnetized target fusion that use long current pulses (>150 ns). We then need to determine the appropriate value of $\Delta\theta$, which is determined by requirements of power flow in the transmission line.

TABLE I. Summary of experimental results.

Electrode	Γ (kg/m^2)	M_{tot} (kg)	Resistive loss%
20 μm mylar	0.02	2.0	55
50 μm steel	0.4	40.0	32
100 μm steel	0.8	80.0	7
250 μm steel	2.0	200.0	<1

IV. POWER FLOW

A. Magnetic insulation

We shall now find the relationship between the value of $\Delta\theta$ and the degree of self-magnetic insulation of the low mass RTL. The condition for magnetic insulation is approximately

$$V < Bgc, \tag{11}$$

where V is the voltage applied to the transmission line. The time for the current pulse to propagate through the transmission line will be short ($\sim R_T/c \sim 10$ ns) relative to the rise time of the pulse (>100 ns) and so it will act approximately as a lumped inductance. The voltage along the transmission line will then be given by

$$V(r)=L_{\mathrm{TL}}(r)\frac{dI}{dt}+\frac{d}{dt}(L_p I), \tag{12}$$

where L_p is the inductance of the Z pinch and $L_{\mathrm{TL}}(r)$ is the inductance of the transmission line between the pinch and a radius, r. The inductance of the Z pinch increases as the pinch collapses. The radius of the pinch can be calculated analytically for the current profile of Eq. (2) with the result $r=r_0(1-\tau^4)$. The pinch inductance is thus

$$L_p \leq \frac{\mu_0}{2\pi} l_p \ln\left(\frac{r_0+g_0}{r_0(1-\tau^4)}\right), \tag{13}$$

where g_0 is the initial gap between the pinch and the return current electrode, r_0 is the initial radius of the pinch plasma, and l_p is the length of the pinch. We place an upper limit on the pinch inductance during the rising portion of the current pulse by using the pinch radius at peak current, i.e., $r=2r_0/3$. Using $g_0=2$ mm, $r_0=3$ cm, and $l_p=3$ cm we find that $L_p \sim 3$ nH.

The inductance of the transmission line is

$$L_{\mathrm{LT}}(r)=\frac{\mu_0}{2\pi}\left\{(g_0-\Delta\theta r_0)\ln\frac{r}{r_0}+\Delta\theta(r-r_0)\right\}. \tag{14}$$

Substituting Eqs. (13) and (14) into Eqs. (12) and Eq. (11), then setting $dI/dt=I_p/t_r$, and using the maximum value of L_p, we find that $\Delta\theta \geq \theta_{\min}$, where

$$\theta_{\min}=\max\left(\frac{g_0\left(\ln\left(\frac{r}{r_0}\right)-\frac{F_L c t_r}{r}\right)+\frac{2\pi L_p}{\mu_0}}{(R_T-r_0)\left(\frac{F_L c t_r}{r}-1\right)+r_0\ln\left(\frac{r}{r_0}\right)}\right), \tag{15}$$

for $r_0 \leq r \leq R_T$, where F_L is the fraction of the peak current needed before the transmission line is magnetically insulated, and t_r is the time to peak current. Defining

$$\alpha=\frac{F_L c t_r}{R_T}, \tag{16}$$

we find

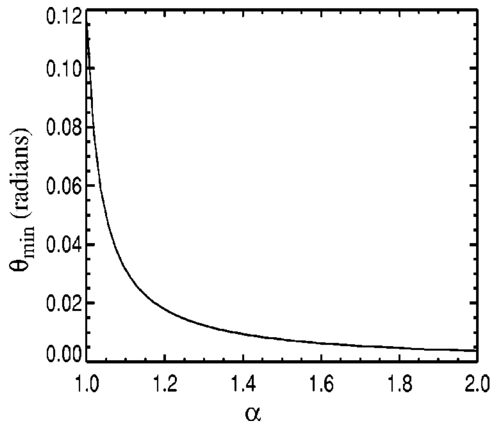

FIG. 7. The calculated minimum angle between the transmission line electrodes is plotted as a function of α.

$$\theta_{\min}=\frac{g_0\left(\ln\left(\frac{R_T}{r_0}\right)-\alpha\right)+\frac{2\pi L_p}{\mu_0}}{(R_T-r_0)(\alpha-1)+r_0\ln\left(\frac{R_T}{r_0}\right)}. \tag{17}$$

Setting $R_T=4$ m, $r_0=3$ cm, and $L_p=3$ nH, the minimum angle is plotted as a function of α in Fig. 7. As can be seen, the minimum angle decreases strongly with increasing α. Substituting this result into Eq. (7), we calculate the transmission line mass which is plotted in Fig. 8 as a function of α. The parameters of the calculation were chosen to be consistent with a high yield capsule suitable for energy. The parameters are $\Delta g=0.1$, $I_p=100$ MA, $\Gamma_n=0.02$ kg/m^2. The mass increases for small angles (large α) because the gap between the electrodes is smaller and less electrode motion can be tolerated to maintain the same Δg. The smallest value of t_r corresponds to the present Z machine and the mass is nearly independent of α because the second term in Eq. (7) dominates. As can be seen the first term of Eq. (7) cannot be ignored as t_r is increased. Clearly the transmission line mass is minimized by decreasing α. Note that α must be greater than zero to maintain a finite value of $\theta_{\min}$. However, increasing $\Delta\theta$ increases the transmission line inductance and thus the voltage needed to drive the current. Using Eqs. (12) and (14), the driving voltage has been calculated. The results are shown in Fig. 9. As can be seen, the voltage is decreased by increasing the rise time of the current pulse and by increasing α.

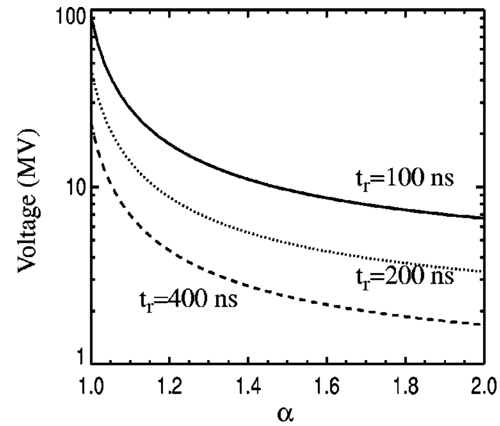

FIG. 9. The required driving voltage is plotted as a function of α for several values of the current rise time.

Since the power is the product of current times voltage, it is clear that the total energy delivered to the transmission line will increase with driving voltage. However, the energy delivered to the pinch is only dependent on the current as given by[15]

$$E_p=\sqrt{3}l_p\left(\frac{\mu_0}{2\pi}\right)I_x^2. \tag{18}$$

Thus the efficiency of delivering energy to the pinch will increase with α. We calculate this efficiency using Eqs. (2) and (11)–(18). The result is plotted in Fig. 10 for two values of the transmission line radius. As can be seen, the efficiency is not a strong function of the transmission line radius. Note that it is independent of the current rise time at a fixed value of α.

Equation (16) indicates that increasing α corresponds to greater current lost before the transmission line becomes self-magnetically insulated. If the transmission line remains uninsulated for too long, surface plasmas may be formed. These plasmas could cause unacceptable leakage current

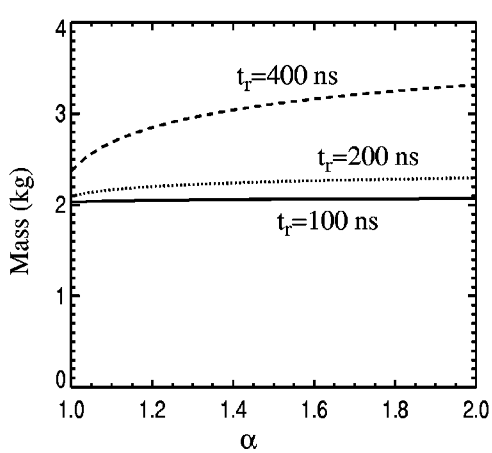

FIG. 8. The RTL mass plotted as a function of α for several values of the current rise time.

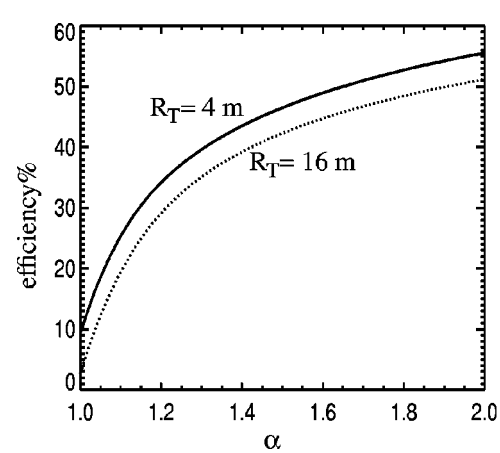

FIG. 10. The efficiency is plotted as a function of α for two values of the transmission line outer radius.

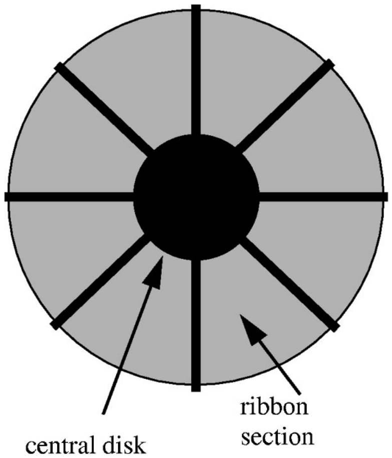

FIG. 11. A schematic of a disk electrode composed of triangular ribbon sections.

later in the pulse. Therefore there is a maximum practical value for α. The existing Z accelerator is self-insulated at about 0.15 of the peak current, i.e., $F_L = 0.15$. This corresponds to $\alpha \sim 1.5$ for a 150 ns rise time. Experiments will be needed to determine if larger values of α can be used and if the value depends on the rise time of the current pulse. Using the value $\alpha = 1.5$, the required driving voltage is approximately 8 MV and the efficiency is approximately 44% for a 150 ns rise time current pulse. The present Z accelerator is driven with about 3 MV, so this is not a very large extrapolation. The efficiency of 44% is quite acceptable. In principle this could be improved if a scheme to recapture the magnetic energy were devised.

B. Ion losses

The results of our experiments indicate that 20 μm mylar electrode may be about the minimum electrode thickness, thus $\Gamma_n \sim 0.02\ \mathrm{kg/m^2}$. A 20 μm foil is fairly strong, but would wrinkle easily. These wrinkles could be smoothed out by applying outward tension at the transition between the permanent electrodes and the low mass RTL. There may be some difficulty removing all of the wrinkles if the electrodes are a complete disk. A convenient alternative is to use trapazoidally shaped ribbons as shown in Fig. 11. This arrangement should make it much easier to remove wrinkles, but the edges could enhance a plasma breakdown process. This could be a concern if such a plasma forms on the anode side of the transmission line and allows ions to be accelerated across the gap. This ion current would not be delivered to the Z pinch and thus is a loss. The magnitude of this effect can be estimated as follows. Assume that some fraction, f_A, of the available anode area forms a plasma that acts as a space-charge-limit ion source. The ion current density is then given by the Child–Langmuir law

$$J = \frac{4}{9}\varepsilon_0 \sqrt{\frac{2e}{m}}\,\frac{V^{3/2}}{d^2}. \tag{19}$$

If we assume $d = r\Delta\theta$, the voltage found from Eq. (12) is simplified to the following form:

$$V \approx \frac{\mu_0}{2\pi}\Delta\theta \frac{I_p}{t_r} r. \tag{20}$$

We then perform the integral

$$I_{\mathrm{loss}} = \int_0^{R_r} 2\pi J r\, dr \tag{21}$$

with the result

$$\frac{I_{\mathrm{loss}}}{I_p} = \frac{8}{27}\left(\frac{e\mu_0 I_p}{\pi m \Delta\theta}\right)^{1/2} \frac{f_A R_T^{3/2}}{c^2 t_r^{3/2}} \approx 0.26 f_A \tag{22}$$

assuming $I_p = 100$ MA, $L_s = 4$ m, and $\Delta\theta = 0.01$, which corresponds to $\alpha = 1.5$. Since plasma will only be formed at the boundaries, $f_A \ll 1$ and the ion loss current should be negligible. This analysis suggests that ribbons could be used. In fact the experiments reported in Sec. III used three sections.

Note that the existing Z accelerator delivers power efficiently through four transmission lines with an outside radius of about 2 m. This implies that most of the anode area of these transmission lines does not become a space-charge-limited source of ions. Experiments will be needed to determine if this benign behavior persists at the high current densities required to drive fusion capsules.

V. DISCUSSION

We have investigated the issue of minimizing the mass of a transmission line which delivers current to a Z pinch. We have shown that electrode thickness needed to provide sufficient inertia against the current induced magnetic field decreases strongly with radius and corresponds to very thin electrodes at the outer portion of the transmission line. We have performed experiments that indicate a minimum electrode thickness is required to avoid excessive resistive losses. These experiments indicate that 20 μm of mylar is sufficient to carry the current with acceptable resistive losses. This result indicates that a transmission line with a mass as little as 2 kg could be used for fusion energy applications. Increasing the RTL mass to a few tens of kilograms would result in negligible resistive losses. Reducing the mass of the transmission line will lower the cost of recycling the RTL, but an even more important effect is to lower the momentum delivered to the reactor vessel and the current feeds. Material close to the fusion explosion will be vaporized. The blast from this material should be effectively stopped by a screen of liquid, e.g., a flibe waterfall. At some distance from the explosion, the material will fragment, but remain solid. It may be more difficult to shield against this shrapnel. It is clear that minimizing the mass of the transmission line at large radii will be an advantage. In fact a significant portion of the outer transmission line will be vaporized by the current driving the Z pinch. Detailed calculations of these processes will be needed to determine if the RTL is a viable concept for obtaining fusion energy with Z pinches.

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