THE LEGEND OF A. F. SHIELDS

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ABSTRACT: The well-known doctoral work of Shields is a tale that is frequently recounted by many authors and has spawned a large, continuing body of research over the last 60 years. Despite the success of Shields' work, the details of his experimental methods and results as reported by others are quite variable. Inconsistencies and misconceptions regarding Shields' work are identified and examined here. Incomplete descriptions by Shields, loss of his original data, and Shields' postgraduate absence from the hydraulic engineering community leave some of the identified inconsistencies open to debate.

INTRODUCTION

Webster defines legend as ''a story handed down for generations among a people and popularly believed to have a historical basis, although not verifiable.'' Over the last six decades the doctoral work of Albert Frank Shields (1936a,b,c) has become legendary.

Shields' work on incipient motion and bed-load transport is a benchmark study that has inspired numerous investigations and is widely applied in fields such as hydraulic engineering, fluvial geomorphology, aquatic biology, physical oceanography, and economic geology. Nevertheless, the scientific literature is rife with misconceptions and errors regarding Shields' methods and results. Although the cause of this confusion is uncertain, inaccessibility of the original work may have been a factor. Shields' dissertation was printed in two German-language versions (Shields 1936a,b) and one gray-literature, English-language translation (Shields 1936c). Consequently, much of what is popularly known about Shields' work is derived from second-hand descriptions in textbooks and journal articles. Incomplete or inaccurate recounting of Shields' work has fostered the legend of A. F. Shields. Verification of these second-hand stories is exacerbated by the apparent loss of Shields' original laboratory data during World War II (Kennedy 1995). The purpose of this article is to identify some of the inconsistencies in the tales of Shields' research, and, where possible, correct previous misconceptions and errors.

EXPERIMENTAL METHODS AND CONDITIONS

Shields (1936b,c) conducted laboratory flume studies examining incipient motion and bed-load transport of noncohesive, nearly uniform grains. He used four sediment types (amber cuttings, brown coal, crushed granite, and crushed barite), providing a range of particle densities (1,060–4,300 kg/m^3) and submerged weights. The grains were subangular to very angular as defined by Russell and Taylor (1937), and Powers (1953), with median sizes ranging from 0.36 to 3.44 mm (Shields 1936b, p. 22). The brown coal experiments were conducted in a nonrecirculating wooden flume that was 14 m long, 0.81 m wide, and 0.3 m tall. Grains of coal similar in size to the bed material were glued to the walls to provide a uniform skin friction across the channel bed and walls. Half of the barite experiments were conducted in the same flume, but with walls made of lacquered wood. The remaining experiments were conducted in a smaller, glass-walled, nonrecirculating flume with dimensions of 14 m × 0.4 m × 0.5 m. The flumes are further described by Kramer (1932, 1935) and Casey (1935a,b), predecessors of Shields (Kennedy 1995).

A planar bed of each sediment type was set at a constant flume slope and exposed to ~16 incremental increases in hydraulic discharge. During the experiments, sediment was fed into the flume by hand at a roughly constant rate with the aid of a distributing board. Flow depth, water-surface slope, bed-load transport rate, and bed-form morphology were measured at each successive discharge after attainment of equilibrium conditions [the definition of which is not provided by Shields (1936c, pp. 32 and 33)]. Experiments for each sediment type were repeated with an average of four different slopes. The brown coal experiments may be the work of H. H. Wheaton (unpublished report, n.d.) (Shields 1936c, p. 18).

Initial Motion

Most authors report that Shields determined critical shear stresses for incipient particle motion by extrapolating paired measurements of shear stress and bed-load transport rate to a zero level of transport [e.g., Vanoni (1964, 1966), Ward (1969), Gessler (1971), Paintal (1971), Mantz (1977), Miller et al. (1977), Church (1978), Andrews (1983), Middleton and Southard (1984), and Lavelle and Mofjeld (1987)]. However, Kennedy [(1995) pp. 770 and 771] contends that Shields measured initial motion through visual observation of the flume bed surface using Kramer's (1932, 1935) criteria for general motion. These two methods of defining incipient motion are respectively termed reference and visual techniques, each of which may yield different values of dimensional and dimensionless critical shear stress for initiation of bed motion (Buffington and Montgomery 1997).

Shields (1936c) discussed both reference and visual methods but did not clearly state which technique he employed. In his discussion of initial motion, Shields (1936c, p. 10) said

> ''Considering the grains in a bed of uniform grain sizes, the question is: When will the grain be dislodged from the bed and set into motion? . . . it is assumed that the movement is small . . . and is experimentally to be determined by extrapolating the bed-load transport curve to the time bed-load movement ceases.''

This description of bed-load extrapolation fits the reference transport method commonly reported for Shields' work. However, Shields [1936c, pp. 10 and 11] added a footnote to this discussion as follows:

> ''. . . In case of mixed bed-load, the beginning of movement cannot definitely be established in this manner, because of the possible removal of only a part of the grain-sizes (for instance, the finer ones). In that case, one must refer to a certain and partly arbitrary determination of this boundary. The so-called weak movement, after Kramer and Casey, corresponds most closely to the beginning of movement.''

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Consequently, for sediment mixtures Shields recommended visual observation of incipient motion using Kramer's (1932, 1935) weak-movement criteria. Kramer (1932, 1935) proposed four levels of visual bed movement as follows:

1. None
2. Weak—(". . . several of the smallest particles are in motion, in isolated spots, and in countable numbers.")
3. Medium—(". . . grains of mean diameter are in motion in numbers too large to be countable . . . movement is no longer local in character. It is not strong enough to affect bed configuration and does not result in transportation of an appreciable quantity of material.")
4. General—(". . . grains up to and including the largest are in motion . . . . It is sufficiently vigorous to change the bed configuration . . . . There is an appreciable quantity of material transported . . . .")

While seemingly straightforward, this second passage from Shields' dissertation is curiously contradicting. In the first sentence of this passage, Shields cautioned that bed-load extrapolation based on partial transport of a sediment mixture is not an accurate measure of bed-surface incipient motion, suggesting that he equated incipient motion with movement of all sizes present. However, in the last sentence he advocated Kramer's (1932, 1935) weak movement as an alternative, which describes selective transport of those grains most easily mobilized within the sediment mix, exactly what Shields warned against in using his bulk bed-load extrapolation method for mixed sediments.

Nevertheless, these two passages from Shields' dissertation offer two definitions of incipient motion (bed-load extrapolation for uniform sizes versus visual observation for mixed grains). Because Shields neglected to explain which method he used and did not present sufficient data to recreate his calculations, the matter of his experimental procedure remains open to debate. However, throughout his dissertation he discussed his approach and results as being representative of uniform grains (Shields 1936c, pp. 11, 14, and 16), suggesting that he employed bed-load extrapolation (the method he described for uniform sediment).

Channel Roughness

Because Shields used an initially planar bed surface, it is commonly thought that his measurements were free from bed-form drag [e.g., Miller et al. (1977), p. 524]. However, as Shields increased discharge during his experiments, his initially planar bed surfaces deformed into a variety of bed morphologies (Shields 1936b,c, his Figs. 6 and 12–14), following a now well-known sequence of lower-stage plane-bed, ripple, dune, and upper-stage plane-bed morphologies (Gilbert 1914; Kennedy 1963, 1975; Simons and Richardson 1966; Vanoni 1974; Middleton and Southard 1984). Although Shields applied a sidewall correction, he did not attempt to correct calculated basal shear stresses for bed-form drag.

Form drag and turbulence caused by unaccounted-for bed forms undoubtedly influenced both bed-shear stress and sediment transport rate during Shields' experiments. If Shields did determine incipient motion by extrapolation of these measurements, then the initial-motion values reported by Shields are affected by the presence of bed forms. Depending on their amplitude and wavelength, bed forms in sand-bed and gravel-bed rivers (i.e., ripples, dunes, and bars) can dissipate 10–75% of the total channel shear stress (Parker and Peterson 1980; Prestegaard 1983; Dietrich et al. 1984; Hey 1988), causing potentially significant overestimation of the actual bed shear if neglected from stress calculations. Bed stresses also may have been overestimated in some experiments due to unaccounted-for particle form drag caused by high values of relative roughness (D_{50}/h, where D_{50} is median grain size and h is flow depth) (Shields 1936c, p. 17).

The sidewall correction used by Shields accounts for momentum diffusion caused by both the proximity of channel walls (i.e., so-called "width-to-depth effects") and differences in skin friction between the channel bed and walls. Shields' sidewall correction partitions the total boundary shear stress into bed and wall components by dividing the cross-sectional flow area along isovel-perpendicular lines into separate subareas of flow acting on the channel bed versus the walls [an approach based on the work of Leighly (1932)]. An effective hydraulic radius for the bed (R') was determined from the corresponding subarea of flow, and bed-shear stress (τ') was calculated as

$$\tau' = \rho g R' S \qquad (1)$$

where ρ = fluid density; g = gravitational acceleration; and S = water-surface slope. Unlike the stress-partitioning models of Einstein (1934), Johnson (1942), and Vanoni and Brooks (1957) that require choice of both a resistance equation and a skin-friction value for the channel walls, Shields' approach relies on case-specific determination of channel isovels.

For experiments with uniform wall and bed roughness (brown coal), Shields developed predictive equations for R' as a function of channel width-to-depth ratio and an assumed isovel structure (Shields 1936c, pp. 3–5, his Figs. 2 and 3). For experiments in smooth-walled flumes (all others), Shields constructed isovels from several vertical velocity profiles measured in the central region of the channel (Shields 1936c, pp. 7 and 8). However, he apparently did not use the constructed isovels to determine R' but instead argued that the flow in the channel center was the most important for sediment transport and that R' in this region could be approximated by the total flow depth as long as the maximum velocity occurred near the water surface (which was the case for these experiments) (Shields 1936c, pp. 8, 9, and 30). Consequently, it appears that in some experiments Shields used cross-sectionally averaged bed-shear stresses that were corrected for sidewall effects, whereas in other experiments he used an uncorrected depth-slope product specific to the central region of the channel.

Sediment Grading

Many authors report that Shields used uniform-size sediments in his experiments [e.g., Rouse (1939a,b, 1949), Ippen and Verma (1953), Neill and Yalin (1969), Richards (1990), Carson and Griffiths (1985), Kuhnle (1993), and Chin et al. (1994)]. Shields also referred to his experiments as representative of uniform-size grains (Shields 1936c, pp. 11, 14, and 16). Although he used closely graded sediments, they were by no means uniform. Graphic standard deviations [$\sigma_g = [\phi_{84} - \phi_{16}]/2$ (Folk 1974)] for Shields' barite, granite, brown coal, and amber grains varied from 0.16 to 0.78ϕ, representing sediments ranging from very well sorted to moderately sorted as defined by Folk (1974); ϕ is the standard $\log_2$ unit of grain-size measurement (Krumbein 1936), and ϕ_{84} and ϕ_{16} are percentiles of the grain-size distribution for which 84 and 16% of the grains are finer.

Mixed-size sediments can exhibit differential mobility and selective transport that, in turn, influence identification of incipient motion thresholds (Shields 1936c; Neill and Yalin 1969; Wilcock 1988). The most easily mobilized particles in a mixed-size bed are those with high protrusions (Kirchner et al. 1990) and low intergranular friction angles (Miller and Byrne 1966), distributions of which are functions of sediment size, shape, rounding, sorting, and packing (Miller and Byrne 1966; Li and Komar 1986; Kirchner et al. 1990; Buffington et al. 1992; Carling et al. 1992). Kramer's (1932, 1935) method of visual observation and Shields' (1936c) method of bed-load extrapolation are both sensitive to the initial sediment flux, which is, in turn, influenced by the distributions of grain protrusion and friction angle associated with a particular sediment mixture. Furthermore, because each size class has a distribution of grain protrusions and friction angles, the first-moved grains are unlikely to represent conditions of general motion for either their size class or the bed surface as a whole. Differences in sediment grading that result in variable selective transport may explain some of the scatter in incipient-motion values reported by Shields [(1936c) his Fig. 6], regardless of the particular initial motion technique used (i.e., reference-based versus visual-based approaches).

Sediment Density

The densities of the four sediment types used in Shields' experiments are most commonly reported as 1,060, 1,270, 2,700, and 4,250 kg/m^3 for amber, brown coal, granite, and barite, respectively, agreeing with values listed in Shields' (1936b,c) figures. However, these values for granite and barite are apparently averages; the granite and barite densities actually ranged from 2,690 to 2,710 kg/m^3 and from 4,190 to 4,300 kg/m^3, respectively (Shields 1936b, p. 22). It is unknown whether Shields used the actual or average densities when constructing his figures and presenting his results.

Shields chose a range of sediment densities to exploit the corresponding range of submerged specific gravities (0.06–3.3), allowing investigation of a wide variety of critical shear stresses and boundary Reynolds numbers while minimizing the range of channel slopes, flow depths, and discharges needed for the experiments (Shields 1936c, p. 29; Kennedy 1995). However, some of the scatter observed in the incipient-motion values reported by Shields (1936c, his Fig. 6) may be the result of using different sediment densities without considering the consequent inertial effects on incipient motion (Ward 1969). Modeling incipient motion by means of a dynamic force balance (as opposed to a static one, as Shields did) allows inclusion of inertial forces and the effect of sediment density relative to that of the fluid (Ward 1969).

Supplemental Data

Shields supplemented his experimental results with data from laboratory studies by Gilbert (1914), Kramer (1932, 1935), Casey (1935a,b), and the U.S. Waterways Experiment Station (USWES) (1935). These additional data are for well rounded to subangular sediments with median grain sizes of 0.17–7.01 mm, graphic sorting coefficients of 0.16–0.82ϕ, and densities of 2,650–2,700 kg/m^3. As with Shields' work, form drag was not accounted for in these supplemental studies, despite the presence of various types of bed forms. Furthermore, Shields apparently used averages or subsets of the data reported by each of these sources, because he presents fewer initial-motion values than were available. For example, Shields evidently used data from only two of Gilbert's (1914) 11 grades of sediment and eight of Casey's (1935a,b) 17 sediment types (only the ''uniform'' sediments). Similarly, Shields distilled Kramer's (1932, 1935) 12 experiments into three initial-motion values, probably representing averages of each of the three sediment mixtures used by Kramer (Kennedy 1995).

It is uncertain how Shields defined incipient motion for these data. The original studies record incipient motion by visual observation; with the exception of Gilbert (1914), all of these additional sources used Kramer's (1932, 1935) visual definitions of bed-surface motion. However, each of these studies also presented data on shear stress and bed-load transport from which Shields could have defined incipient motion by means of his bed-load extrapolation method. As discussed previously, Shields recommended bed-load extrapolation for defining incipient motion of uniform-size sediments and visual observation for mixed grains. Shields referred to the work of Kramer (1932, 1935), Casey (1935a,b), and USWES (1935) as being representative of mixed-size sediments (Shields 1936c, p. 17) but at the same time claimed that his incipient-motion curve (constructed, in part, with these supplemental data) was representative of uniform-size sediments (Shields 1936c, p. 14). Consequently, it is uncertain whether Shields used a reference-based or visual-based definition of incipient motion for these data. Vanoni (1964) claimed that Shields' incipient-motion values for Kramer (1932, 1935) and USWES (1935) were based on their reported thresholds for weak movement as defined by Kramer (1932, 1935).

The issue of methodology was investigated further here by calculating visual-based and reference-based incipient-motion values from data available for these studies and comparing them to the initial-motion values reported by Shields (1936b,c) (Table 1). Shields (1936b,c) expressed incipient grain motion as a dimensionless ratio of the critical bed-shear stress (τ_c') to submerged grain weight per unit area

$$\tau_c^* = \frac{\tau_c'}{(\rho_s - \rho)gD} \tag{2}$$

where ρ_s = sediment density; D = characteristic grain size; and τ_c^* = corresponding dimensionless critical shear stress. For his own experiments, Shields scaled τ_c' by the median grain size of his hydraulically unworked sediment mixtures ($D = D_{50m}$) [(cf. Fig. 16 and Table 2 of Shields (1936b,c)]. However, Gilbert (1914) and USWES (1935) reported unworked mixture means ($\bar{D}_m$), rather than medians. Mean and median sizes are equivalent only when (1) grain-size distributions are symmetrical about the median; or (2) mixtures are very well sorted ($\sigma_g < 0.35\phi$). Differences between D_{50m} and $\bar{D}_m$ cause corresponding differences in τ_c^* values calculated from (2), because τ_c' is not grain-size specific [compared to that of Day (1980) or Parker et al. (1982)] but, rather, is based on a bulk measure of sediment transport for both of the incipient-motion methods discussed here.

Shields undoubtedly used $D = \bar{D}_m$ for Gilbert's (1914) data, because Gilbert (1914) does not provide sufficient information to determine D_{50m} values. Nevertheless, Gilbert's (1914) sediment mixtures are very well sorted (Table 1), indicating that the $\bar{D}_m$ and D_{50m} values are approximately equivalent, as are their dimensionless critical shear stresses. In contrast, poorer sorting and asymmetrical grain-size distributions cause a 3–20% difference between D_{50m} and $\bar{D}_m$ values for the USWES (1935) data, resulting in a corresponding difference in calculated τ_c^* values (Table 1). Because it is unknown if Shields corrected the USWES (1935) mean grain sizes to median ones, dimensionless critical shear stresses for the USWES (1935) experiments were calculated here in terms of both D_{50m} and $\bar{D}_m$ (Table 1). The generic symbol τ_c^* is used in Table 1 for the dimensionless critical shear stresses reported by Shields, signifying the uncertainty in both incipient-motion definition (i.e., visual-based versus reference-based) and D assignment (i.e., $D = D_{50m}$ versus $\bar{D}_m$).

Regardless of whether D is defined as the mean or median particle size, each of the laboratory studies examined here derived D from their unworked sediment mixtures. Recognition of grain-size distribution type (i.e., surface, subsurface, or unworked laboratory mixture) is important, because it influences calculated values of dimensionless critical shear stress and their use (Buffington and Montgomery 1997). In particular, scaling critical shear stresses by D of an unworked laboratory mixture may not accurately describe thresholds for bed-surface mobility. Hydraulic reworking of sediment mixtures prior to incipient-motion measurement may result in surface grain-size distributions different from that of the unworked mixture, causing erroneous results when critical shear stresses for wa-

TABLE 1. Sidewall-Corrected Incipient-Motion Values Determined from Visual and Reference Techniques Compared to Dimensionless Critical Shear Stresses Reported by Shields

Source (1)	D_{50m} (mm) (2)	σ_g (ϕ) (3)	$\tau^*_{c_{v50m}}$, weak (4)	$\tau^*_{c_{v50m}}$, medium (5)	$\tau^*_{c_{v50m}}$, general (6)	$\tau^*_{c_{r50m}}$ (7)	τ^*_c (Shields 1936b) (8)
Gilbert (1914)	(4.94)	<0.26	(0.049)	(0.050)	(0.050)	(0.040)	0.050
	(7.01)	<0.22	(0.051)	(0.046)	(0.058)	(0.031)	0.059
Kramer (1935)	0.51	0.74	0.029	0.038	0.043	—[a]	0.032
	0.53	0.81	0.035	0.046	0.055	—[a]	0.038
	0.55	0.62	0.033	0.040	0.050	—[a]	0.033
Casey (1935a,b)	0.17	0.41	0.064	0.073	0.079	—[a]	0.067
	0.26	0.42	0.048	0.053	0.058	—[a]	0.051
	0.67	0.16	0.034	0.039	0.044	—[a]	0.034
	0.87	0.17	0.032	0.037	0.040	—[a]	0.034
	0.93	0.19	0.033	0.036	0.039	—[a]	0.037
	1.27	0.20	—[a]	—[a]	—[a]	—[a]	0.040
	1.74	0.19	0.035	0.039	0.043	—[a]	0.041
	2.47	0.22	0.031	0.033	0.036	0.029	0.038
USWES (1935)	0.18 (0.21)	0.32	0.061 (0.053)	0.074 (0.064)	0.191 (0.164)	0.230 (0.190)	0.051
	0.28 (0.31)	0.53	0.050 (0.045)	0.138 (0.124)	0.258 (0.233)	0.104 (0.094)	0.045
	0.34 (0.35)	0.37	0.046 (0.045)	0.086 (0.084)	0.204 (0.199)	0.110 (0.110)	0.038
	0.43 (0.51)	0.82	0.044 (0.037)	0.056 (0.047)	0.094 (0.080)	0.042–0.110 (0.036–0.083)	0.034
	0.48 (0.52)	0.53	0.038 (0.036)	0.047 (0.043)	0.055 (0.051)	0.035–0.100 (0.032–0.084)	0.036
	0.45 (0.54)	0.66	0.039 (0.033)	0.059 (0.049)	0.070 (0.059)	0.041 (0.034)	0.036

Note: Shields' (1936b,c) sidewall correction applied to data in Columns 4–7. Values in parentheses are for $\bar{D}_m$, rather than D_{50m}. Visually based dimensionless critical shear stresses for Gilbert (1914) are derived from data in his Table 10, whereas reference-based values are derived from data in his Tables 4G and 4H. Gilbert's (1914) observations of few, several, and many grains moving are respectively assumed equal to Kramer's (1932, 1935) three levels of motion. All visually based incipient-motion values (Columns 4–6) are averages of multiple experiments, and are based on data for first, sustained occurrence of each type of motion for experiments with increasing discharge. Reference-based incipient-motion values (Column 7) are based on composite data sets for given grain size, except where multiple stress-transport curves are observed [e.g., Figs. 1(d, e g, and h)].

[a] Unavailable data.

ter-worked sediments are scaled by grain-size percentiles of the unworked distribution. Because each of the data sources made their measurements after attainment of equilibrium conditions, there was an opportunity for sediment mixtures to undergo hydraulic reworking.

Shields presented τ^*_c values for each source only in graphic form, plotting τ^*_c as a function of critical boundary Reynolds number (R^*_c) (Shields 1936b,c, his Fig. 6). Furthermore, he did not report the specific D_{50m} or $\bar{D}_m$ value used to calculate each τ^*_c and R^*_c pair. Consequently, grain sizes listed in Table 1 are based on a sensible match of the D_{50m} or $\bar{D}_m$ values used by each source and the τ^*_c and R^*_c pairs reported by Shields. The match between grain size and dimensionless critical shear stress was made by rearranging (2) to solve for τ'_c

$$\tau'_c = \tau^*_c(\rho_s - \rho)gD \tag{3}$$

and inserting the right-hand side of (3) into the definition of R^*_c

$$\mathsf{R}^*_c \equiv \frac{u^*_c D}{\nu} = \sqrt{\frac{\tau'_c}{\rho}}\left(\frac{D}{\nu}\right) = \sqrt{\frac{\tau^*_c(\rho_s - \rho)gD}{\rho}}\left(\frac{D}{\nu}\right) \tag{4}$$

allowing comparison of reported and back-calculated R^*_c values for a given τ^*_c and R^*_c pair and a candidate grain size. u^*_c is the critical shear velocity for incipient motion ($u^*_c \equiv \sqrt{\tau'_c/\rho}$), and ν is the kinematic viscosity of the fluid, set equal to $1.2\ \text{m}^2/\text{s}$, matching the value probably used by Shields (1936b,c, his Fig. 7).

Visually based values of dimensionless critical shear stress ($\tau^*_{c_{v50m}}$ or $\tau^*_{c_{v\bar{m}}}$) presented in Table 1 for weak, medium, and general motion (Kramer 1932, 1935) were determined from bed-surface observations and shear-stress measurements reported by each source, and reference-based values ($\tau^*_{c_{r50m}}$ or $\tau^*_{c_{r\bar{m}}}$) were estimated by extrapolating paired measurements of bed-load transport rate (q_b) and dimensionless shear stress (τ^*_{50m} or $\tau^*_{\bar{m}}$) to a zero transport rate (Fig. 1). Note, however, that Shields' method of determining critical shear stress by extrapolating bed-load transport to zero is flawed because most stress-transport relations are power functions [e.g., Einstein (1950), Paintal (1971), and Parker et al. (1982)] (Fig. 2), with transport rates approaching zero only when shear stress goes to zero (Paintal 1971; Lavelle and Mofjeld 1987). Nevertheless, the stress-transport curves of Fig. 1 represent a best guess of how Shields might have employed his bed-load extrapolation approach; the curves are individually fitted by eye, with $\tau^*_{c_{r50m}}$ and $\tau^*_{c_{r\bar{m}}}$ values estimated from the intersection of these curves with the y-axis at zero transport rate.

Many of the $\tau^*_{c_{r50m}}$ values could not be calculated for the experiments conducted by Kramer (1932, 1935) and Casey (1935a,b). Like Shields, Kramer (1932, 1935) measured bed-load transport rates during his experiments (Kramer 1932, p. 807), but did not report them in any publications. Similarly, Casey (1935a,b) reported bed-load transport data for only two of his 17 sediment mixtures. Prior to World War II the unpublished transport data were available upon request from the Prussian Experiment Institute (Johnson 1943) but may have been lost during the war, as were Shields' data (Kennedy 1995).

To recreate Shields' methods, all shear-stress calculations were corrected for momentum diffusion caused by proximity of channel walls (i.e., so-called ''width-to-depth effects'') using Shields' approach for uniformly rough boundaries. Because Gilbert (1914), Casey (1935a,b), and USWES (1935) used smooth-walled flumes, differences in skin friction between the bed and walls should also be considered in the sidewall correction [e.g., Einstein (1934), Johnson (1942), Einstein and Barbarossa (1952), Vanoni and Brooks (1957), and Houjou et al. (1990)]. However, Shields' approach for this latter correction involved experiment-specific measurements of vertical velocity profiles. Because few vertical velocity profiles were measured by the supplemental sources, Shields probably did not attempt this latter correction. As with Shields' experiments, no stress corrections were made for form drag caused by bed forms and relative roughness.

To assess whether Shields applied a sidewall correction, as

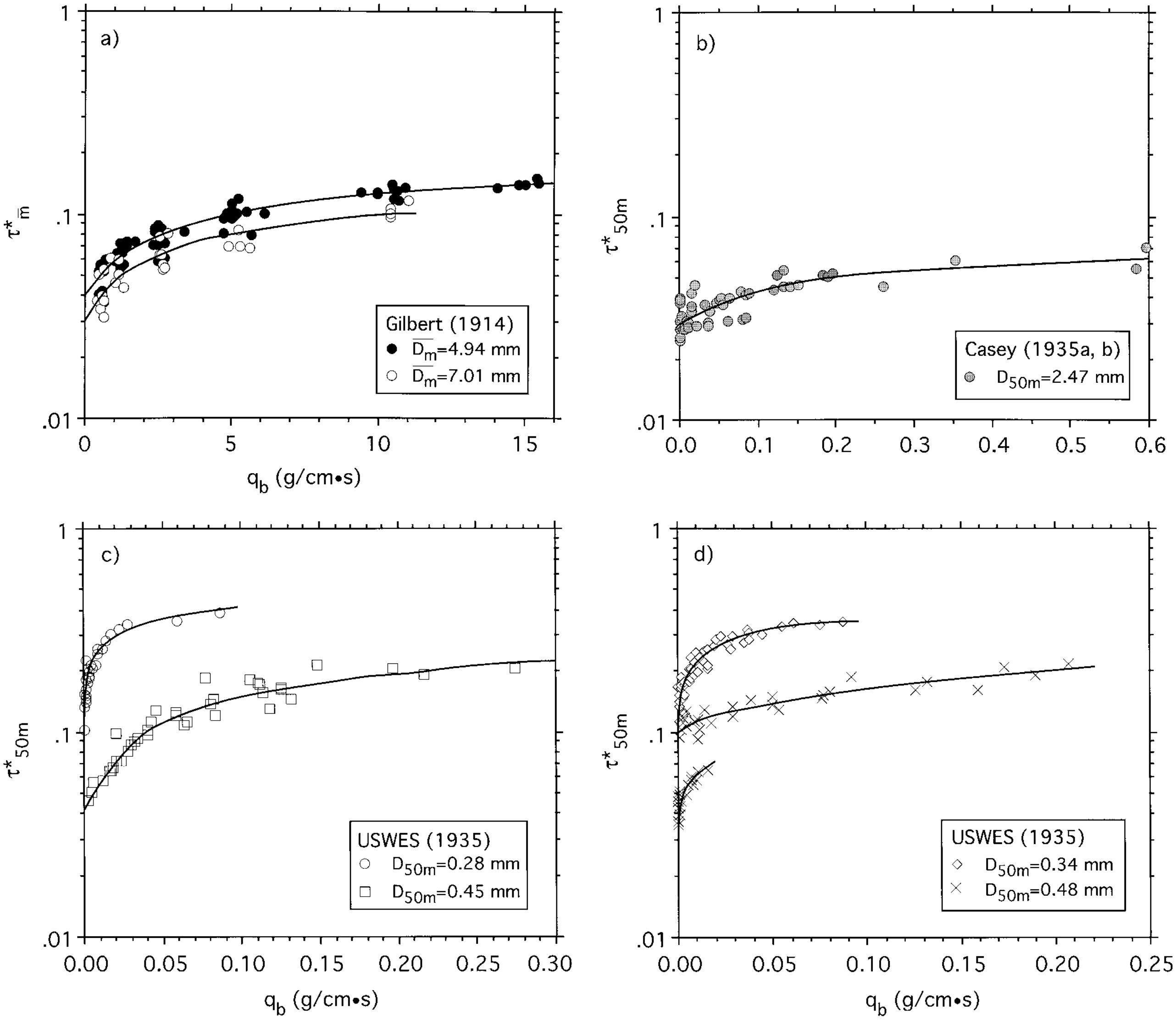

FIG. 1. Dimensionless Shear Stress as Function of Bed-Load Transport Rate (q_b) for Laboratory Flume Experiments by: (a) Gilbert; (b) Casey; (c–h) USWES. (f–h) Are the Same Experiments as (c–e) but with Dimensionless Shear Stress Based on $\bar{D}_m$, Rather Than D_{50m}

assumed, the above analysis was repeated with uncorrected shear stresses (Table 2).

Results of these dimensionless critical shear stress comparisons indicate that Shields did, in fact, use Kramer's (1932, 1935) weak-movement criteria for all but Gilbert's (1914) data. For the Kramer (1935), Casey (1935a,b), and USWES (1935) experiments, the visually based values of dimensionless critical shear stress ($\tau^*_{c_{v50m}}$ and $\tau^*_{c_{v\bar{m}}}$) for weak movement are in fairly close agreement with Shields' generic dimensionless critical shear stresses (τ^*_c), whereas the medium-movement and general-movement values are typically higher [Figs. 3(a–d); Tables 1 and 2]. Furthermore, comparison of weak-movement values derived from mean versus median grain sizes indicates that the dimensionless critical shear stresses reported by Shields for USWES (1935) are for their mixture means ($\tau^*_{c_{v\bar{m}}}$), rather than medians ($\tau^*_{c_{v50m}}$) [Figs. 3(c,d); Tables 1 and 2]. For Gilbert's (1914) data, Shields apparently defined dimensionless critical shear stress based on thresholds of general motion, rather than weak movement [Fig. 3(e); Tables 1 and 2].

Use of uncorrected shear stresses produces almost perfect agreement between Shields' τ^*_c values and Casey's (1935a,b) dimensionless critical shear stresses for weak movement [Fig. 3(b); Table 2], and slightly improves the agreement with Kramer's values [Fig. 3(a); Table 2]. However, uncorrected shear stresses degrade the agreement between the other data sources and Shields' reported values. Consequently, it appears that Shields corrected for sidewall effects in some studies, but not others.

Contrary to popular belief, Shields did not use bed-load extrapolation to define dimensionless critical shear stresses for these supplemental data. Reference-based dimensionless critical shear stresses ($\tau^*_{c_{r50m}}$ and $\tau^*_{c_{r\bar{m}}}$) are lower than τ^*_c values reported by Shields for Casey (1935a,b) [Fig. 3(b)] but are generally higher for the USWES (1935) data [Figs. 3(c and d)] and are either higher or lower for the Gilbert (1914) data, depending on whether Shields' sidewall correction is applied [Fig. 3(e)].

The results of this analysis suggest that Shields may have used multiple incipient-motion definitions when combining his data (probably reference-based) with the supplemental data (visually based). Furthermore, the dimensionless critical shear stresses calculated by Shields for the supplemental studies combine incipient motion thresholds for weak and general movement, mean and median grain sizes, and stresses corrected for proximity of walls in some cases, but not in others. Use of multiple incipient-motion definitions, multiple degrees of roughness correction, and multiple characteristic grain sizes for scaling critical stresses may explain some of the scatter of Shields' compiled results (Shields 1936b,c, his Fig. 6). In particular, differences within and between methods of defining

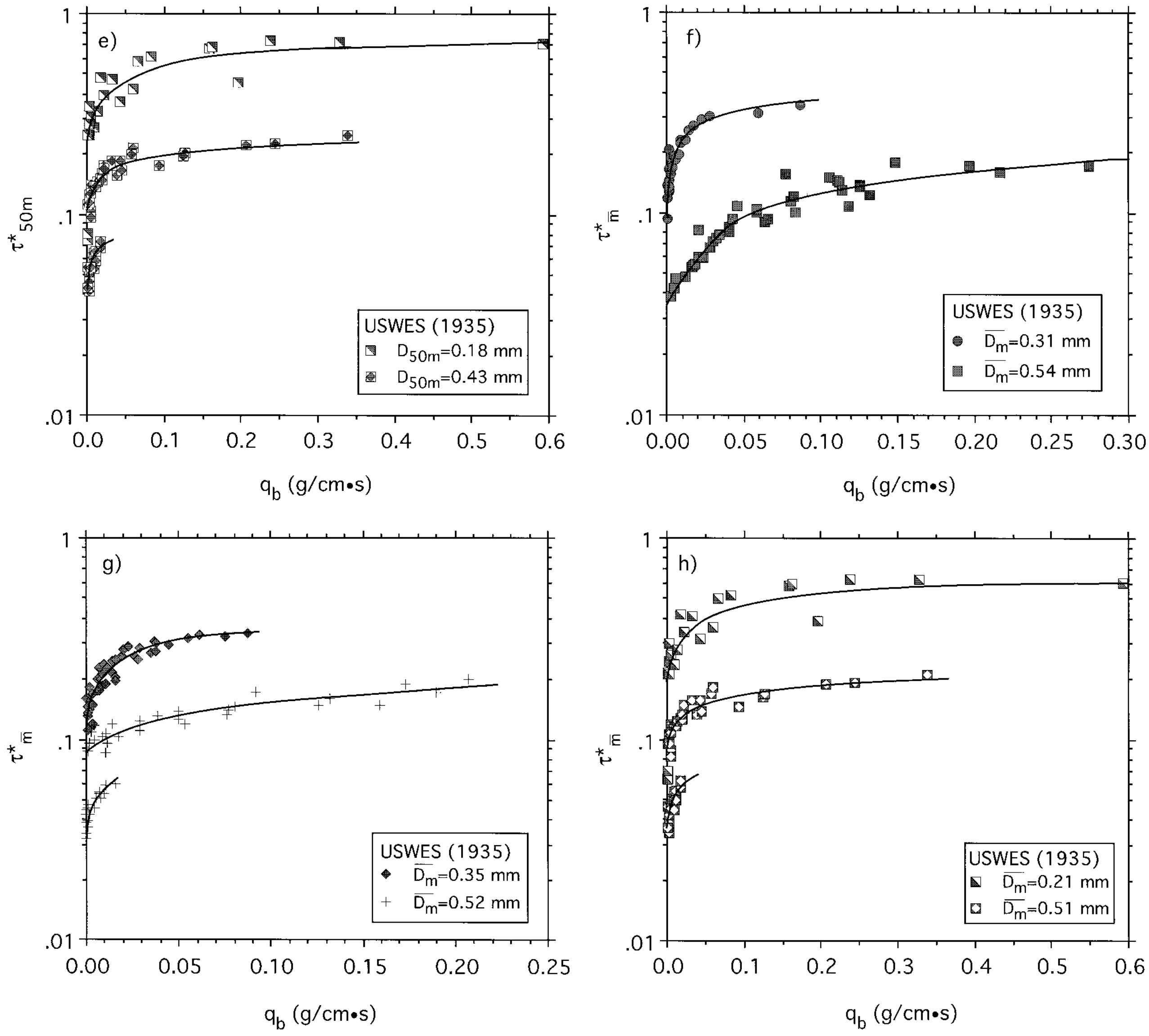

FIG. 1. (*Continued*)

incipient motion can cause systematic biases in calculated dimensionless critical shear stresses (Buffington and Montgomery 1997). Mixture-specific differences in both sediment sorting (0.16–0.82ϕ) and grain rounding (very angular to well rounded) also may have contributed to the scatter of Shields' compiled data by creating variable grain protrusions and friction angles that, in turn, may have caused mixture-specific incipient-motion thresholds.

"EVOLUTION" OF SHIELDS' CURVE

Shields demonstrated that the dimensionless critical shear stress of the median grain size of unworked laboratory mixtures ($\tau^*_{c_{50m}}$) varies as a function of critical boundary Reynolds number (R^*_c) (Fig. 4). The boundary Reynolds number represents both the thickness of the viscous sublayer and the hydrodynamic conditions of the flow around the surface grains. Based on analogy with Nikuradse's (1933) findings, Shields identified three hydrodynamic boundary conditions (smooth, transitional, and rough) and hypothesized that $\tau^*_{c_{50m}}$ attains a constant value of 0.060 at $R^*_c \geq 1{,}000$ (Shields 1936c, p. 16) (Fig. 4). Shields also speculated an abrupt drop in $\tau^*_{c_{50m}}$ at even higher R^*_c values (Shields 1936c, p. 16) based on known form-drag experiments [e.g., Wieselsberger, cited by Schlichting (1968, p. 17)]. As discussed above, the dimensionless critical shear stresses that Shields reported for Gilbert (1914) and USWES (1935) are actually for the mean grain size ($\tau^*_{c_{\bar{m}}}$), which differs from that of $\tau^*_{c_{50m}}$ only for the USWES (1935) experiments.

Many additions and revisions of Shields' incipient-motion curve have been made over the last 60 years, providing further insight and detail about the processes influencing particle motion [see review by Buffington and Montgomery (1997)]. However, Shields' original data also have "evolved" over time because of drafting errors and personal interpretations by later authors.

Rouse's Interpretation

Shields represented the relationship between $\tau^*_{c_{50m}}$ and R^*_c as a wide band, emphasizing the variability in incipient-motion values for any particular boundary Reynolds number (Fig. 4). Note that Shields' hypothesized value of 0.060 at $R^*_c \geq 1{,}000$ is shown as the top of his data envelope rather than an average value. Rouse promoted Shields' work in the United States, originally preserving the representation of the data by a wide band (Rouse 1939b) but subsequently fitting a line through the data (Rouse 1939a, 1949) (Fig. 4). Rouse's fit of the data roughly bisects Shields' original data envelope but diverges toward the upper envelope at higher R^*_c values, approaching $\tau^*_{c_{50m}} \approx 0.060$ at $R^*_c \geq 1{,}000$ as suggested by Shields (Fig. 4).

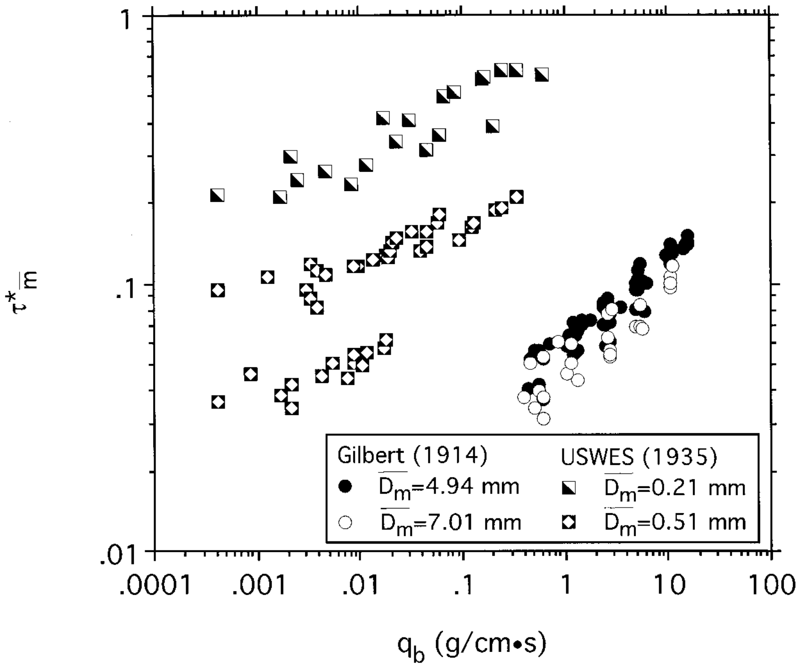

FIG. 2. Data from Figs. 1(a and h) Replotted with log-log Axes, Demonstrating Commonly Observed Power-Law Relationship for Stress-Transport Data

TABLE 2. Uncorrected Incipient-Motion Values Determined from Visual and Reference Techniques Compared to Dimensionless Critical Shear Stresses Reported by Shields

Source (1)	D_{50m} (mm) (2)	σ_g (ϕ) (3)	$\tau^*_{c_{v50m}}$, weak (4)	$\tau^*_{c_{v50m}}$, medium (5)	$\tau^*_{c_{v50m}}$, general (6)	$\tau^*_{c_{r50m}}$ (7)	τ^*_c (Shields 1936b) (8)
Gilbert (1914)	(4.94)	<0.26	(0.055)	(0.056)	(0.058)	(0.083)	0.050
	(7.01)	<0.22	(0.073)	(0.085)	(0.074)	(0.077)	0.059
Kramer (1935)	0.51	0.74	0.029	0.039	0.045	—[a]	0.032
	0.53	0.81	0.036	0.048	0.057	—[a]	0.038
	0.55	0.62	0.033	0.042	0.052	—[a]	0.033
Casey (1935a,b)	0.17	0.41	0.067	0.076	0.083	—[a]	0.067
	0.26	0.42	0.050	0.055	0.062	—[a]	0.051
	0.67	0.16	0.036	0.042	0.047	—[a]	0.034
	0.87	0.17	0.035	0.041	0.045	—[a]	0.034
	0.93	0.19	0.037	0.042	0.046	—[a]	0.037
	1.27	0.20	—[a]	—[a]	—[a]	—[a]	0.040
	1.74	0.19	0.041	0.046	0.051	—[a]	0.041
	2.47	0.22	0.038	0.041	0.046	0.033	0.038
USWES (1935)	0.18 (0.21)	0.32	0.062 (0.054)	0.077 (0.066)	0.204 (0.175)	0.280 (0.220)	0.051
	0.28 (0.31)	0.53	0.052 (0.047)	0.148 (0.133)	0.285 (0.257)	0.110 (0.100)	0.045
	0.34 (0.35)	0.37	0.047 (0.046)	0.090 (0.087)	0.229 (0.222)	0.127 (0.120)	0.038
	0.43 (0.51)	0.82	0.046 (0.038)	0.059 (0.049)	0.102 (0.086)	0.043–0.120 (0.036–0.082)	0.034
	0.48 (0.52)	0.53	0.040 (0.037)	0.049 (0.045)	0.058 (0.053)	0.037–0.120 (0.034–0.099)	0.036
	0.45 (0.54)	0.66	0.041 (0.034)	0.062 (0.052)	0.075 (0.062)	0.050 (0.040)	0.036

Note: See Table 1 footnotes.

Henderson's Interpretation

Henderson's (1966) fit of the data is similar to Rouse's but more accurately bisects the original data envelope at higher R^*_c values, putting $\tau^*_{c_{50m}}$ at 0.056 for $R^*_c \geq 1{,}000$ (Fig. 4). This value of 0.056 is widely cited as ''the Shields' parameter'' outside of the United States [e.g., Novak and Nalluri (1975), Philipps (1980), and Carson and Griffiths (1985)], whereas Americans generally adhere to the 0.060 value suggested by Shields [e.g., Rouse (1939a,b, 1949), Vanoni (1966), Little and Mayer (1976), and Wiberg and Smith (1987)]. The difference between 0.060 and 0.056 is academic, because both values are subjective extrapolations of Sheilds' data. As Shields (1936c, p. 16) puts it, ''. . . an estimation of this constant value can only be obtained through very uncertain extrapolation.''

Musical Chairs

As Shields' data have been redrafted over the years some of the data have exchanged places with one another or moved to new positions altogether, much like a game of musical chairs. These changes in data position are most likely the result of drafting errors or misidentification of data points in photocopies of Shields' original incipient-motion curve. For example, in later renditions of the Shields curve, Rouse (1939a, 1949) mislabels the first USWES (1935) point as belonging to Kramer (1932, 1935)—a mistake that has been perpetuated by many subsequent authors [e.g., Vanoni (1964), Paintal (1971), Miller et al. (1977), Yalin and Karahan (1979), Blatt et al. (1980), and Middleton and Southard (1984)]. Vanoni (1966, 1975) erroneously shifted the last Gilbert (1914) point from

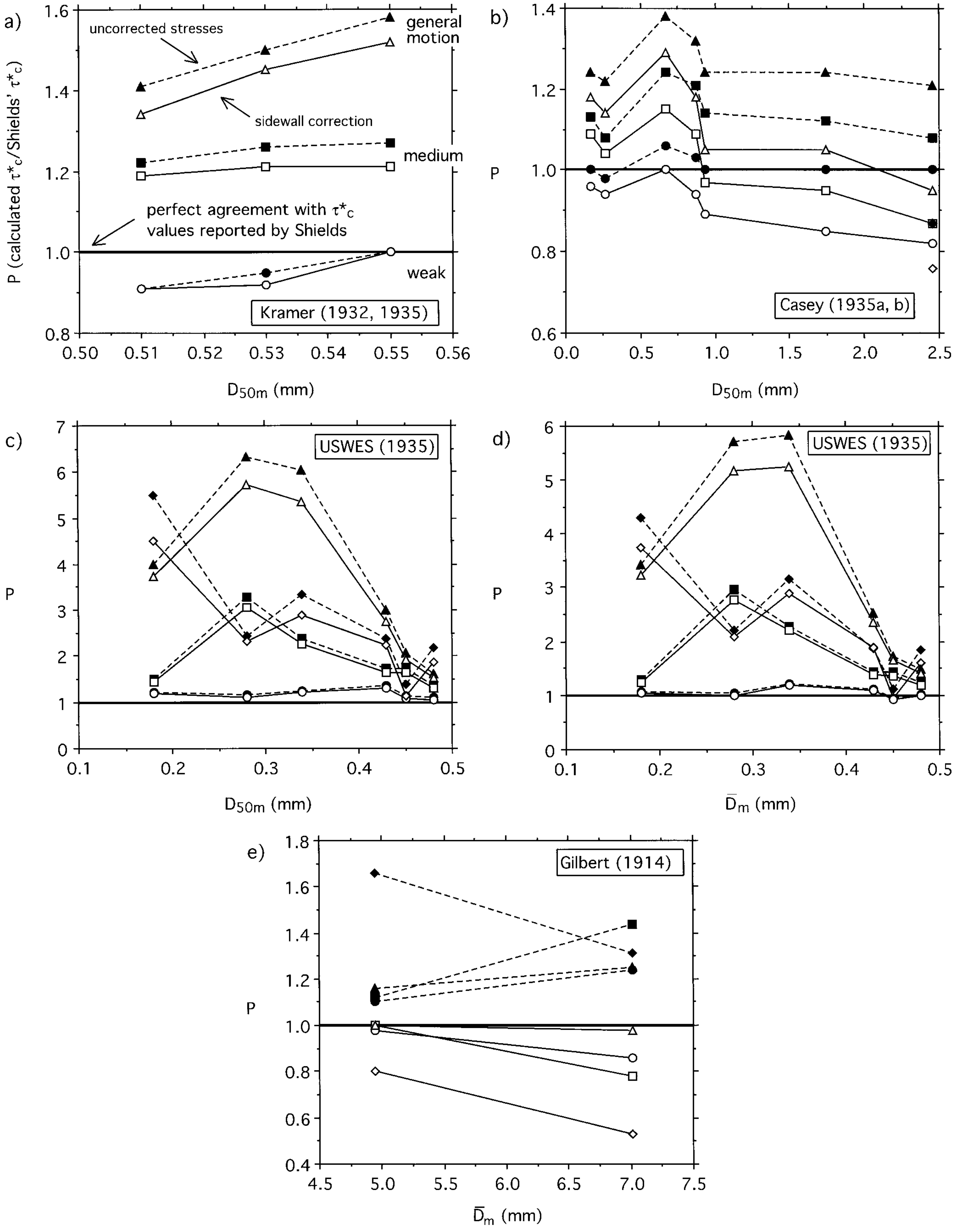

FIG. 3. Ratio of Dimensionless Critical Shear Stresses (*P*), Comparing Calculated Dimensionless Critical Shear Stresses (Columns 4–7, Tables 1 and 2) to Those Reported by Shields (Column 8, Tables 1 and 2) for Sediments Used by: (a) Kramer; (b) Casey; (c, d) USWES; (e) Gilbert. Heavy Black Line ($P = 1$) Represents Perfect Agreement with Shields' Values. Circles, Squares, and Triangles Represent Kramer's Visual-Based Definitions of Weak, Medium, and General Movement, Respectively (Columns 4–6, Tables 1 and 2); Diamonds Represent Bed-Load Extrapolation (Column 7, Tables 1 and 2; Fig. 1); Open Symbols with Solid Lines Have Been Corrected for Sidewall Effects (Table 1), While Closed Symbols with Dashed Lines Have Not (Table 2). (d) Is Same as (c) but with Dimensionless Critical Shear Stress Based on $\bar{D}_m$, Rather Than D_{50m}

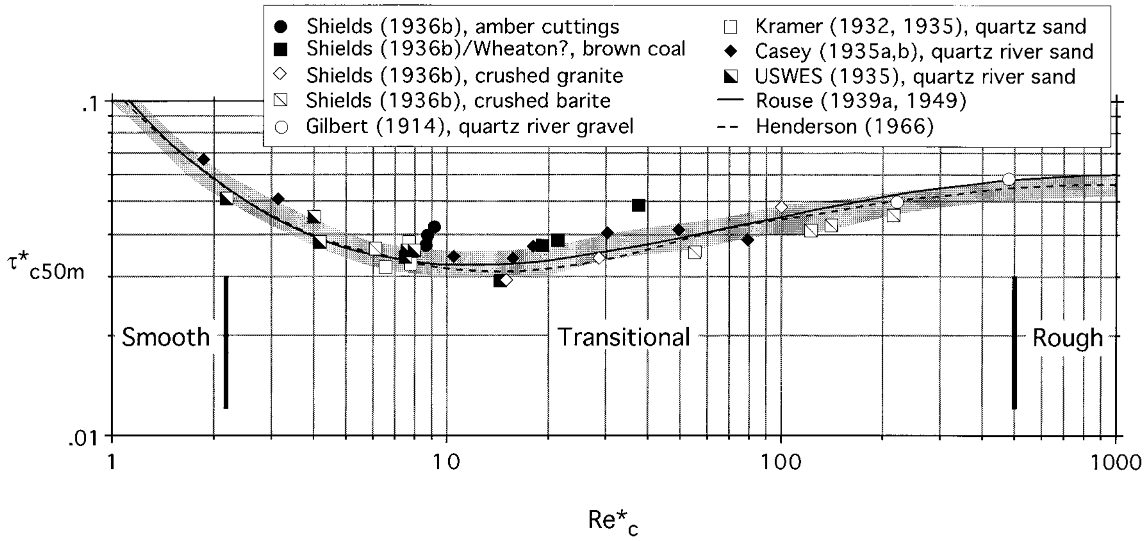

FIG. 4. $\tau^*_{c_{50m}}$ **as Function of** R^*_c**, Redrafted from Shields (1936b). Shaded Band with Irregular Boundaries Is Data Envelope and Curve Fit as Defined by Shields. Solid and Dashed Lines Are Later Fits of These Data by Subsequent Authors. Note that Gilbert and USWES Values Are Actually for** $\tau^*_{c_{\bar{m}}}$**, Which Differs from that of** $\tau^*_{c_{50m}}$ **Only for USWES**

TABLE 3. Incipient-Motion Values and Sediment Characteristics of Flume Studies Used by Shields

Source (1)	$\tau^*_{c_{50m}}$ (2)	R^*_c (3)	D_{50m} (mm) (4)	σ_g (φ) (5)	ρ_s (kg/m^3) (6)	Particle characteristics (7)
Shields (1936b)	0.037	8.6	1.56	0.59	1060	Very angular amber cuttings
	0.040	8.7	1.56	0.59	1060	Very angular amber cuttings
	0.042	9.1	1.56	0.59	1060	Very angular amber cuttings
Shields (1936b) and Wheaton (?) (unpublished report, n.d.)	0.029	14.3	1.77	0.78	1270	Subangular brown coal
	0.037	19.3	1.77	0.78	1270	Subangular brown coal
	0.038	21.3	1.88	0.72	1270	Subangular brown coal
	0.049	37.4	2.53	0.56	1270	Subangular brown coal
Shields (1936b)	0.029	14.9	0.85	0.23	2700	Very angular crushed granite
	0.034	28.5	1.23	0.23	2700 [2710]	Very angular crushed granite
	0.048	100	2.44	0.23	2700 [2690]	Very angular crushed granite
	0.036	6.1	0.36	0.30	4250 [4300]	Angular crushed barite
	0.035	54.5	1.52	0.35	4250 [4200]	Angular crushed barite
	0.041	121	2.46	0.22	4250 [4190]	Angular crushed barite
	0.043	140	2.76	0.41	4250 [4200]	Angular crushed barite
	0.046	216	3.44	0.16	4250 [4200]	Angular crushed barite
Gilbert (1914)	(0.050)	(221)	(4.94)	<0.26	2650 [2690]	Subrounded to subangular gravels from the Sacramento and American rivers
	(0.059)	(473)	(7.01)	<0.22	2650 [2690]	
Kramer (1932, 1935)	0.032	6.5	0.51	0.74	2650 [2700]	Well-rounded sand
	0.038	7.6	0.53	0.81	2650 [2700]	Well-rounded sand
	0.033	7.7	0.55	0.62	2650 [2700]	Well-rounded sand
Casey (1935a,b)	0.067	1.9	0.17	0.41	2650	Subangular to subrounded river sand
	0.051	3.1	0.26	0.42	2650	Subangular to subrounded river sand
	0.034	10.4	0.67	0.16	2650	Subangular to subrounded river sand
	0.034	15.8	0.87	0.17	2650	Subangular to subrounded river sand
	0.037	18.1	0.93	0.19	2650	Subangular to subrounded river sand
	0.040	30.2	1.27	0.20	2650	Subangular to subrounded river sand
	0.041	49.2	1.74	0.19	2650	Subangular to subrounded river sand
	0.038	79.6	2.47	0.22	2650	Subangular to subrounded river sand
USWES (1935)	(0.051)	(2.2)	(0.21)	0.32	2650	Subangular to angular Mississippi River sand
	(0.045)	(4.0)	(0.31)	0.53	2650	Subrounded to subangular Okay Creek sand
	(0.038)	(4.1)	(0.35)	0.37	2650	Subrounded to subangular Mississippi River sand
	(0.034)	(7.5)	(0.51)	0.82	2650	Angular to subrounded creek sand
	(0.036)	(7.6)	(0.52)	0.53	2650	Subrounded to rounded Mississippi River sand
	(0.036)	(7.9)	(0.54)	0.66	2650	Subangular to subrounded creek sand

Note: Values in rounded parentheses are for $\overline{D}_m$, rather than D_{50m}; see Table 1 note. ρ_s values in square brackets (Column 6) are actual sediment densities as reported by the original source, as opposed to those reported in Shields' [1936b,c, Fig. 6 (nonbracketed values)]; it is uncertain which of these two ρ_s values were used in Shields' calculations. Roundness terms (Column 7) from Russell and Taylor (1937) and Powers (1953).

$R^*_c \approx 470$ to $R^*_c \approx 550$, slighlty elongating the Shields' curve. He also slightly shifted the positions of many other data points and attributed two of the USWES (1935) points between $R^*_c = 7$ and 8 as belonging to Kramer (1932, 1935) [cf. Fig. 4 with Vanoni's (1966) Fig. 2-E.2. or Vanoni's (1975) Fig. 2.43]. Paintal (1971) dropped Gilbert's (1914) data, mislabeled the granite and barite points as amber, and dropped two of USWES's (1935) points. Mantz (1977) dropped one of Gilbert's (1914) data points and two of USWES's (1935), and shifted the positions of many other points. Miller et al. (1977, their Fig. 1) dropped two barite points and one of Casey's (1935a,b), referred to granite as graphite, and mislabeled a

Kramer (1932, 1935) value as amber. This discouraging litany continues in many other reports [e.g., Mizuyama (1977), Yalin (1971), and Yalin and Karahan (1979)].

The data points and sediment types shown in Fig. 4 and Table 3 have been carefully redrafted from Shields' original presentation of the data (Shields 1936b, his Fig. 6; Rouse 1939b). Despite careful examination of Shields' original plot, the clustering and overlap of some points [e.g., those of Kramer (1932, 1935), USWES (1935), and Shields' amber grains in Fig. 4] cause the potential for misidentification of sediment type and the omission of data masked by other points. For example, Shields employed six types of barite mixtures, but there are only five observable barite points in his incipient-motion curve (Shields 1936b, his Fig. 6) (Fig. 4). Nevertheless, a more accurate redraft of Shields' data is not possible without obtaining his original laboratory measurements, which apparently have been lost (Kennedy 1995). Future investigators should beware that the first value of the *y*-axis in the original Shields (1936b) plot is mistakenly labeled as 0.010, rather than 0.020.

DISCUSSION AND CONCLUSIONS

Comparison of Shields' (1936a,b,c) original work with the diverse stories and data reported in his name by subsequent investigators allows clarification of some inconsistencies and misconceptions. However, in many instances Shields' descriptions of his work are vague or incomplete, leaving some matters unresolved. For example, Shields probably used a reference-transport technique (i.e., bed-load extrapolation) to determine incipient-motion thresholds for the four sediment types he investigated. However, without recourse to his original laboratory measurements (presumably now lost), we will never really know how his data were analyzed. Furthermore, even if we assume that Shields did, in fact, use bed-load extrapolation to determine incipient-motion thresholds for his experiments, we do not know how he fitted the data. This is an important piece of information, because differences in curve-fitting techniques for reference-based incipient-motion studies can produce different results (Paintal 1971; Diplas 1987; Wilcock 1988; Ashworth and Ferguson 1989; Ashworth et al. 1992; Wathen et al. 1995). For example, Paintal (1971, his Fig. 8) found a several-fold difference in $\tau^*_{c_{r50m}}$ values depending on whether stress-transport relations are fitted with a linear or nonlinear curve.

Although some aspects of Shields' work remain enigmatic, popular misconceptions and errors concerning his work can be dispelled through scrutiny of his original publications and data. The conclusions drawn from this investigation of Shields' doctoral work are as follows:

1. Shields probably defined incipient-motion thresholds for the four sediment types that he examined by extrapolating stress-transport relations to a zero transport rate, as is popularly believed. In contrast, he defined incipient-motion values for supplemental data sources using two differing thresholds of visually based movement [i.e., Kramer's (1932, 1935) weak versus general motion]. Consequently, Shields may have mixed reference-based and visual-based incipient-motion approaches, as well as mixed subdefinitions of visual motion (i.e., weak versus general), contributing to the scatter of his reported dimensionless critical shear stresses.
2. A variety of bed forms and relative roughnesses were present during Shields' experiments. Because he did not account for the form drag caused by these roughness elements, values of dimensionless critical shear stress determined from stress-transport extrapolation may have been overestimated. Furthermore, because sediment transport is influenced by bed morphology and consequent velocity structure (Middleton and Southard 1984), it is uncertain whether extrapolation of bed-load transport data from channels with ripples and dunes has relevance for determining incipient motion of a lower-stage plane-bed channel, as was intended by Shields.
3. Shields' method of defining initial motion by extrapolating stress-transport curves to a zero transport level is flawed. Most stress-transport relations are power functions that do not attain zero transport until still-water conditions are achieved (Paintal 1971; Lavelle and Mofjeld 1987). Use of stress-transport data to determine critical mobility conditions requires specification of a reference transport rate that is relevant for the particular study goal (Paintal 1971).
4. The sediments used by Shields and his other data sources were not uniform in size but rather mixtures with σ_g values ranging from 0.16 to 0.82ϕ. Furthermore, the sediments varied from well rounded to very angular. Mixture-specific differences in grain protrusion and friction angle (functions of grain size, sorting, shape, and rounding) likely led to selective transport and differences in incipient-motion thresholds that may explain some of the scatter of dimensionless critical shear stress values reported by Shields.
5. Shields' dimensionless critical shear stresses are not grain-size specific [compared to those of Day (1980) or Parker et al. (1982)] but, rather, are derived from bulk measures of sediment movement, with corresponding critical shear stresses scaled by either the mean or median grain size of the sediment mixture. Because mean and median grain sizes can be somewhat different for the same data set, dimensionless critical shear stresses determined from differing characteristic grain sizes (i.e., mean versus median) may explain some of the scatter of Shields' results. Furthermore, predicting bed-surface mobility from Shields' dimensionless critical shear stresses is cautioned because his values combine critical shear stresses for water-worked sediments with grain size percentiles of unworked mixtures; worked and unworked sediments may have different incipient motion thresholds, depending on the degree of textural alteration caused by hydraulic working of the bed.
6. Although Shields hypothesizes that $\tau^*_{c_{50m}}$ attains a constant value of 0.060 at $\mathsf{R}^*_c \geq 1{,}000$, he cautions that this conclusion is very tentative; the scatter of his data invite alternate interpretations, such as Henderson's (1966) 0.056 value. The specific value is academic and by no means sacred. What is more relevant is the selection of a defendable value of dimensionless critical shear stress for one's particular study goals (Buffington and Montgomery 1997).
7. Despite the wide success and familiarity of Shields' work, almost every redraft of his incipient-motion data contains errors. Although scientifically discouraging, this observation illustrates how often-told tales can become legendary.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

D = characteristic grain size;
$\bar{D}_m$ = mean grain size of hydraulically unworked laboratory sediment mixture;
D_{50} = median grain size;
D_{50m} = D_{50} of hydraulically unworked laboratory sediment mixture;
g = gravitational acceleration;
h = flow depth;
P = ratio of calculated dimensionless critical shear stress to that reported by Shields;
q_b = bed-load transport rate (weight per channel width per unit time);
R' = hydraulic radius of bed;
R_c^* = critical boundary Reynolds number (Eq. 4);
S = water-surface slope;
u_c^* = critical shear velocity, $u_c^* \equiv \sqrt{\tau_c'/\rho}$;
ν = kinematic viscosity;
ρ = fluid density;
ρ_s = sediment density;
σ_g = graphic sorting coefficient, $\sigma_g = (\phi_{84} - \phi_{16})/2$ (Folk 1974);
τ' = bed-shear stress;
τ_c' = shear stress applied to bed at incipient motion;
τ_c^* = dimensionless critical shear stress of D [Eq. (2)] for unspecified method of determining incipient motion;
$\tau_{c_{\bar{m}}}^*$ = dimensionless critical shear stress of $\bar{D}_m$ based on unspecified method of determining incipient motion;
$\tau_{c_{r\bar{m}}}^*$ = dimensionless critical shear stress of $\bar{D}_m$ for low, reference value of q_b (Shields' reference q_b is 0);
$\tau_{c_{v\bar{m}}}^*$ = dimensionless critical shear stress of $\bar{D}_m$ determined by visual observation of grain movement;
$\tau_{c_{50m}}^*$ = dimensionless critical shear stress of D_{50m} based on unspecified method of determining incipient motion;
$\tau_{c_{r50m}}^*$ = dimensionless critical shear stress of D_{50m} for low, reference value of q_b (Shields' reference q_b is 0);
$\tau_{c_{v50m}}^*$ = dimensionless critical shear stress of D_{50m} determined by visual observation of grain movement;
$\tau_{\bar{m}}^*$ = noncritical value of dimensionless shear stress for $\bar{D}_m$;
τ_{50m}^* = noncritical value of dimensionless shear stress for D_{50m};
ϕ = $\log_2$ unit of grain size measurement, $\phi = -\log_2 D$ for D (mm) (Krumbein 1936); and
ϕ_{nn} = ϕ value for which *nn*% of sizes are finer.