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Equation of State and Electron Transport Effects in Exploding Wire Evolution*

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Abstract

Accurate MHD modeling of a single exploding Al wire is a prerequisite to being able to model a wire array. We have compared our simulation results to the high-quality laboratory measurements from exploding-wire experiments recently done at Cornell University. Exploding wire simulations have identified regions where our transport models must be most accurate. Improvements we have made in these regions (*e.g.*, solid density near melt, and the metal-insulator transition) have proven to be critical for achieving accurate simulations.

I. INTRODUCTION

The detailed evolution of each exploding wire comprising a cylindrical array determines the energy and power densities ultimately achievable in a z-pinch implosion of the array. Presumably, once the relationship between exploding wire evolution and z-pinch dynamics is understood sufficiently, one could design a z-pinch experiment that would produce the optimum behavior for a given application. Before attempting to learn how exploding wire evolution affects z-pinch behavior it is crucial to be able to accurately model the single exploding wire. Toward that end we have been doing single-exploding-wire simulations in the z-r plane with the MHD code MACH2 [1,2] and with Sandia's new code ALEGRA [3]. We have so far limited our study to Aluminum because its theoretical description is simple compared to higher-Z metals. Only the radial evolution is considered. Our standard configuration corresponds to recent, well diagnosed exploding aluminum wire experiments from Cornell [4].

The transition to a coronal state is a key feature inferred from the wire voltage measured in the experiments. In this state most current flows through the rapidly-expanding, low-density ionized vapor at large radius. This transition is rapid under the conditions considered, and coincides with a voltage collapse.

The importance of good exploding wire data cannot be overstated. Experimental details of the exploding wire evolution provide both qualitative (voltage collapse) and quantitative (time of voltage collapse, peak voltage, and core expansion rate) features that simulations must match if they are to be considered accurate.

Plotting either the conductivity or pressure parametrically with radius across the wire at a given time as a function of density and temperature shows graphically the mutual relations between the EOS and the electrical conductivity during the exploding wire's evolution. This view has shown us that the path in phase space that an exploding wire takes in its evolution from solid metal to high-temperature plasma is quite sensitive to the EOS and conductivity models used. Our EOS choices and efforts to refine the electrical conductivity have been guided by comparing the simulations with the Cornell data. We have maintained as a constraint the consistency of our models with known EOS and conductivity data from independent experiments [5] and with modern physical descriptions appropriate in the parameter regime of the metal-insulator transition [6].

II. MODEL DESCRIPTION

A. Brief Description of MACH2

The MACH2 Code solves the time-dependent magnetohydrodynamics (MHD) equations for on block-structured grid composed of arbitrary hexahedrals. The code is of the ALE (Arbitrary Lagrangian-Eulerian) variety which allows the grid to move independently of the magnetofluid. The physical processes included in the model are diffusion, Lagrangian hydrodynamics, and advection. Details of the algorithms are provided in references 1 and 2.

The geometrical domain is block structured which allows for the modeling of complex geometries. MACH allows for numerous boundary conditions in the 2-D plane which are specified by the user. An important feature is that new EOS/conductivity/opacity models are easily implemented.

B. Brief Description of ALEGRA

Developed by Sandia National Laboratories, ALEGRA is an Arbitrary Lagrangian-Eulerian Finite Element Code in 1D, 2D, or 3D. It emphasizes large distortion and shock propagation problems. Designed to run on distributed-memory parallel computers, it utilizes adaptive mesh refinement techniques. Physics options include Hydrodynamics, Solid dynamics, Structural dynamics MHD, and Radiation MHD. Material models include various Equation of State models, Electron Transport and Opacity models, Constitutive, Yield, Plasticity, Fracture, and Burn models.

ALEGRA produces accurate results for numerous problems with known solutions; *e.g.*, impact generated shocks, and cold diffusion.

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III. ELECTRICAL CONDUCTIVITY MODEL

The fidelity of our calculations depends strongly upon an accurate knowledge of the material resistivities over a wide range of temperatures and densities. Of particular importance for these simulations is the regime defined by temperatures below a few eV and densities below solid, especially in the vicinity of the metal-insulator transition. A previous work [7] describes the importance of improvements we have made in electrical conductivity, building on the Lee-More model [8] and incorporating information from the experimental data of DeSilva [5]. The Modified Lee-More, or Lee-More-Desjarlais (L-M-D) [6] conductivity tables provided a vast improvement for doing meaningful MHD simulations in the ultra-high current density regime of 1-10 MA/cm.

Quantum mechanical molecular dynamics simulation tools, recently made available by University of Vienna [9-11], have been applied to refining the EOS and electrical conductivity modeling for densities near the metal-insulator transition. Our simulations show that exploding wire behavior is very sensitive to conductivity in this regime.

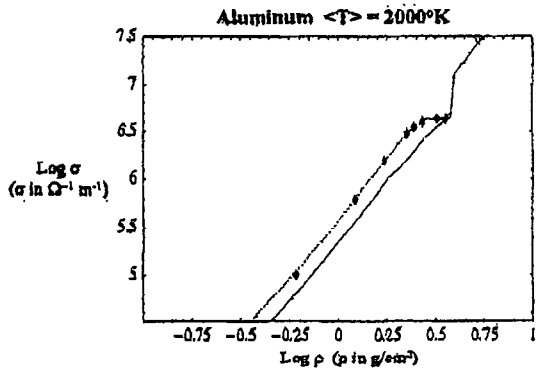


Figure 1. Conductivity isotherm (2000°K) from original Lee-More-Desjarlais conductivity model (lower curve) and the same isotherm based on molecular dynamics simulations. Points are from VASP [9-11] simulations.

IV. SIMULATIONS OF CORNELL PULSE 1588

The simulations discussed below all attempt to model Cornell single-exploding, 13-μm diameter, Aluminum wire shot 1588 [4]. Every simulation was energized by applying the Cornell current waveform as a B_ϕ boundary condition at the outer radius of the problem. MACH simulations were arbitrary Lagrangian-Eulerian (ALE) in a way that preserved the number of radial cells at the outer edge of the radially expanding problem. This is accomplished at the expense of radially stretching the cells near the axis. ALE allows sufficient resolution of the corona formation process with a manageable number of cells (32-128). The simulation space expands with the corona reaching out past 1 mm after 500 ns. The ALEGRA simulations were run Eulerian using multiple blocks, each zoned with interval numbers appropriate for the region; *e.g.*, the block that covers the region where the corona begins forming has more intervals than in a large block covering the outer region of the corona late in the pulse. In spite of these different approaches, and major algorithm differences, excellent agreement was observed between MACH and ALEGRA simulations with comparable resolution and the same EOS and conductivity model.

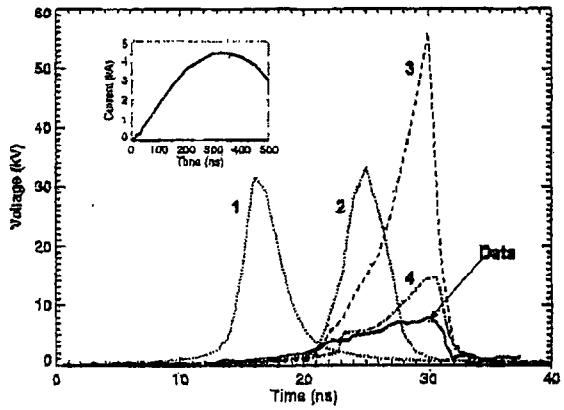


Figure 2. (1-4) Simulation Voltage waveforms. The solid curve is the measured voltage from the Cornell experiment. Inset is the Current waveform measured in the experiment, also used to drive the simulations.

Curves 1 and 2 of Fig. 2 illustrate the large discrepancy between MHD simulations and data when the SESAME 23714 Aluminum conductivity [12] (1) (where V/10 is plotted), or the optimized Lee-More conductivity model (2) are used in the calculation. No corona forms in (1), while a weak corona forms early in (2). The corona is weak in the sense that the current transfer from core to corona is gradual rather than rapid. This corresponds to a symmetric voltage pulse rather than one that collapses.

The first Lee-More-Desjarlais conductivity model (3) brings the simulation voltage in line with the features of the data, but gives too great a resistivity for liquid Aluminum. The recent Lee-More-Desjarlais model (4) with its extended region of high conductivity improves this significantly. The energy deposited in the wire, shown in Fig. 3 for the experiment and the simulation (4 of Fig. 2) also indicates reasonable agreement. Approximately 25 mJ was deposited in the wire prior to corona formation.

A more complex comparison with the experiment is the detectable diameter of the dense wire core and its expansion rate. The experiment [4] starts with radiographs of the exploding wire at different times. The radial edge-to-edge width of the radiographic image corresponds to the detectable diameter of the wire core. The edge image density is essentially indistinguishable from the noisy

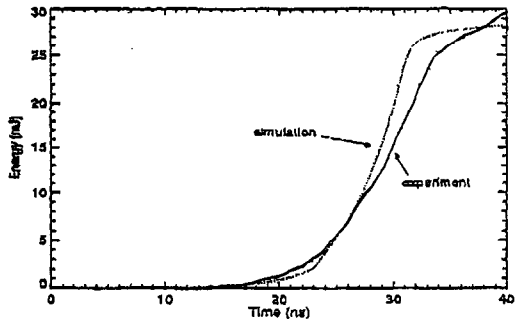


Figure 3. Comparison of deposited energy for Cornell experiment and simulation.

background density of the exposed film. The radiographs, produced by x-rays projected through the circular cross section of the wire, correspond to the Abel transform of the wire's radial density function at each exposure time. Hence, we inspect the Abel transforms of the simulation $\rho(r)$ snapshots for the roughly 50% point (taking into account an assumed background noise level of the exposed film, equivalent to 100 kg/m^2) to ascertain an expansion velocity in a manner that can be compared to the experiment. The result in Fig. 4 shows remarkably good agreement with the experimental points. Like the experiment there is little discernible core expansion before 100 ns. After this time the detectable diameter expands at a near-constant rate of 2.6 μm/ns, close to the experimental value of 2.0 μm/ns.

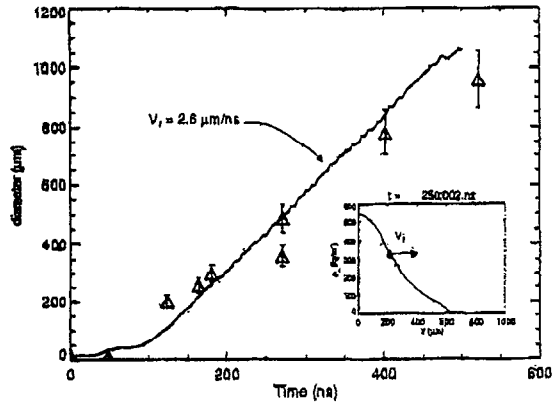


Figure 4. History of simulation core diameter. Data points (triangles with error bars) are from experiment [4]. Inset shows the Abel Transform of a simulation $\rho(r)$ snapshot with a radial velocity vector indicated for the 50% point.

Next, we examine simulation data for details of the corona formation. ALEGRA's "void" treatment [3] allows a highly resolved view of the hot, low-density corona plasma in an exploding wire. As seen in Fig. 5, sequential

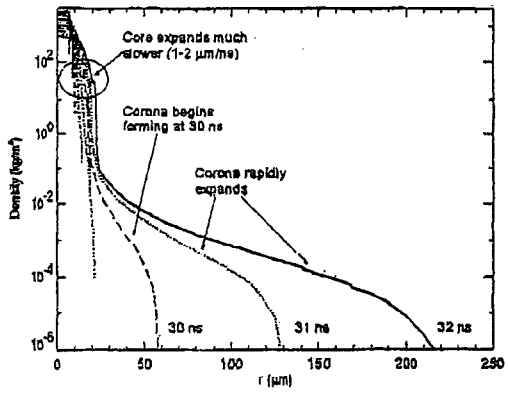
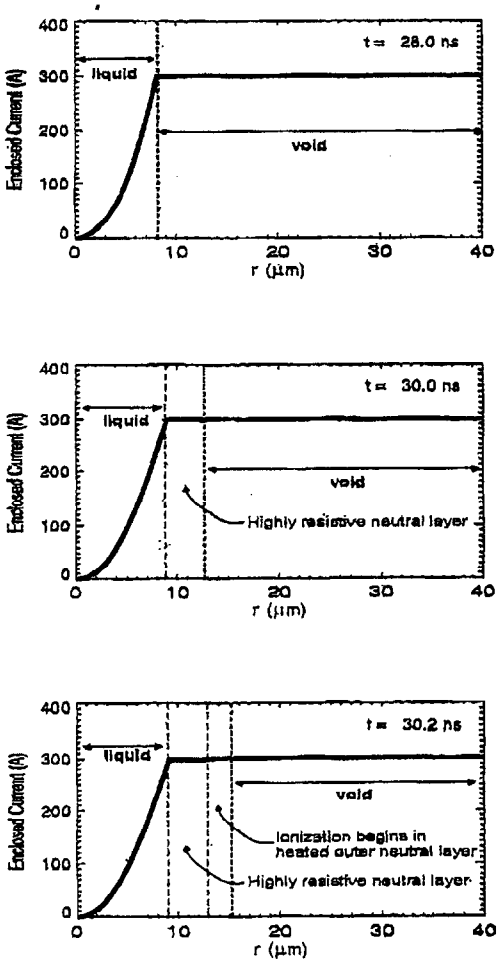


Figure 5. Sequential snapshots (1 each ns) of radial density profiles, from 20 to 32 ns showing slowly expanding core and rapidly expanding corona.

snapshots of radial density profiles clearly show the corona formation, beginning near 30 ns.



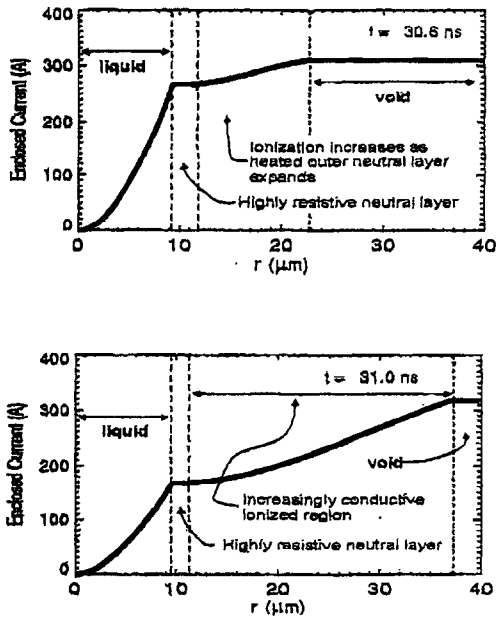


Figure 6. Sequence of 5 snapshots of enclosed current showing the details of the corona formation.

The electrical nature of the corona formation is most clearly seen in the snapshots of the enclosed current profiles. As illustrated in Fig. 6 there are five qualitative stages of corona formation. (1) pre-corona (before 29 ns). During this stage the wire expands slowly (likely more slowly than our existing EOS models allow) and is entirely solid or liquid. (2) Neutral gas layer evolves (30 ns). The slightest conductivity of this layer permits magnetic field diffusion and subsequent Joule heating, which leads to increased temperature, ionization, and conductivity of stage (3) (30.2 ns). The conductivity of the outer layer increases and expands in (4) (30.6 ns). Finally, (5) the corona is fully established when the conductivity of the outer layer reaches the value for which the resistance equals, and subsequently exceeds with continued expansion, that of the more conductive inner core region. At 31 ns the resistance of the wire is equally divided between 60 Ω for the core ($0 < r < 10$ μm) and 60 Ω for the corona (11 μm $< r < 37$ μm).

V. CONCLUSIONS

Advances we are making in EOS and electron transport have enabled accurate exploding wire simulations. Simulations of a single exploding wire show strong dependence on detailed structure of electrical conductivity and EOS.

Our ALEGRA and MACH simulations that use the recent L-M-D electron transport model and EOS with vapor dome and Maxwell constructions yield wire voltage that is in reasonable agreement with the Cornell measurements. The simulation voltage exhibits the correct timing and the voltage collapse (rapid corona formation) is resolved. The core diameter and expansion rate from the simulation also agree well with the experiment. This agreement was achieved without invoking any *ad hoc* surface conditions; only pure Al was considered.

We believe that the remaining discrepancy between the simulation and measured voltages is partially related to inadequate solid binding energy in our EOS and inaccurate specific heat. We are developing improved EOS tables. Refinements of L-M-D below solid density are also in progress.

VI. REFERENCES

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