# Simulations of Ultrasonic Transit Time in a Fully Developed Turbulent Flow using a Ray-Tracing Method

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## ABSTRACT

The work describes the effect of turbulent-like fluctuations in fully developed flow on the trajectories of a single path, transit time ultrasonic flow meter in a plane channel. A ray-tracing method was used to simulate the trajectories of the ultrasonic pulse. Results obtained for flows at Mach Number from 0.01 to 0.1, and Reynolds Numbers, $Re_\tau$ from 193 to 22494 suggest that as Mach Number increases, the deviation in the transit time measurement due to turbulent-like fluctuations increases. The maximum measurement error for low Reynolds Number flows may be as large as 0.26% at Mach Number of 0.1.

## 1 INTRODUCTION

Ultrasonic flow meters were developed in the 1940s to measure volumetric flow rate non-intrusively [1]. This capability is advantageous because, unlike with other types of flow meters, there is a negligible pipeline pressure drop introduced by the instrument. There are several techniques that can be employed in an ultrasonic flow meter [1], but the present work discusses the transit time method only.

Non-uniformities in the velocity profile can affect the measurement uncertainty. These non-uniformities can derive from two sources: flow gradients (spatial inhomogeneities) and flow unsteadiness (temporal inhomogeneities). If the mean velocity profile in a pipe is not axisymmetric, the velocity profile along the pulse path may not be a representative calculation, thus the mean velocity obtained from the transit time difference may be substantially different from the actual mean velocity [2-4]. Regulations for gas flow measurement are in place [5] and an uncertainty of ±0.7% is required for multi-path flow meters.

Yeh and Mattingly, [3] used analytical velocity profiles to simulate fully developed turbulent flow in a pipe and computational solutions of the Reynolds Averaged Navier-Stokes (RANS) equations to simulate the flow downstream of a single elbow. They found that assuming a straight-line trajectory was adequate for Mach Number, *Ma,* less than 0.1. However, wherever the velocity flow profile was asymmetric, careful consideration of both the location and orientation of the flow meter were critical for optimum performance. Zanker, [4] used analytical asymmetric velocity profiles to simulate installation effects on ultrasonic flow meters. His results showed that consideration of the orientation of the flow meter with respect to the velocity profile was critical for optimum performance. Moore, [2] verified the conclusions of Yeh and Mattingly, and Zanker by carrying out CFD simulations, using RANS, of a four path ultrasonic flow meter, in a liquid, downstream of a single elbow, double elbow, contraction, and expansion. The simulations were verified experimentally and it was concluded that optimum measurement performance depends on the location of the flow meter. In addition, in asymmetric flow, the orientation of the flow meter must also be considered.

While the effects of mean flow gradients have been investigated by several researchers, it is not known how unsteadiness of the flow affects the measurements. In particular, in this work, we concentrate on the effects of the unsteadiness due to turbulence, which are always present in flows in pipes at moderate or high Reynolds numbers. A ray-tracing method proposed by Pierce [6] is used to calculate transit times for pulses that propagate into turbulent-like velocity profiles generated to match actual fluctuating profiles of the types found in ducts and pipes. The error due to flow unsteadiness incurred in the measurement of the transit times (and therefore, of the average flow rate), is then quantified as a function of the Reynolds and Mach Number of the flow.

## 2 PROBLEM FORMULATION

The transit time required by an ultrasonic pulse to traverse an enclosure such as that shown schematically in Fig. 1, depends on its trajectory and the fluid velocity distribution across the enclosure. Thus, simulation of this phenomena requires consideration of these two elements.

In this work, we artificially generate velocity profiles using the empirical formulae suggested by Dean [7] for the time-averaged velocity profile. Then unsteady turbulent fluctuations based on Direct Numerical Simulation (DNS) data obtained by Moser *et al* [8] are superimposed on the mean velocity profile. Finally sound waves are propagated through these velocity fields and comparison of the transit times obtained accounting for turbulent-like fluctuations with the same obtained assuming mean flow is done.

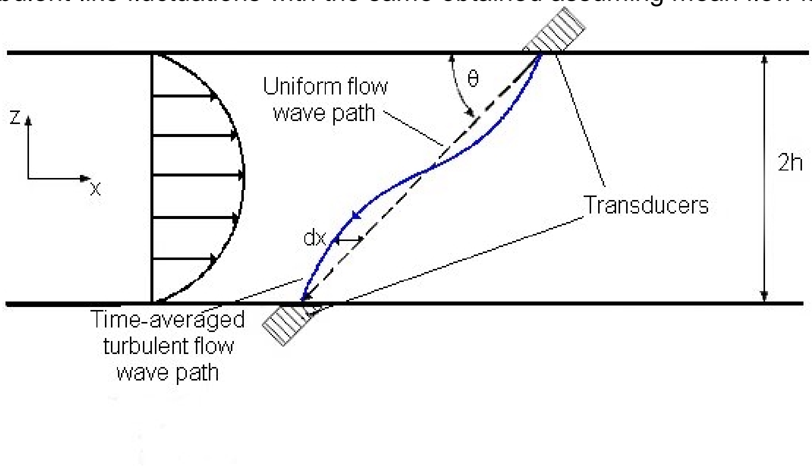


Fig. 1 - Schematic of the channel section used to simulate ultrasonic transit times in fully developed turbulent flow.

### 2.1 Ultrasonic Flow Meter

Transit time ultrasonic flow meters use two or more transducers to transmit ultrasonic pulses in opposite directions; in the simplest design, one pulse propagates downstream, the other upstream. The transit time principle operates by measuring the difference in the time required for the pulses to travel up and downstream. The component of flow velocity along the path adds to or subtracts from the speed of sound in the fluid in the downstream and upstream measurements respectively. By calculating the difference in the transit times of the pulses, one can obtain an estimation of the mean flow velocity along the pulses trajectory.

The propagation velocity of the ultrasonic sound wave is the sum of the speed of sound and the flow velocity in the propagation direction. Therefore, the transit time of the upstream, $t_1$, and downstream, $t_2$, traveling waves can be expressed as:

$$t_1 = \int_1^2 \frac{\mathrm{d}\mathbf{s}}{c + \mathbf{u} \bullet \mathbf{n}} \tag{1}$$

$$t_2 = \int_2^1 \frac{\mathrm{d}\mathbf{s}}{c - \mathbf{u} \bullet \mathbf{n}} \tag{2}$$

where $\mathbf{s}$ is the ultrasonic path length, $c$ the speed of sound in the fluid (assumed constant), $\mathbf{u}$ is the flow velocity vector at point $\mathrm{d}s$ along the path and $\mathbf{n}$ is the direction normal to the wave front. It can be shown that the mean velocity measured by the ultrasonic flow meter, $\bar{u}_m$, assuming uniform velocity, is given by:

$$\bar{u}_m = \frac{L\Delta t}{2 t_1 t_2 \cos\theta} \tag{3}$$

where $\theta$ is the angle of inclination of the acoustic propagation with respect to the axial direction of the flow (see Fig. 1), $L$ is the distance between the transducers and $\Delta t$ is the difference in the transit times.

A ray-tracing method outlined by Pierce [6] was used to simulate the propagation of the ultrasonic pulses through the fluid media. According to this formulation, the propagation of the ultrasonic pulse is represented by:

$$\frac{d\mathbf{s}}{dt} = c\mathbf{n} + \mathbf{u} \tag{4}$$

$$\frac{d\mathbf{n}}{dt} = -\sum_{k=1}^{3} n_k \left[\nabla - \mathbf{n}(\mathbf{n} \bullet \nabla)\right] u_k \tag{5}$$

where the diffraction terms have been neglected and the speed of sound, $c$, is assumed constant.

If the speed of sound, $c$, is infinitely greater than each of the components of the velocity of the fluid, $\mathbf{u}$, then the trajectory can be assumed linear with direction specified by $\theta$, since as $Ma \to 0$, $d\mathbf{s}/dt \to c\mathbf{n}$ and $d\mathbf{n}/dt \to 0$. Thus, averaging the transit times simulated in an ensemble of velocity profiles with turbulent-like fluctuations gives the same as the transit time simulated using the time-averaged velocity profile (*i.e.*, the effect of the turbulent fluctuations disappears).

However, as *Ma* increases the trajectory changes according to the local velocity gradient. Variation in either the local velocity gradient or *Ma* results in a curved trajectory. Thus, the average of the transit times obtained from an ensemble of velocity profiles with turbulent-like fluctuations is not the same as the transit time obtained from the time averaged velocity profile. The shift due to the unsteadiness and non-linearity of the ultrasonic pulse propagation is evaluated in this paper.

### 2.2 Velocity Profile

The turbulent velocity profiles were modeled by considering an ensemble of velocity profiles with turbulent-like fluctuations. The method of simulating the velocity profile had three steps. First, the time averaged turbulent velocity profile was assigned; then, random distributions of the velocity fluctuations were superposed on the mean velocity profile. Finally, the random velocity fluctuations were modulated in such a way that their root-mean-square distributions of the velocity fluctuations, $u'^{+}$, $v'^{+}$, and $w'^{+}$ matched the distributions computed by Moser *et al* [8]. Scaling methods were used to generate profiles at varying Reynolds Numbers and no Mach Number scaling was applied.

The mean velocity profile used in this work was divided into three regions, corresponding to the viscous sublayer, logarithmic region, and the channel core; they are given by

$$u^+ = z^+ \qquad \text{for } z^+ < 5 \tag{6}$$

$$u^+ = W(\eta)\left[\frac{1}{\mathrm{K}}\left(\ln z^+ + A\right) + \frac{\Pi}{\mathrm{K}}\omega(\eta)\right] + \left[1 - W(\eta)\right]\left[\frac{\eta^{1/7}}{u_\tau}\frac{u_0}{\bar{u}}\right] \qquad \text{for } z^+ > 5 \tag{7}$$

where $u^+$ is stream wise velocity normalized in wall units. Equation (6) corresponds to the viscous sublayer, whereas (7) matches the logarithmic law and wake regions [the terms in the first square bracket in (7)] to the core region of the channel [the last term in (7)] through a hyperbolic tangent matching function, $W(\eta)$ [7, 9]. The following definitions and constant values are used in the equations above:

$$\omega(\eta) = 2\sin^2\left(\frac{\pi\eta}{2}\right), \quad \eta = \left(1-|z|\right)\Big/h \qquad W(\eta) = \frac{1}{2}\left\{1 + \tanh\left[\frac{\beta(\eta - b)}{(1-2b)\eta + b}\right]\Big/\tanh(\beta)\right\},$$

$$z^+ = \frac{u_\tau(1-|z|)}{\nu} \qquad u_\tau = \frac{c_f^{1/2}\,\mathrm{Re}_b\,\mu}{2\sqrt{2}h}$$

$$c_f = 0.073\,\mathrm{Re}_b^{-\frac{1}{4}} \qquad \mathrm{K} = 0.4,\ A = 5,\ \Pi = 0.14,\ \beta = 4,\ b = 0.2,\ h = 1$$

The ratio between maximum velocity in the channel, $u_o$, and actual mean velocity, $\bar{u}$, is obtained from the empirical correlation $u_o/\bar{u} = 1.28\,\mathrm{Re}_b^{-0.0116}$ [7]. The resulting velocity profile is shown in Fig. 2 for $\mathrm{Re}_\tau = 400$.

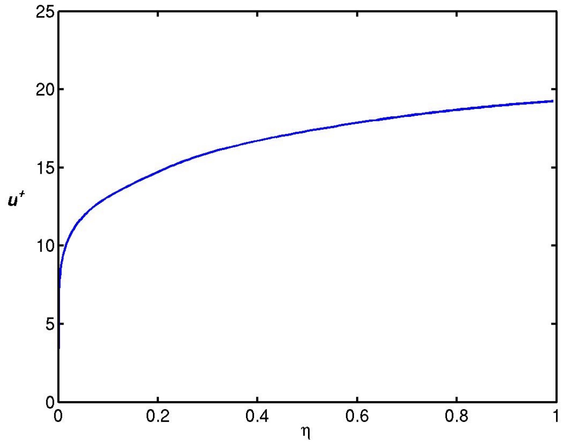


Fig. 2 - Velocity profile at $\mathrm{Re}_\tau = 400$ from Eqns. (6) and (7).

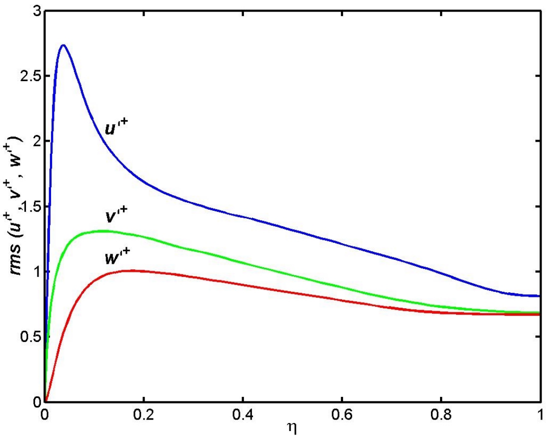


Fig. 3 - Root mean square of turbulent velocity fluctuations $u'^+$, $v'^+$ and $w'^+$. $\mathrm{Re}_\tau = 400$, [8].

To avoid grid effects in the evaluation of the velocity gradients required in (5) and to account more realistically for the varying sizes of the turbulent eddies as the wall is approached, the random velocity profiles were filtered using a box filter over a variable interval $l_m^+$ prescribed using a mixing-length hypothesis:

$$l_m^+ = \mathrm{K} z^+ \left[ 1 - \exp\left( -\, z^+ \big/ A^+ \right) \right] \tag{8}$$

This method allowed us to take into account the fact that the scale of the turbulent eddies (*i.e.*, the scale over which velocity gradients are significant) varies with distance from the wall. Fig. 4 compares the fully developed mean-flow distribution (Fig. 4-Top) with one turbulent-like profile (Fig. 4-Bottom).

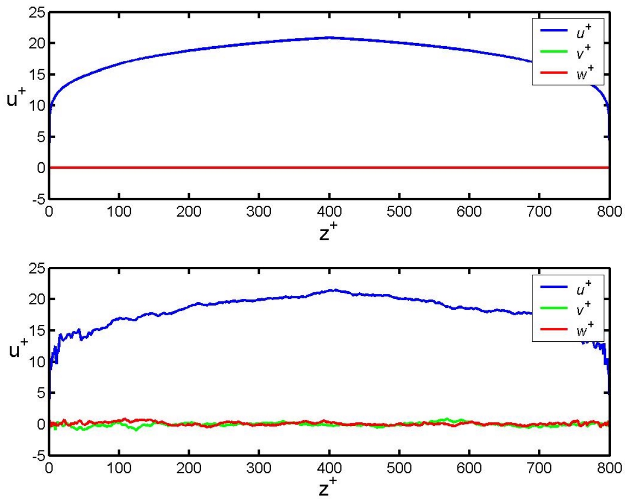


Fig. 4 - Fully developed turbulent velocity flow. Top: without instantaneous turbulent fluctuations and mixing length simulations; Bottom: with instantaneous turbulent fluctuations and mixing length simulations.

Typically, commercial ultrasonic flow meters measure instantaneous transit times, which are then averaged over multiple measurements. In order to simulate this technique, the transit times $t_1$ and $t_2$ were calculated for multiple realizations (*i.e.*, different random-number sequences), and then averaged.

### 2.3 Calculation of trajectories

Given that the ultrasonic wave front is spherical, there is an infinite number of normal unit vectors (*i.e.,* the emission unit-vector) and each could be considered by the ray-tracing method. However, of importance to the problem considered here is only the emission unit-vector that, when ray-traced, will arrive at the detection site.

For the uniform velocity profile case, it can be shown that the emission unit-vector aligned with the geometrical path between the two ultrasonic transducers, $L$, will always hit the detection site (see Fig. 1). Similarly, for the time-averaged velocity profile case, there is a unique emission unit-vector that will always arrive at the detection site. However, due to the random nature of the turbulent velocity profiles, the propagation of the same emission unit-vector would not always result in an arrival at the detection site. Thus, an iterative process was used to find the emission unit-vector that arrived at the detection site for each turbulent velocity profile considered. This iterative process was accomplished using a bisecting method while the time advancement of equations (4) and (5) was accomplished using a 3rd-order Runge-Kutta method.

Since in this work, we considered at least 1000 turbulent velocity profiles at any given Reynolds and Mach Number setting there will be an ensemble of transit times that will result. The results of those realizations were statistically averaged to obtain a representative result, which was then compared with the time-averaged velocity profile case to estimate the error in the ultrasonic flow meter due to turbulent fluctuations. In what follows we will show the results obtained for the time-averaged velocity profile case and contrast them with the results obtained for the turbulent velocity profiles.

## 3 RESULTS

The method described above was used to model the ultrasonic pulse propagation, using ray-tracing, in representative velocity profiles. For these results, a 45$^o$ acoustic propagation inclination angle was used [see equation (3)]. In what follows, these definitions of error, $E$, and deviation, $D$, are used:

$$E = \frac{\bar{u}_m - \bar{u}}{\bar{u}} * 100 \tag{9}$$

$$D = \left| \frac{t_i - t_{i\_uniform}}{t_{i\_uniform}} \right| * 100 \tag{10}$$

where $i = 1,2$ for down and upstream transit time respectively, $t_{i\_uniform}$ is that calculated in uniform flow.

### 3.1 Ultrasonic Pulse Propagation Through Time-Averaged Velocity Profiles

The estimation of mean flow velocity using equation (3) assumes uniform velocity in the channel, for which case, the trajectories of both ultrasonic pulses are (a) straight and (b) of length equal to $L$. In reality, however, these trajectories are curved by the gradients in the velocity profile and their lengths increase as a function of increasing Mach number, $Ma$. Using the ray tracing method, the trajectory deviations from the uniform flow case can be estimated.

Fig. 5 shows the percentage streamwise offset of the ultrasonic trajectories from the uniform-flow's straight-line trajectory, $dx/2h$ *100, as a function of channel height, $z/h$. The results reveal that the ultrasonic pulses are dragged downstream by the flow in the first half of their trajectory with the trend reversing in the second half. These changes lead to an S-shaped trajectory, which coincides with the straight-line trajectory at three points: its beginning, its end, and its mid-point. The observed S-shaped deflection increases in magnitude with increasing Mach Number.

An inequality in the length change for the upstream and downstream paths is apparent, and thus the associated transit times are affected by the increased length in the ultrasonic pulse trajectory (see Fig. 7). Given that in equation (3) the ideal path length, $s$, was replaced by the distance between the ultrasonic transducers, $L$, the path-length inequality can only be expressed through the time term in equation (3), $\Delta t / t_1 t_2$. It can be shown that for the curved trajectory case the time term becomes,

$$\frac{t_1 - t_2 + (\alpha_2 - \alpha_1)}{(t_1 + \alpha_1)(t_2 + \alpha_2)} \tag{11}$$

where, $\alpha_1$ and $\alpha_2$ represent the increased transit time in the upstream and downstream paths, respectively. In the special case of similar increase in path length (*i.e.,* $\alpha_1 = \alpha_2$ ) and $\bar{u} << c$ (*i.e.,* $t_1 \cong t_2$) the expression reduces to the original one and thus there is no error introduced. However, the non-linear interaction between the velocity profile and the pulse propagation equations leads to $\alpha_1 \neq \alpha_2$ for all velocity profiles other than uniform.

Fig. 6 shows how the resulting error in transit time difference varies as a function of Reynolds and Mach Numbers. As noted before, the error increases for increasing Mach Number, but reduces with increasing Reynolds Number – something advantageous for most ultrasonic flow meter applications where the Reynolds Numbers tends to be larger than 50,000. In fact, results show that for applications with bulk Reynolds Numbers larger than 10,000 (*i.e.,* $\mathrm{Re}_\tau \geq 3{,}000$), the dependency in Reynolds Number disappears and the error only growths as a function of Mach Number.

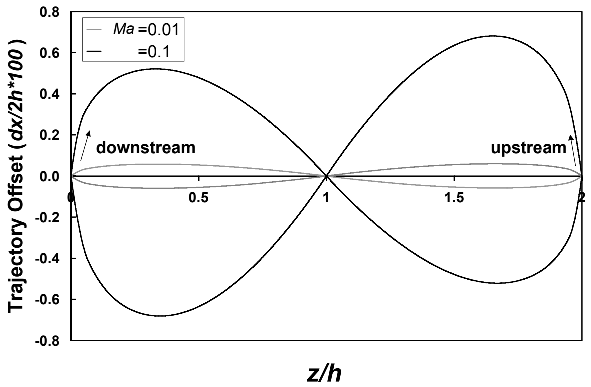


Fig. 5 – Percentage offset of trajectory from straight line in time averaged velocity profile. $\mathrm{Re}_\tau = 193$.

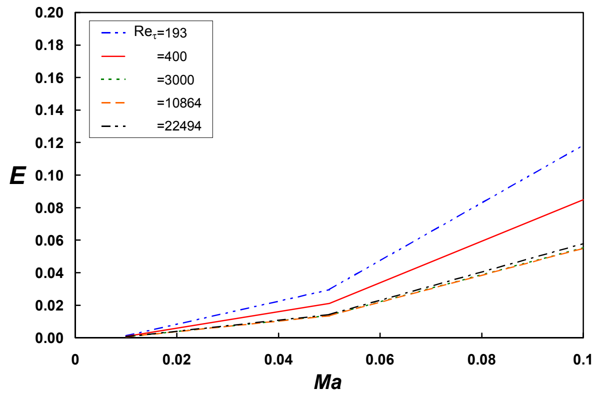


Fig. 6 – Error in the mean velocity measurement utilising ray-tracing method, using (9), $193 < \mathrm{Re}_\tau < 22494$.

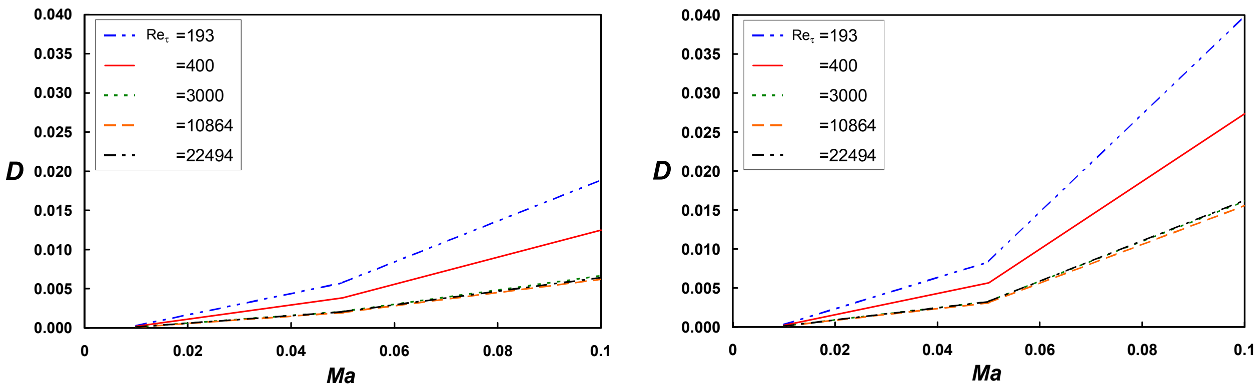


Fig. 7 – Deviation of the transit time from the case of uniform velocity profile, using (10), $193 < \mathrm{Re}_\tau < 22494$. Left: Downstream; Right: Upstream.

### 3.2 Turbulent-like Velocity Profiles

Calculation of the trajectories in turbulent-like velocity profiles results in a trajectory offset from the case of uniform flow shown in Fig. 8. The ultrasonic pulse follows an irregular trajectory in the turbulent-like profile, the offset of which depends on the local velocity gradient. Thus the average of an ensemble of trajectory transit times can be used to calculate the mean flow velocity through (3).

The average of the ensemble, $\overline{t_i}$, was calculated using,

$$\overline{t_i} = \frac{\sum_{j=1}^{N} t_{i\_j}}{N} \tag{12}$$

where $N$ is the number of realizations. Since a finite number of realisations were used, the standard deviation of the ensemble, $\sigma_{t_i}$, was used to estimate the standard uncertainty, $\sigma_E$, of the error in (9),

$$\sigma_E^2 = \frac{L}{2\cos\theta}\left[\frac{\sigma_{t_1}^2}{t_1^2} + \frac{\sigma_{t_2}^2}{t_2^2}\right] k_p^2 * 100 \tag{13}$$

where

$$\sigma_{t_i}^2 = \frac{N\sum t_i^2 - \left(\sum t_i\right)^2}{N(N-1)} \tag{14}$$

and $k_p$ is Student's t-factor at 95 % confidence level.

The standard uncertainty of the error, shown in Fig. 9, increases linearly with increasing *Ma* and decreases with increasing $\mathrm{Re}_\tau$. The calculated error in the mean velocity at $\mathrm{Re}_\tau = 193$ and $22494$ is shown in Fig. 10 and Fig. 11 respectively. The time-averaged error lies within the error bars of the ensemble of the turbulent-like error, it is possible that if an infinite number of turbulent-like profiles are used the time-averaged results may be recovered.

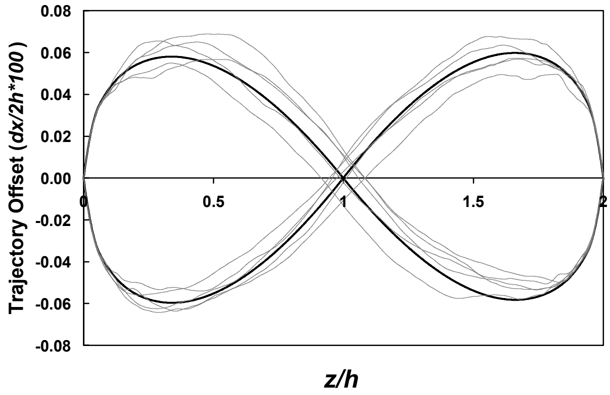


Fig. 8 - Percentage offset of trajectory from straight line in five turbulent-like velocity profiles at $\mathrm{Re}_\tau = 193$, $Ma = 0.01$.

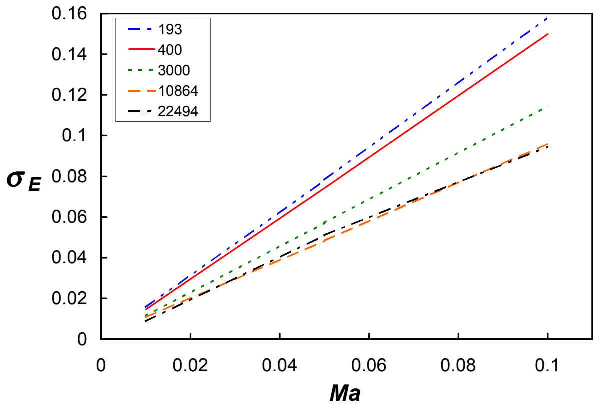


Fig. 9 - Standard uncertainty of the error. $\mathrm{Re}_\tau = 193$ to $22494$.

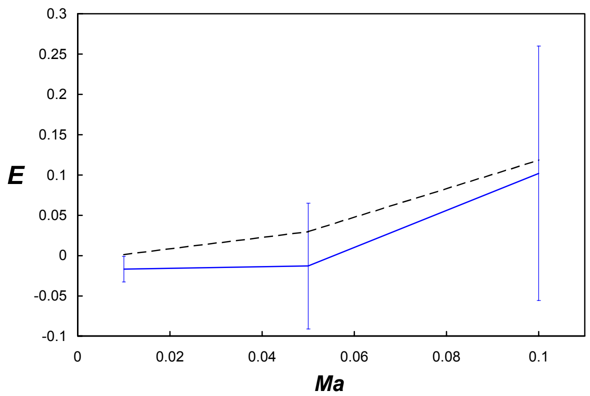


Fig. 10 - Error in the mean velocity measurement. Dashed - Time-averaged, Solid – ensemble-averaged. $\mathrm{Re}_\tau = 193$.

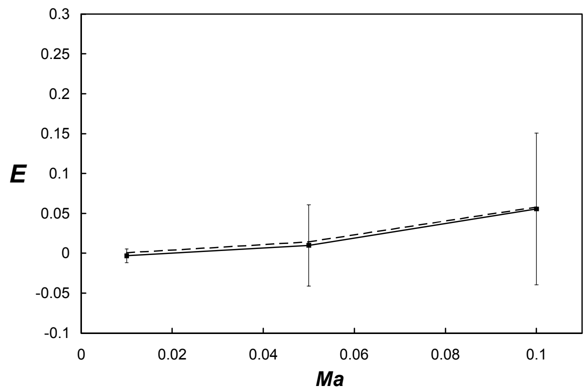


Fig. 11 - Error in the mean velocity measurement. Dashed - Time-averaged, Solid – ensemble-averaged. $\mathrm{Re}_\tau = 22494$.

## 4 CONCLUSION

The work describes the effect of turbulent-like fluctuations in fully developed flow on the trajectories of a single path, transit time ultrasonic flow meter in a plane channel. Generally, multi-path ultrasonic flow meters are used in industry, which use a weighted average of the individual path measurements to calculate velocity flow rate. The industry standard for gas flow requires 0.7 % uncertainty [5] for multi-path flow meters although tighter tolerances of 0.1 % are often reported [10].

First, the case of time-averaged velocity profiles was considered. The deviation of the calculated transit time increases with increasing *Ma* due to the enlarged change of the path trajectory from a straight line. The magnitude of the change in the down and upstream trajectories are generally not equal. This results in an associated error in the measured mean velocity which is proportional to *Ma*. For the range of *Ma* and $Re_\tau$ considered the maximum error is 0.12 %.

Second, the case of an ensemble of turbulent-like velocity profiles was considered. Similar trends were found and it appears that if an infinite number of realizations were to be used the error would tend to that of the time-averaged case. For 1000 realizations a maximum error, with 95% confidence, of 0.26 % was observed.

The general effect of consideration of turbulent-like velocity profiles is a small change in the error of the measured mean velocity. When the claimed uncertainty of the flow meter is 0.7% the error associated with flow with turbulent-like fluctuations is small. However, when the uncertainty of the flow meter is 0.1% the consideration of turbulent-like fluctuations may be required.

## 5 ACKNOWLEDGEMENTS

Financial support for this research was provided by the Process Measurements Division of the Chemical Science and Technology Laboratory of NIST, under award number 60NANB1D0044.

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