## TELE-METROLOGY AND ADVANCED ULTRASONIC FLOW METERING

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# ABSTRACT

In an effort to expand the range of the national standards, NIST is experimenting with telemetrology concepts to annex flow calibration facilities throughout the U.S. An essential part of this approach is the use of sensors capable of assessing the dynamic properties of the flow-testing environment. Specifically, we advocate the use of an advanced ultrasonic flow meter (AUFM) to provide a detailed description of the flow field entering the flow meter being calibrated. The AUFM couples multi-path ultrasonic sensing capabilities with pattern recognition software to provide likely flow fields and their probability of existence. The knowledge encoded in the pattern recognition software is derived from training exercises that use computational fluid dynamic and experimental results to teach a flow field recognizer (FFR) via learning algorithms. The performance of the FFR is proportional to the number and variety of example flow fields used during training, and to the flow detection features used for classification. Although the purpose of this research is to reduce the uncertainty of flow calibration devices, the use of AUFM could prove beneficial in field applications where installation effects can lead to gross errors when ultrasonic signals are evaluated using conventional integration schemes.

# **INTRODUCTION**

The ever-expanding needs for flow traceability have forced the National Institute of Standards and Technology (NIST) to look for novel ways to increase the operational range of the Nation's flow standards while reducing their uncertainty. In order to attain these goals, NIST is experimenting with tele-presence<sup>1,2</sup> concepts that will enable calibration facilities throughout the U.S. to be occasionally operated as national flow standards. For a flow calibration facility to be classified as a national standard it must undergo rigorous periodic evaluations. This also requires NIST to confirm that all testing strictly adheres to the procedures stated in the calibration report. Any secondary metrology

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laboratory whose calibration facilities are to serve as a national standard would be required to satisfy these elements.

The NIST use of a tele-metrology site will be based on a number of considerations and is likely to necessitate that the facility undergoes some degree of modification. The foremost selection criteria would be that the facility must offer a measurement capability that is outside of NIST's existing capacity since there would be reduced value in having more than one standard covering the same need. NIST metrologists would then evaluate the candidate facility to determine its suitability as a national standard. At a minimum, this would involve examination of the facility's principle of operation, sensitivity, reproducibility,<sup>3</sup> and schedule availability. Additionally, implementation of the telepresence concept would require the availability of adequate information technology infrastructure at the site. Once a candidate facility has been chosen, instrumentation would be added or upgraded as necessary. For most flow calibration facilities, this would involve the addition of NIST pressure, temperature, and time instrumentation. All instrumentation will be periodically calibrated according to a schedule similar to what is done for NIST's in-house instrumentation. Finally, tele-presence capabilities will be added to the remote site so that NIST personnel can maintain *metrological control* over the facility while it is being used as a national standard. Although tele-presence capability is not essential, it will provide substantial time and cost savings while allowing customers to participate in the calibration of their flow sensors.

Implementation of the aforementioned steps at an existing calibration facility would lead to its Internet accessibility and possible use as a national standard. However, these steps have limited potential to reduce the uncertainty of any calibration facility under consideration. Improvements in the level of uncertainty may require a reevaluation of the way in which flow calibrations are currently conducted. Specifically, the unique installation effects of each facility should also be addressed.

Consider the conventional flow calibration facility shown in Figure 1. Generally speaking, all such facilities will make use of some type of *fluid displacement system* (*e.g.*, compressor, blower, reservoir, pump, etc.) to establish the flow and some means of controlling the flow rate. The device to be calibrated referred to as the *meter under test* (MUT), provides an indication of flow rate based on the interaction of its sensors with the flow field passing through it. The mass that has passed through the MUT is then directed to a *flow determination system*, which makes a bulk assessment of the flow rate used for calibration. If all mass that has passed through the MUT is accounted for by the flow determination system, the ratio in output between the two readings will constitute the calibration coefficient of the MUT at the established flow rate. For optimal performance, flow calibration facilities rely on the assumption that the flow field interacting with the MUT conforms to the design specifications of its sensing elements. However, research has shown that small deviations from this assumption can lead to significant errors in the MUT calibration.<sup>4</sup> Furthermore, the problem is exacerbated because it is difficult to detect the degree to which these deviations may be present.

These flow field induced calibration errors can be quantified by determining the combined fluid delivery characteristics of all the calibration facility elements upstream of the MUT. NIST has experimentally conducted this type of assessment in its large flow rate calibration facilities by measuring the dynamic characteristics of our testing environments (*i.e.*, velocity distributions in our test sections have been measured using laser Doppler velocimetry). However, these evaluations are costly and only provide data that relates to the condition of the calibration facility at the time of the evaluation. This paper advocates a way to determine the flow characteristics of the flow-testing environment by introducing a *flow assessment unit* upstream of the MUT (see Figure 2). This permanent addition to the calibration facility will serve two purposes: (1) it would provide an *in situ* determination of the quality of the flow field present during testing, and (2) it would provide an

additional independent means of measuring the flow rate being delivered to the MUT. This approach will allow calibration facilities to better determine the flow rate passing through the MUT and characterize the suitability of the established flow field.

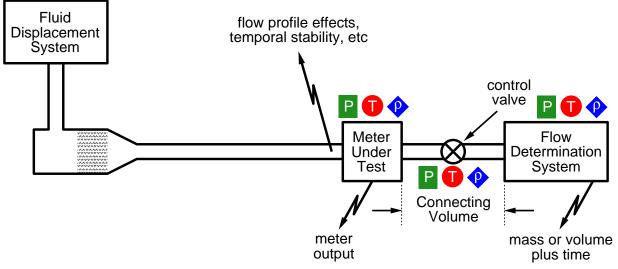
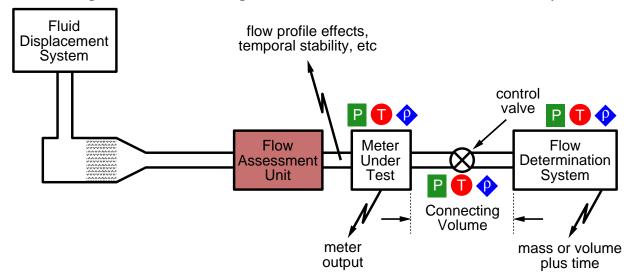


Figure 1. Schematic diagram of a conventional flow calibration facility.



# Figure 2. Schematic diagram of a flow calibration facility incorporating the flow assessment unit concept.

The remainder of this paper will chronicle the ongoing work that has been done at NIST and elsewhere towards the development of flow assessment units. Specifically, the next section introduces the reasoning behind the selection of multi-path ultrasonic sensing technology for this task. This is followed by an introduction to the physical concepts behind ultrasonic sensing phenomena and the mathematical modeling utilized to predict the performance of prospective flow assessment units. In closing, the artificial intelligence ideas being applied to enhance the diagnostic capabilities of the planned ultrasonic flow assessment unit are described.

#### ULTRASONIC FLOW ASSESSMENT UNITS

The design of the transducer arrangements and signal processing algorithms for time of travel ultrasonic flow meters has generally been based on the assumption of 1-D axisymmetric velocity profiles. These hypothetical profiles seldom occur in field applications, where meter installation effects often lead to skewness and circulation velocity components that can produce significant errors.<sup>4</sup> Ultrasonic flow meter manufacturers have made use of various acoustic transducer arrangements and signal processing algorithms to improve the performance of their products in field applications. The results of these efforts have led to the increased use of multi-path ultrasonic flow meters, which have been shown to be less sensitive to installation effects.<sup>5</sup> However, these flow meters rarely provide the detailed flow field descriptions required of a flow assessment unit. To attain that level of detail more complex sensing arrangements need to be considered.

An alternative method, which is not subject to the limitation of requiring an assumed velocity profile, is acoustic tomography;<sup>6-8</sup> a sensing methodology where the response of a large array of ultrasonic sensors is used to reconstruct the 3-D profile present in the flow meter. In theory, acoustic tomography provides a potentially attractive procedure for sensing a complex time-dependent flow field given the non-intrusive nature of its detection and the completeness of its results. However, the limited resolution of ultrasonic travel time and ray space, can lead to some practical problems. For example, to avoid the interference among acoustic rays, the travel time between all sensor pairs cannot be determined at the same time. This requires a compromise between the minimum number of acoustic paths needed for adequate spatial reconstruction and the maximum time allowed for the deconvolution of their signals within an appropriate physical time scale. Other practical issues include the angular coverage, and incorporation of interface-boundary scattering effects. The couplings among the propagation physics, environmental uncertainties, and signal structure can also affect the reliability of the reconstructed data. At present, these practical limitations can lead to non-unique results and poor time resolution.

Here a different approach is proposed for the deconvolution of the signals provided by the proposed flow assessment unit – one based on the use of pattern recognition algorithms. This approach shall be implemented by installing a customized multi-path ultrasonic flow meter upstream of the MUT. This advanced ultrasonic flow meter (AUFM) will be equipped with a large number of sensing paths (between 8 and 12 ultrasonic pairs) to acquire a more complete description of the flow field. The output of the AUFM will consist of an array of path-averaged cordal velocities (i.e., indicated velocities),  $V_1$ , that will be fed to a *flow field recognizer*. The flow field recognizer will make use of machine learning concepts to determine the association between the  $V_I$ 's and the flow features of interest to the user. Some users might be interested in determining the flow field profile, while others might be interested in a low uncertainty determination of the volumetric flow rate. Depending on the desired AUFM response, the user will "train" the flow field recognizer to establish the correct association by providing examples of  $V_1$ 's with their corresponding responses a priori. The uncertainty in the recognition process will depend on: (1) the ability of the AUFM sensor array to measure the quantity of interest (*i.e.*, features selected for recognition), (2) the uniqueness of the inputoutput transformation (i.e., separability of the classification process), and (3) the number and relevance of the training exercises provided to the flow field recognizer.

The advantages of pattern recognition methods over tomographic deconvolution mathematics are numerous. First, the time required for flow field classification is less than that required for

deconvolution of tomographic data because no matrix inversion is required. Second, pattern recognition methods are designed to establish robust associations between sensor data and response classes which makes them less sensitive to noise that could otherwise hinder tomographic deconvolution (*e.g.*, inversion of a noisy tomographic matrix may lead to a non-realizable solution). Third, the use of pattern recognition methods requires no physical knowledge of the process being classified – only cause-effect relations need to be known.

In what follows, we will first discuss the mathematical modeling of ultrasonic flow meter responses and the creation of training examples for flow field recognizers. These will be followed by examples showing the application of these ideas to the determination of the mean volumetric flow rate and a quadrant based description of the flow field.

## ULTRASONIC FLOW METER MODELS

Travel-time ultrasonic flow meters measure the difference between upstream and downstream pulse propagation times for each ultrasonic sensor pair. This propagation time difference is related to the speed of sound in the media, c, and the fluid velocity between the sensors, V, according to the following equations,

$$\int_{1}^{2} dt = t_{12} = \int_{1}^{2} \frac{ds}{c + \mathbf{V} \cdot \mathbf{e}} \approx \frac{s}{c + V_{s}} \quad downstream$$

$$\int_{2}^{1} dt = t_{21} = \int_{1}^{2} \frac{ds}{c - \mathbf{V} \cdot \mathbf{e}} \approx \frac{s}{c - V_{s}} \quad upstream$$
(1)

In equation (1), t is time, s is the distance along the ultrasonic path between sensors 1 and 2, e is the unit vector tangent to s, and  $V_s$  is the average fluid velocity vector along the path 1-2. For the geometrical arrangement shown in Figure 3, equation (1) reduces to

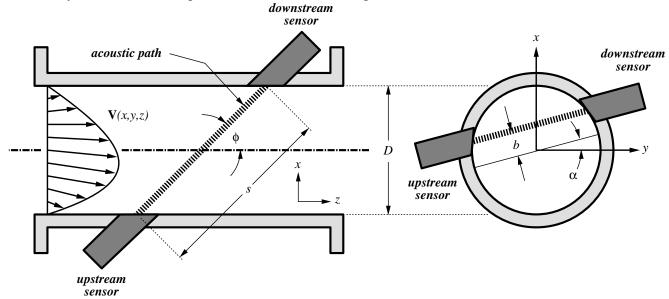
$$V_{I} = \frac{V_{s}}{\cos(\phi)} = \frac{s}{2\cos(\phi)} \left(\frac{1}{t_{12}} - \frac{1}{t_{21}}\right)$$

$$c = \frac{s}{2} \left(\frac{1}{t_{12}} + \frac{1}{t_{21}}\right)$$
(2)

where  $V_1$  is the sensor pair indicated velocity,  $t_{12}$  and  $t_{21}$  are the downstream and upstream propagation travel times, respectively, and  $\phi$  is the sensor pair axial angle.

Recently,<sup>4</sup> the response of ultrasonic meters has been mathematically predicted with the help of ray-tracing theory.<sup>9</sup> Given the geometrical arrangement of ultrasonic paths in a meter, the response of the sensors can be modeled provided that the velocity profile entering the sensor is known. Using Figure 3 as an example, the configuration parameters for dual-sensor ultrasonic flow meters included in the model are: the diameter of the pipe D, the ultrasonic path axial offset distance b, its azimuthal angle  $\alpha$ , and its axial angle  $\phi$ . In addition, since the fluid velocity V depends on the piping upstream

and downstream from the meter, the correction factors will be functions of both the meter installation and the dynamic conditions present at the time of testing.





Equation (3) represents the ray tracing model used to assess the response of each one of the ultrasonic sensor pairs ( $\nabla$  denotes vector gradient).

$$\frac{ds}{dt} = c\mathbf{e} + \mathbf{V}$$

$$\frac{d}{dt} \left( \frac{\mathbf{e}}{c + \mathbf{V} \cdot \mathbf{e}} \right) = -\frac{\nabla c + e_x \nabla u + e_y \nabla v + e_z \nabla w}{c + \mathbf{V} \cdot \mathbf{e}}$$
(3)

This ray theory approximation assumes that the vector gradients are slowly varying functions of space, but avoids the normally assumed straight acoustic path between the ultrasonic sensors. From equation (3), it can be seen that both gradients in the sound speed and in the velocity affect the propagation direction of the acoustic ray  $\mathbf{e}$ . This is especially significant in gas metering applications where the speed of the fluid can approach the magnitude of the sound speed, leading to considerable acoustic path deflections for Mach numbers larger than 0.1.<sup>4</sup> Therefore, equation (3) shall be utilized to avoid errors that would result from use of the simpler straight acoustic path model.

#### **SELECTED PIPE FLOW PROFILES**

Thus far we have shown that equation (3) can be used to estimate the time of travel resulting from the interaction between an ultrasonic pulse and the flow field under consideration. Using this result, equation (2) can be employed to determine the indicated velocities that would be provided by a prospective AUFM. To complete the mathematical evaluation of the AUFM, a representation of the three-dimensional velocity field of interest, including its gradients, is required. Such a representation

can be obtained by various means: experimental evaluation, computational fluid dynamics, or superposition of analytical profiles.

Experimental evaluation, which is by far the most costly option, requires the detailed profiling of the velocity field using instrumentation that yields limited flow disruption. Candidate technologies would be: Doppler global velocimetry,<sup>10</sup> particle image velocimetry, or laser Doppler anemometry. Using any of these techniques, the completeness of the results would be limited by the spatial resolution of the interrogation matrix and the temporal stability of the flow during the measurements. Nonetheless, the data provided by these methods would render the most accurate representation of the flow dynamics for the foreseeable future.

Computational fluid dynamics (CFD) is rapidly becoming an economical alternative to experimental methods because it is capable of providing an unrivaled level of completeness in its results. With ever cheaper and faster computing power, and with the proliferation of a variety of ready-to-use commercial products, simulations of complex flow meter installations are now common. However, deficiencies in the modeling of turbulence physics still plague industrial-scale CFD results. This necessitates experimental validation to ascertain the quality of the numerical results even in the simplest piping configurations.

An economical alternative for velocity profile construction can be obtained by superimposing analytical functions that resemble the velocity distributions encountered in generic flow meter installations. Such an approach has been successfully used to ascertain the uncertainties of point-velocity integration methods for determination of volumetric flow rate<sup>11</sup> and to estimate the sensitivity of ultrasonic flow meter prototypes.<sup>4</sup> Following the approach proposed by Salami in reference 11, one can formulate a large number of artificial flow profiles by adding analytical functions that resemble common flow disturbances. For example, a family of flow profiles, based on a power-law representation of the fully developed pipe flow, can be created as follows,

$$U(r,\theta,z) = U_1 + \sum_{i=2}^{n} U_i = (1-r)^{1/k} + \sum_{i=0}^{n} \text{flow disturbance}_i$$
(4)

In equation (4), U is the streamwise velocity component,  $r, \theta, z$  are the radial, azimuthal, and streamwise coordinates, respectively. The first right-hand-side (RHS) term in the equation represents the fully developed flow profile for the exponent k, while the second RHS term represents a summation of the superimposed flow disturbances that will yield the flow fields of interest. Similar functions can be created for the radial and azimuthal components of the velocity vector field which, for example, could be used to introduce streamwise vorticity similar to that seen downstream of a single elbow.<sup>4</sup>

Equation (5) is an example of an analytical streamwise flow field with a single disturbance superimposed over the power-law profile (for k = 9 or Re  $\approx 725,000$ ).

$$U(r,\theta) = U_1 + U_2 = (1-r)^{1/9} + \frac{1}{5}r(1-r)^{1/9}\sin\theta$$
 (5)

As shown in Figure 4, this combination of a diametral eddy and a power-law distribution leads to a one sided skewness in the velocity profile which could be interpreted as the flow field produced by unequal side-by-side sources feeding a single pipe.

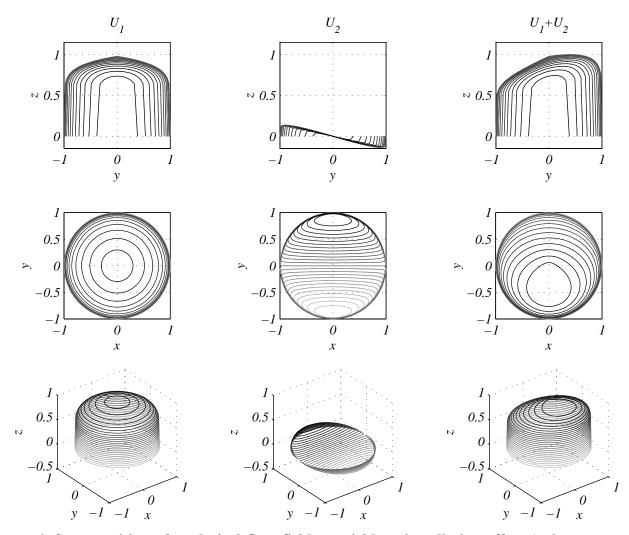


Figure 4. Superposition of analytical flow fields to yield an installation effect (only streamwise velocity components shown for clarity). *Top row* – side view, *Center row* – end view, *Bottom row* – three-dimensional profiles.

## ANALYSIS OF ULTRASONIC SIGNALS

There are many ways by which the indicated velocities,  $V_I$ 's, provided by a multi-path ultrasonic flow meter can be transformed into useful results. In the case of volumetric flow rate determinations, the most common methodology involves the use of numerical integration techniques, which typically require the assumption of a velocity distribution inside the meter. Using these techniques, the volumetric flow rate can be expressed as,

$$\dot{V} = A \overline{V} = \frac{\pi D^2}{4} \sum_{i=1}^p w_i V_{I_i}$$
(6)

where  $\dot{V}$  is the volumetric flow rate, p is the total number of ultrasonic paths, the  $w_i$ 's are the integration method weight factors associated with each ultrasonic path, and D is the internal diameter of the meter. An adverse result of the use of this methodology is that deviations from the assumed profile may lead to errors in the estimation of the volumetric flow rate due to the inadequacy of the postulated  $w_i$ 's. One way to overcome this problem is by applying flow profile correction factors,  $C_I$ , to each acoustic path. Making such correction, equation (6) becomes,

$$\dot{V} = A \,\overline{V} = \frac{\pi \, D^2}{4} \sum_{i=1}^p C_{I_i} \, w_i \, V_{I_i} \tag{7}$$

Unfortunately, the selection of the  $C_I$ 's is unique for each velocity profile and this leads to a problem of immense proportions – especially if treated experimentally.

Here the use of pattern recognition algorithms is suggested as an alternative to the conventional analysis tools.<sup>12</sup> For the case of volumetric flow rate determinations, the array of  $V_I$ 's (or *feature-vector*) will be provided as input to a pattern recognition program (or *flow field recognizer*, FFR) which will select a *response category*,  $C_I$ 's, based on a transformation function encoded in its *association matrix*, T. In mathematical terms this process can be described as,

$$[C_I] = T[V_I] \tag{8}$$

where  $[V_I]$  and  $[C_I]$  are the transpose of the vectors  $V_I = (V_1, V_2, ..., V_p)$ ,  $C_I = (C_1, C_2, ..., C_q)$ , and *T* is a matrix of dimensions  $q \ge p$ .

As with most other pattern classification algorithms, the FFR association matrix is established through a training process. The training process consists of inputting a series of feature-vectors along with each vectors' corresponding response category. During training, the FFR algorithm establishes a memory that, when the FFR program is put into operation, will be used to classify previously unseen feature-vectors into the proper response category.

There are three main steps in the development of a FFR using the pattern classification program: (1) selecting the training and testing data (*i.e.*, sets of feature-vectors vs. response category), (2) training and testing the FFR, and (3) exporting the FFR's memory. The training and testing data can be obtained form experimental evaluation of the AUFM or from its mathematical modeling as previously described. Prior to training and testing the data read by the FFR program will be organized in lines containing the feature-vector (*i.e.*, the  $V_I$ 's) followed by the vector's corresponding response category (*e.g.*, fully developed flow, elbow flow, reducer flow, etc.). After loading the data, the FFR program automatically divides the input data into the training and testing sets.

The recognizer's ability to establish an effective vector-to-category transformation function is dependent on a number of crucial training parameters. Given the optimum set of parameters for a given set of training and testing data, the recognizer will classify the testing patterns to the minimum level of uncertainty. Normally, this would not be the case for a generic set of parameters; consequently, the objective of performing training and testing is to find a set of parameters that yield the highest level of recognition performance. Using the FFR program, this search for the highest performance level can be carried-out manually or through an automatic parameter optimization. In most cases, it will be more expedient to use the FFR program's automatic parameter optimization capabilities. Exporting the recognizer's memory is the third and final step in the classifier development process. The recognizer's memory data is outputted in a format that can be read-in by the AUFM diagnostic program during operation.

The key to the success of this process resides in the ability of the selected sensor arrangement to provide a feature-vector with enough information to perform the classification. For example, suppose that the AUFM had only one ultrasonic pair. Then, it would be difficult for the FFR to yield the correct selection of the  $C_I$ 's, even in symmetric flows, given that the sensing arrangement cannot provide enough information to discern differences between the various profiles. However, if the AUFM had four cordal paths in relevant locations, the FFR may be capable of discerning the differences between the profiles and provide the correct  $C_I$ 's for each case of interest.

If the desired output from the AUFM is a quadrant based description of the flow field, the ultrasonic sensor arrangement would need to be more complex (*e.g.*, 8 to 12 sensor pairs may be required). For such an application, the feature-vector will be enlarged in proportion to the increase in sensor pairs and the response category would need to become more complex to accommodate the detailed description required. In all likelihood, the response category would not involve the selection of  $C_I$ 's given that no volumetric flow determination would be made. In its place there could be integral assessments of localized quantities like the quadrant volumetric flow rate (using a modified version of equation 7), or the circulation in that specific zone (in the case that vortical structures where of interest).

#### SUMMARY

The World's expanding needs for flow traceability have forced metrologists to look for novel ways to increase the operational range of flow standards while reducing their uncertainty. The telepresence concept has been suggested as a means by which to provide worldwide access to existing flow facilities. The technologies needed for the accurate remote calibration of flow meters has been briefly outlined and their use to incorporate secondary metrology laboratories into the family of national standards has been discussed. Specifically, the use of flow assessment units as a means to determine the quality of the testing environment has been suggested. A design for an advanced ultrasonic flow meter has been discussed in the context of its use as a flow assessment unit. The main feature of the AUFM is the use of a flow field recognizer to make detailed determinations of the flow field properties using pattern recognition algorithms. A training process for the flow field recognizer has been discussed and its application to the determination of volumetric flow rate and quadrant flow descriptions has been provided. A significance benefit of this approach is that the end user will not need to know the physics of the ultrasonic detection or flow classification processes given that the pattern recognition algorithm operates only on cause-effect relations.

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## DISCLAIMER

Identification of specific commercial products is made for technical completeness but does not constitute an endorsement, nor does it indicate that the products are preferred for the application.

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