The Legend of A. F. Shields[a]

Discussion by Marcelo H. García,[2] Member, ASCE

The author should be commended for his critical review of the pioneering work of Albert F. Shields on initiation of motion. The hydraulic engineering community has long accepted the work of Shields without questioning inconsistencies and misconceptions about the way the experimental data were obtained and interpreted. Interestingly, as correctly pointed out by the author, most of the inconsistencies are the result of work by others and not by Shields himself. However, in his analysis of Shields' work, the author raises some questions that do merit discussion. It is the view of the writer that Shields' work on initiation of motion has become somewhat of a legend, not because of the accuracy of the values that estimate the conditions for initiation of motion, but rather because the principles of similarity are so eloquently presented with a very simple (yet full of physics) dimensionless diagram. One should also not forget the context of Shields' experimental work. He was trying to find the dimensionless parameters that would lead to dynamic similarity and, hence, would facilitate the design and operation of river models. Incipient motion was simply one step toward the object of characterizing bed-load transport of sediment materials having a wide range of densities.

This discussion is concerned with (1) the conclusion by the author that ''Shields' method of defining initial motion by extrapolating stress-transport curves to a zero transport level is flawed''; (2) the analysis of Shields' work in the context of river mechanics; and (3) the application of Shields' similarity principles to movable-bed river modeling.

A typographical error can be found in the first column of page 379 where it says ''and ν is the kinematic viscosity of the fluid, set equal to 1.2 m^2/s.'' It should be 0.012 cm^2/s or 1.2×10^{-6} m^2/s.

CONDITIONS FOR INITIATION OF MOTION AND SEDIMENT TRANSPORT

The author argues that Shields extrapolated his observations of bed-load transport versus shear stress to a zero transport level in order to obtain the critical shear stress for initiation of motion. However, as pointed out by Kennedy (1995), an inspection of Shields' work shows absolutely no indication that this was the method he used. Unfortunately, Shields (1936c) does not give much explanation of how he obtained critical shear stress values. The experiments conducted by Taylor and Vanoni (1972), however, provide insight that can be used to shed light on Shields' work. Taylor and Vanoni (1972) conducted a set of well-controlled flume experiments to elucidate the effect of water temperature on low-transport, flat-bed flows. They found that Shields' data on incipient motion corresponded very well with a plot of a dimensionless sediment transport contour $q_* = 10^{-2}$ (see their Fig. 10), where q_* is a dimensionless volumetric bed-load transport rate per unit width given by $q_* = q_s/u_* D_g$, where q_s is the volumetric bed-load transport per unit width, u_* is the shear velocity, and D_g is the geometric mean size of the sediment grains. For a grain size D_g = 0.5 mm, water temperature T = 25°C, and kinematic viscosity ν = 0.00935 cm^2/s, the critical shear velocity according to Shields is u_* = 1.64 cm/s. The number of grains N being transported per unit width can be estimated by assuming nearly spherical particles,

$$q_s = \frac{\pi}{6} \cdot D_g^3 N = 10^{-2} u_* D_g \quad (5)$$

Solving for N gives 12.5 grains/cm·s or 383 grains/ft·s. These values suggest that there was measurable sediment transport taking place for ''incipient'' motion, which contradicts the argument advanced by the author, as well as others in the literature [e.g., Paintal (1971)]. Taylor and Vanoni (1971) found also that near the condition for incipient transport, $q_s \sim \tau^{17.5}$, clearly indicating that a small reduction in bed shear stress causes a large reduction in transport rate. For instance, a 4% reduction in bed shear stress τ would cause a drop in the bed-load transport rate of 50%. This is indicative of a highly nonlinear behavior for low-transport conditions and initiation of motion, which would explain, at least in part, some of the scatter observed in Shields' data. The above analysis suggests that there was indeed measurable sediment transport for the incipient motion conditions observed by Shields.

A SHIELDS REGIME DIAGRAM

The work of Shields can be extended to obtain a ''regime'' diagram for rivers. Alluvial rivers can broadly be divided into two types: sand-bed streams and travel-bed streams. Sand-bed streams typically have values of median bed sediment D_{50} varying between 0.1 and 1 mm. The sediment tends to be relatively well sorted, with values of geometric standard deviation σ_g varying from 1.1 to 1.5. Gravel-bed streams typically have values of median size of the surface bed sediment D_{50} of 15 to 200 mm or larger; the substrate is typically finer by a factor of 1.5 to 3. The geometric standard deviation of the substrate σ_g is usually quite large, with values in excess of 3 being common.

Two dimensionless parameters provide an effective delineator of rivers into the above two types. The first of these is the Shields stress τ^*, defined

$$\tau^* = \frac{\tau_b}{\rho g R D} = \frac{HS}{RD} \quad (6)$$

where H and S are the uniform flow depth and bottom slope, respectively; and $R = (\rho_s - \rho)/\rho$ is the submerged specific gravity of the sediment. In (6), D should be interpreted as a median size of the bed material exposed on the surface in the case of gravel-bed streams. The second of these is the particle Reynolds number R_p defined as

$$\mathsf{R}_p = \frac{\sqrt{RgD}D}{\nu} \quad (7)$$

which is the dimensionless surrogate for grain size.

The Shields diagram is not especially useful in the form of Fig. 4 because to find τ_c^*, one must know $u_* = \sqrt{\tau_c/\rho}$. The relation can be cast in explicit form by plotting τ_c^* versus R_p, noting the internal relation

$$\frac{u_* D}{\nu} = \frac{u_*}{\sqrt{RgD}} \frac{\sqrt{RgD}D}{\nu} = (\tau^*)^{1/2}\mathsf{R}_p \quad (8)$$

A useful fit of Shields' data is given by Brownlie (1981):

$$\tau_c^* = 0.22\mathsf{R}_p^{-0.6} + 0.06\exp(-17.77\mathsf{R}_p^{-0.6}) \quad (9)$$

With this relation, the value of τ_c^* can be computed readily when the properties of the water and the sediment are given. Fig. 5 shows a plot of the data used by Shields, as presented in Table 3, in the $\tau^* - \mathsf{R}_p$ space. The fit given by (9) is also included.

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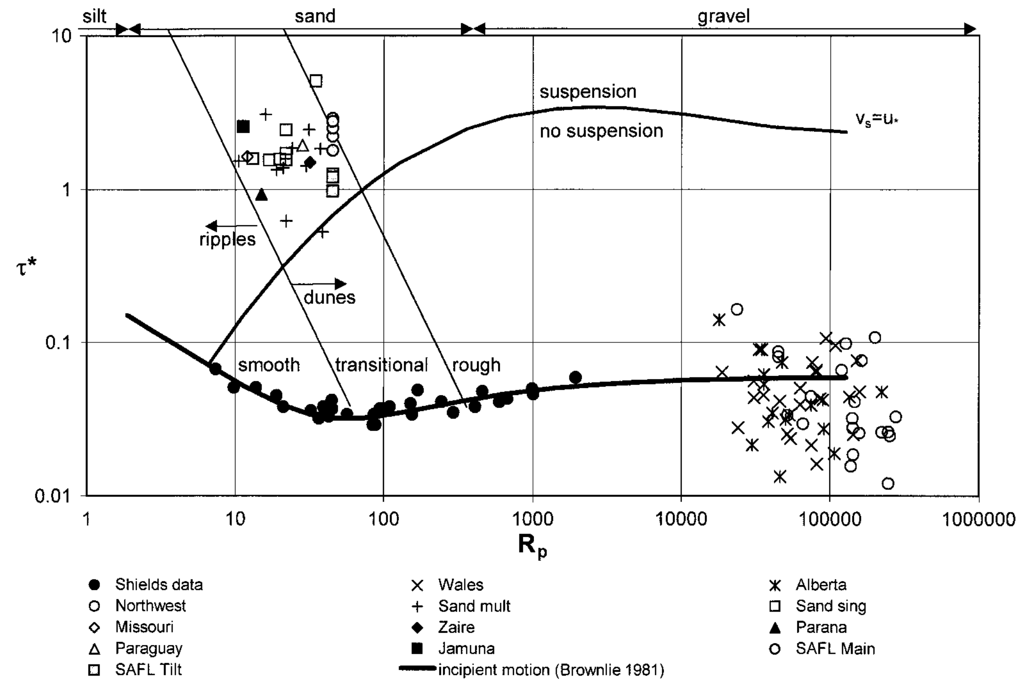

FIG. 5. Shields Regime Diagram

Fig. 5 also shows a plot of the values of τ^* evaluated at bank-full flow versus R_p for six sets of field rivers: (1) gravel-bed rivers in Wales, U.K. (Wales); (2) gravel-bed rivers in Alberta, Canada (Alberta); (3) gravel-bed rivers in the Pacific Northwest (Northwest); (4) single-thread sand-bed streams (Sand sing); (5) multiple-thread sand-bed streams (Sand mult); and (6) large sand-bed rivers. Also shown in the diagram are lines for (7) the onset of significant suspension, given by the condition of $u_*/v_s \cong 1$, where v_s is the sediment fall velocity; and (8) the transition from a smooth boundary to a hydraulically rough bed. It can be seen from Fig. 5 that sand-bed rivers plot in a very different place from gravel-bed rivers. In the case of gravel-bed rivers, Shields stress τ^* at bank-full conditions tends to be low, with typical values that are less than twice the Shields stress for the onset of motion of the D_{50} size of the surface material. Dunes are often poorly developed even at flood flows, with the dominant bed form being bars. Sand-bed streams tend to be quite different. Shields stresses at bank-full conditions are typically 20 to 60 times the Shields stress at the onset of motion, so that dunes develop prominently for a considerable part of every flood hydrograph, and sediment transport is intense at flood flows. It is also clear from the Shields regime diagram that in sand-bed streams, bed sediment is transported mainly in suspension, while gravel is predominantly transported as bed load. For gravel-bed streams, hydraulically rough flow conditions are the norm, while sand-bed streams are always in the transition from smooth to rough condition. This has important implications, since Manning's equation, which is commonly used to estimate flow discharge in streams, applies only for fully developed turbulence and hydraulically rough flow conditions. While this would be the case for gravel-bed streams, the Shields regime diagram indicates that for the data shown, including large and small sand-bed rivers, fully rough conditions do not normally exist. More data for bank-full flow conditions would be needed to assess whether this behavior can be observed in all sand-bed rivers.

APPLICATION OF SHIELDS REGIME DIAGRAM TO RIVER MODELING

The Shields regime diagram shown in Fig. 5 can be quite useful for the design and operation of movable-bed river models. To ensure similarity, model and prototype parameters should satisfy the following conditions:

$$(\tau^*)_{\text{model}} = (\tau^*)_{\text{prototype}} \tag{10}$$

and

$$(R_p)_{\text{model}} = (R_p)_{\text{prototype}} \tag{11}$$

Most Froude scale models usually satisfy condition (10) but pay little attention to condition (11). If this is the case, there will be scale effects that could eventually lead to misinterpretation of the results, as well as flawed designs.

The Shields regime diagram can readily be used to assess whether laboratory conditions are indeed representative of the field conditions. For instance, Shields stresses observed in a recent study of countermeasures to protect bridge piers from scour conducted at the St. Anthony Falls Laboratory (SAFL), University of Minnesota, are plotted in Fig. 5 (Parker et al. 1998). It is clear from the diagram that the laboratory conditions used to study bridge pier scour fall well within the range of Shields stresses observed in sand-bed streams for bank-full flow conditions.

The Shields regime diagram can also be used to test new, emerging technologies in the field of hydraulic modeling, such as ''micromodeling'' of rivers and streams (Davinroy 1999). For instance, the diagram could readily show that ripples will develop in a river model even though dunes have been observed in the field for the same flow conditions (i.e., Shields stress).

The author should be complimented for having pointed out very clearly many of the inconsistencies and misconceptions about Shields' work in a very thought-provoking paper. By placing the work of Shields in the context of river mechanics, the discusser hopes that the legend of Shields will go on into the next millennium as a milestone in the field of sediment transport. Long live the legend of Shields!

ACKNOWLEDGEMENTS

The discusser would like to thank Gary Parker for having motivated his interest in the Shields regime concept with the help of a diagram he produced in the early 1980s but was not published until very recently (García 1999). Much of the field data in Fig. 5 was kindly given to me

by Gary Parker. Thanks also to José Rodriguez and Juan Fedele for their help in preparing this discussion.

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Discussion by Emmett M. Laursen,[3] Life Member, ASCE

This paper plus that of Kennedy (1995) together paint a quite full picture of the trials and tribulations of research, communication, interpretation, use, and misuse; and there is no end to the story. Both papers should be a part of many different hydraulic engineering courses: they illustrate the difficulties in doing research, analyzing, writing, reading, etc. Hopefully, my subsequent words will add perspective rather than confusion and controversy.

It seems to me that one is interested in the critical tractive force for one of two reasons: (1) selecting riprap; or (2) predicting sediment transport. In neither case is a precise number required—anything approximately correct is usually good enough. In the first case, riprap sizing, one does not know the ''right'' flood frequency, flood magnitude, velocity of flow, particle shear, geometric placement of the riprap, or whatever other factors might affect the stability of the riprap. Besides, one should probably select the next larger size of standard, readily available material—because it is likely to be significantly cheaper.

In the second case, sediment transport, an approximate value is also usually sufficient. A very wrong value is not likely to change the predicted life of a reservoir by a factor of 2—shorter or longer.

One is only interested in whether the life is probably 10, 100, or 1,000 years. Only in special, unusual situations would one build a reservoir that would last just 20 years. More than 200 years from now, a city will need a different water-supply reservoir than it needs today. Only the reservoir with a predicted life of 1,000 years (or 500 years) might be a ''safe'' decision—and that would be half full of sediment in its half-life, when more rather than less water will be needed.

To make prediction chancier is the fact that one cannot know when the really big flood will come along. Tom Maddock Jr. told me a tale of building a little dam in New Mexico in the 1930s depression (when it seemed to be a good idea to do good public works and employ some people). They built the dam and invited the governor and all the local people for the dedication. The evening before, a whopper of a thunderstorm struck the watershed and when the flash flood subsided, the reservoir was full of sediment. It was quite an embarrassment to most everyone involved—and a lesson Tom never forgot (he told me the tale about 40 years later, and I learned the lesson he had learned, without the embarrassment).

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That's enough of the big picture; one can go on and on about variations of the theme, but to no great purpose. There is, however, purpose in looking at details (where the devil hangs out). Often the definition of critical tractive forces is something like: ''the flow (velocity, total shear, particle shear . . .) when a sediment particle is about to move.'' No one can see or measure when a particle is about to move. What the researchers did was stop opening the value when they first saw a particle move. This might be when an unstably balanced particle resting upon another particle tipped over. The conditions (shear, etc.) when this happened depended on the flume, the preparation of the sand bed, the technique of operation, the sand, the light, the eyes of the observer: everything. The wonder is that researchers were in basic agreement, not that they were not in perfect agreement.

My definition of critical tractive force is: ''The local particle shear over an area larger than the particle, smaller than the channel, when the most exposed particle moves.'' I like the particle shear, rather than the total shear, because the ratio of the total shear to the particle shear varies from 1.0 to 10 or more, and because the sum of form resistance due to the pressure around the dune, and the surface resistance due to the surface texture (smooth, fine sand, coarse sand) is remarkably constant (Henry et al. 1967). The surface resistance is greater for coarser sand, but this causes the boundary layer to be thicker and the velocity to be less near the boundary when the flow separates at the crest of a dune, thus decreasing the energy loss along the separated, free-stream surface (and affecting slightly the pressure distribution on the dune.)

In the second case, sediment load, it should be readily apparent that τ_c (the critical tractive force) is really, and simply, a coefficient helping to determine the graphical (and/or mathematical) shape of the particular sediment-transport equation. The value of τ_c will be different for each equation; that these τ_cs are in the same ballpark as τ_ss for the first case is truly remarkable.

The last detail that will be discussed here is the complexity of mixed sediment. On a windy day in the downtown area, the buildings are large-sized particles that don't move ($\tau < \tau_c$); but they do affect the flow conditions affecting the small particles (trash, hats, etc.): high velocities down the streets, vortices downstream of corners, stagnation points with piles of paper, etc. The flow conditions are very complex; transport can even be in an upward spiral vortex (a small-scale tornado or dust devil). In a stream, rocks that are not moving can half-bury themselves through the scour mechanism. Large rocks on the surface can move more easily on a sand-and-gravel, planar bed than on top of similar large rocks.

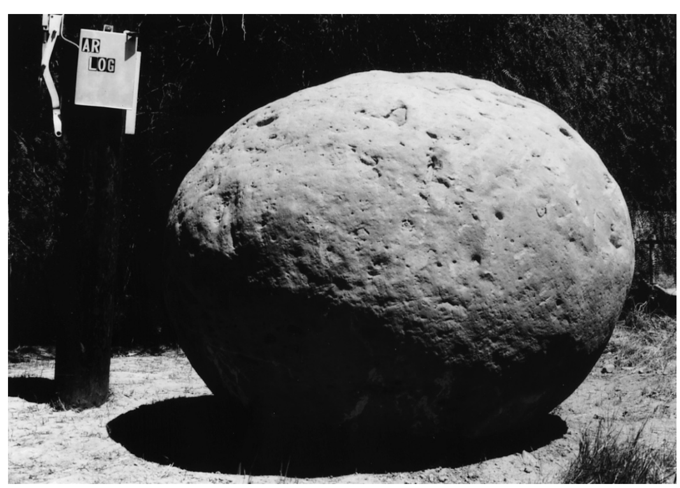

FIG. 6. Large Sediment Particle Found on Streambed after Sizable Flood of Santa Clara River in California (Courtesy of Margaret Petersen, Honorary Member, ASCE)

Fig. 6 is a 17-ton, 6-ft-diameter, almost spherical boulder found in a California river near the coast after a large flood. There were other, smaller boulders scattered on the sandy bed. It is obvious that this boulder would move almost as easily as a cue ball on a pool table, and that the velocity giving rise to the hydrodynamic force propelling the boulder would be relatively greater than the near boundary velocity around the usual sand and gravel particles of the usual bed load.

CONCLUSIONS

The lessons to be learned are (using critical tractive force as an example):

1. What is the practical, real problem being solved? How well do you need to know the critical tractive force? How well do you know other things that affect the solution to design problem?
2. How was your table (or equation or graphical curve) or τ_c values determined? How were the measurements made? How were the parameters chosen? Are the conditions of the experiment similar to the conditions of your problem?
3. What do you think will happen if you use a wrong value for τ_c—either too small or too large? How much would it cost to be more confident that nothing bad could happen? How might you change the design of the project, to make it less vulnerable? What risks are you (and your clients) willing to accept?

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Discussion by Claude Michel[4]

In his paper the author identifies some incomplete descriptions arising from Shields' work. The author recalls the data that have given rise to the well-known Shields curve (Fig. 4 and Table 3). This curve relates the dimensionless critical shear stress (τ^*_{c50m}) to the critical boundary Reynolds number (R^*_c).

When trying some curve fitting on all data from Table 3, one finds that the following relationship fits fairly well the 34 pairs from columns 2 and 3:

$$\tau^*_{c50m} = \frac{0.054}{\mathsf{R}^*_c}\left(\frac{8.8 + \mathsf{R}^{*1.6}_c}{3.6 + \mathsf{R}^{*0.6}_c}\right) \qquad (12)$$

The efficiency coefficient *E*, defined by Nash and Sutcliffe (1970) as

$$E = 1 - \frac{\sum_{i=1}^{34}[(\tau^*_{c50m})_{\text{observed}} - (\tau^*_{c50m})_{\text{computed}}]^2}{\operatorname{var}[(\tau^*_{c50m})_{\text{observed}}]}$$

provides a measure of the quality of such relationships. In the case of (12), *E* is equal to 0.73.

Clearly, in Fig. 4, Shields' experimental data are those that depart the most from the mean curve. Actually, these data are very different from the others as to the ratio ρ_s/ρ, where ρ_s is sediment density and ρ is water density. It seems that some improvement for (12) is possible by taking into account $\rho_s/\rho - 1$ as in

$$\tau^*_{c50m} = \frac{0.060}{\mathsf{R}^*_c}\left(\frac{10.9 + \mathsf{R}^{*1.6}_c}{4.9 + \mathsf{R}^{*0.6}_c}\right)\left(\frac{\rho_s}{\rho} - 1\right)^{-0.05} \qquad (13)$$

with an efficiency coefficient *E* equal to 0.82. Eq. (13) has yet to be tested against a larger set of data such as the one collected by Buffington and Montgomery (1997). Note that τ^*_{c50m} is very sensitive to ρ_s; increasing ρ_s by only 2% for amber cuttings and brown coal (seven first lines in Table 3) yields a relationship similar to (12) with *E* equal to 0.81. In order to preserve the simple form of (12), one may be tempted to define τ^*_{c50m} as

$$\tau^*_c = \frac{\tau'_c}{(\rho_s - \rho)^{0.95}\rho^{0.05}gD}$$

instead of

$$\tau^*_c = \frac{\tau'_c}{(\rho_s - \rho)gD}$$

The physical explanation of this new formula remains to be found. This short exercise might demonstrate that the width of the band around Shields' curve could partially be reduced by choosing adequate additional variables. A special concern is related to the slope *S* of the flume. In all analyses linked with Shields' curve, *S* is indissolubly linked to R', the effective hydraulic radius for the flume bed. Other combinations of variables could yield better relationships. Finally, it must be noted that an equation such as (12) is implicit as regards the product $R'S$ or the variable *D*. If one is interested in the determination of *D* corresponding to incipient motion under known hydraulic conditions, (12) has to be rewritten into the form of a relationship only involving *D* (or R^*_c) as a dependent variable in place of τ^*_{c50m} as in

$$\frac{8.8 + \mathsf{R}^{*1.6}_c}{3.6 + \mathsf{R}^{*0.6}_c} = \frac{g^{0.5}(R'S)^{1.5}}{0.054\nu\left(\frac{\rho_s}{\rho} - 1\right)} \qquad (14)$$

where ν is the kinematic viscosity of the fluid (equal to 1.2 mm^2/s). This equation has to be solved with respect to R^*_c, before obtaining *D* by

$$D = \frac{\nu\mathsf{R}^*_c}{\sqrt{gR'S}}$$

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Closure by John M. Buffington[5]

The writer thanks the discussers for their insightful discussions and valuable addendums to this investigation of Shields' doctoral work.

CLOSURE TO DISCUSSION BY GARCÍA

Bed-Load Transport Rates and Definition of Incipient Motion

García demonstrates that the data used by Shields correspond with measurable transport rates, suggesting that Shields did not determine critical shear stresses for incipient motion

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by extrapolating paired measurements of bed-load transport and shear stress to a zero rate of transport. This closure further examines the transport rates associated with Shields' data and demonstrates that Shields could have used his bed-load extrapolation method despite the fact that his reported values coincide with measurable transport rates.

Taylor and Vanoni (1972) demonstrated that a family of Shields curves can be obtained as a function of dimensionless bed-load transport rate, supporting the argument that definition of ''incipient motion'' requires specification of a reference transport rate (Kramer 1935; Neill and Yalin 1969; Paintal 1971; Ackers and White 1973; Parker et al. 1982; Wilcock 1988). Moreover, Taylor and Vanoni (1972) found that, on average, the data used by Shields correspond with a dimensionless bed-load transport rate of $q^* \approx 10^{-2}$ ($q^* \equiv q_b/u^*D_g$, where q_b is the volumetric bed-load transport rate per unit width of channel, u^* is bed shear velocity, and D_g is geometric mean grain size). However, further analysis indicates that these data span a range of q^* values between 10^{-4} and 10^{-1} (Fig. 7). For comparison, the reference dimensionless transport rate recommended by Parker et al. (1982) ($W^* = 0.002$) is roughly equivalent to $q^* = 10^{-5}$ for uniform-sized sediments; the Shields curve corresponding to $q^* = 10^{-5}$ has values of $W^* \approx 0.002$ to 0.005 for boundary Reynolds numbers (R^*) between 1 and 1,000.

The family of Shields curves in Fig. 7 was determined from a plot of normalized dimensionless bed-load transport rate ($q_N^* = q^*/10^{-4}$) versus normalized dimensionless shear stress ($\tau_N^* = \tau^*/\tau_r^*$, where τ_r^* is the reference value of τ^* at $q^* = 10^{-4}$) using data from Casey (1935a,b), USWES (1935), Paintal (1971), and Taylor and Vanoni (1972) (Fig. 8). Johnson's (1942) sidewall correction was applied to each study. Following the method of Taylor and Vanoni (1972), values of τ_r^* for each data source were determined from plots of τ^* versus q^*. To avoid unreliable extrapolation of τ_r^*, only those data sets that extended below $q^* = 10^{-3}$ were used. The normalized data collapse reasonably well and support Taylor and Vanoni's (1972) findings that $q_N^* \approx \tau_N^{*17.5}$ for values of q_N^* between 0.01 and 100 ($10^{-6} < q^* < 10^{-2}$) (Fig. 8). At larger transport rates, the slope of the transport function declines from 17.5 to 1.5. Fig. 8 was used to predict values of τ_N^* as a function of q^*, and these values were, in turn, applied to the Taylor and Vanoni (1972) data to calculate paired values of τ^* and R^* for each sediment type

$$\tau^* = \tau_N^* \tau_r^* \tag{15a}$$

$$\mathsf{R}^* = \frac{u^* D_g}{\nu} = \sqrt{\frac{\tau^*(\rho_s - \rho) g D_g}{\rho}} \frac{D_g}{\nu} \tag{15b}$$

allowing prediction of Shields curves as a function of q^* (Fig. 7). In (15b) ν is kinematic viscosity, g is gravitational acceleration, and ρ_s and ρ are sediment and fluid densities, respectively.

Using the q^* contours plotted in Fig. 7, specific transport rates were determined for each of Shields' values and combined with García's (5) to estimate the number of grains being transported for Shields definition of incipient motion. Results indicate that the number of grains in motion ranged from 0.02 to 365 per centimeter per second, with a median value of 11. These transport rates span Kramer's (1935) definitions of weak to general motion and would yield measurable transport even for experiments of short duration.

Although Shields' data coincide with measurable bed-load transport rates as indicated by the Taylor and Vanoni (1972) curves, it is possible to obtain similar values of critical dimensionless shear stress (τ_c^*) by extrapolating paired measurements of shear stress and bed-load transport rate to a zero level of transport. For example, some of the extrapolated τ_c^* values

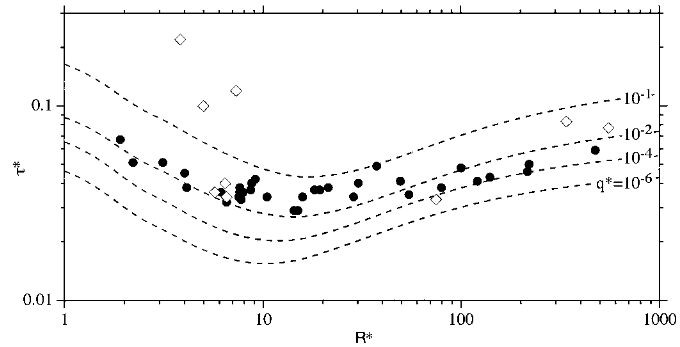

FIG. 7. Dimensionless Shear Stress (τ^*) versus Boundary Reynolds Number (R^*) for Data Used by Shields (1936b) (Table 3) (Circles) and for Values Determined from Bed-Load Extrapolation ($\tau^*_{c_{\overline{rm}}}$, Table 2, Column 7) (Diamonds). Dashed Lines Are Contours of Dimensionless Bed-Load Transport Rate (q^*) Predicted from Fig. 7 and Data of Taylor and Vanoni (1972)

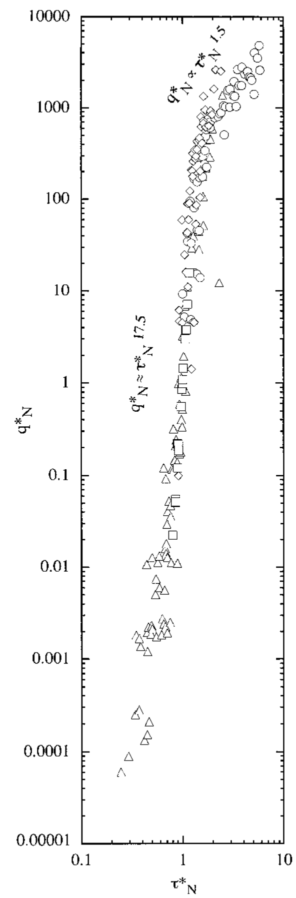

FIG. 8. Normalized Dimensionless Transport Rate (q_N^*) versus Normalized Dimensionless Shear Stress (τ_N^*) for Data of Casey (1935a,b) (Diamonds), USWES (1935) (Circles), Paintal (1971) (Triangles), and Taylor and Vanoni (1972) (Squares)

for the supplemental data sources are similar to visually based values for the same experiments (Tables 2 and 3) and in many instances agree reasonably well with those data reported by Shields (Fig. 7). By definition the extrapolated τ_c^* values should correspond with a zero bed-load transport rate, yet the Taylor and Vanoni curves suggest that these τ_c^* values coincide with a variety of nonzero dimensionless transport rates (Fig. 7). This highlights the erroneous nature of τ_c^* values determined from extrapolating stress-transport data to a zero transport rate. Because most stress-transport relationships are power functions (Figs. 2 and 8), bed-load transport rates will approach zero only when stress goes to zero (Paintal 1971; Lavelle and Mofjeld 1987). Consequently, extrapolation to a zero level of transport should yield $\tau_c^* = 0$. Nevertheless, nonzero values can be obtained through erroneous analysis of the data (as was purposefully done in Fig. 1 to approximate how Shields might have employed his bed-load extrapolation approach). Although determining τ_c^* values by extrapolating stress-transport relationships to a zero level of transport is flawed, this method could have been used by Shields and could have produced values comparable to those reported for the supplemental data sources (Fig. 7). Without recourse to Shields' original data this question of methodology cannot be resolved. Regardless of the method employed, García correctly points out that Shields' data correspond with measurable transport rates (Fig. 7).

Shields Regime Diagram

Fig. 5 is an excellent summary of differences between sand-bed and gravel-bed rivers with regard to transport thresholds, style of sediment transport (bed-load versus suspended), and channel morphology. Many gravel-bed rivers exhibit a near-bank-full threshold for significant sediment transport, while sand-bed channels transport sediment at most discharges, exhibiting a continuous relationship between transport rate and discharge [Henderson (1963); also see review by Montgomery and Buffington (1997)]. Consequently, bank-full values of τ^* for gravel-bed rivers plot close to the Shields curve (Fig. 5), while those of sand-bed rivers are far in excess of the critical values for incipient motion [see also Buffington and Montgomery (1999a, Fig. 12)]. Although there is considerable scatter among the τ^* values for gravel-bed rivers (Fig. 5), these values are within the range of critical dimensionless shear stresses reported for other gravel-bed rivers (Buffington and Montgomery 1997). This scatter might be reduced if τ^* values were calculated from bed stresses corrected for channel roughness (banks, bars, particle form drag) [e.g., Buffington and Montgomery (1999b)]. Similarly, the τ^* values for sand-bed channels might be reduced if they were calculated from boundary stresses corrected for bed-form drag (ripples, dune, bars), rather than calculated from the total boundary shear stress.

García also correctly points out that Shields' study of incipient motion was part of a larger investigation of bed-load transport and application of similarity principles to the study of alluvial rivers. In fact, Shields presented his incipient motion data as a reference point in a regime diagram similar to that of Fig. 5.

CLOSURE TO DISCUSSION BY MICHEL

Inclusion of a relative density term is a useful additional factor for explaining some of the variation of dimensionless shear stress values in Shields diagrams (Ippen and Verma 1953; Ward 1969). In particular, Ward (1969) modeled incipient motion by means of a dynamic force balance, rather than the static one used by Shields, and included a relative density term defined as $\rho_s/(\rho_s - \rho)$. Ward (1969) conducted incipient motion experiments on particles with a broad range of densities immersed in both water and oil. A standard Shields plot resulted in considerable scatter of data, while inclusion of his relative density term collapsed the data toward a unified relationship. A variety of other factors also may account for scatter of data in Shields plots [see review by Buffington and Montgomery (1997)].

Natural grains found in alluvial rivers typically exhibit a narrow range of densities (2,100–3,000 kg/m^3) and require either a high metal content or a large degree of porosity for deviation from this range. Consequently, the influence of variations in particle density on incipient motion and bed-load transport will likely be small in many natural alluvial rivers. However, differences in particle density can be important in studies of placer formation or transport of pollutants and contaminants.

Erratum. In Eq. (4) of the original paper, ν should be defined as $1.2 \cdot 10^{-6}$ m^2/s.

APPENDIX. REFERENCES

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