NIST Measurement Services:

NIST Calibration Services for Gas Flow Meters

Piston Prover and Bell Prover Gas Flow Facilities

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John D. Wright and George E. Mattingly

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ABSTRACT

This document provides a description of the small and medium range gas flow calibration facilities at the National Institute of Standards and Technology (NIST) Fluid Flow Group, as reported in NIST Special Publication 250 [1] for Test Nos. 18010C-18040C and 18050S, Flow Rate Measurements. The Fluid Flow Group can perform gas meter calibrations and special tests at flows between 3.7×10^{-5} m^3/min and 1.4 m^3/min (0.001 standard cubic feet per minute (scfm) and 51 scfm, reference temperature and pressure are 293.15 K and 101325 Pa) using positive displacement techniques. The flow rate of gas passing through the meter under test is determined from pressure, temperature, volume, and transit time measurements of a displaced volume of gas. Two types of displacement devices are used: mercury-sealed piston provers and bell gasometers, or bell provers. This report describes the techniques used for calibrating meters and presents the uncertainty analysis associated with such calibrations.

Key Words: air flow; bell prover; calibration; conservation of mass; density; flow meter; flow rate; piston prover; standards; uncertainty; volume.

1. INTRODUCTION

In fluid flow rate measurements, no "standard" devices or artifacts are available comparable to those for length or mass measurements. Instead, reference flow rates must be derived from measurements that can be related to more fundamental standards, such as length, mass, and time standards. For primary flow rate calibrations, this is accomplished by collecting, under conditions of steady flow and fluid properties, a measured mass or volume of the flowing fluid over a measured time interval, with all measured quantities (i.e., temperature and pressure) referenced to established standards. A gas calibration flow facility consists of a fluid source (e.g., air compressor or another compressed gas source), sufficient conduit length for insertion of the test flowmeter under accepted and controllable conditions, and a means for diverting and timing the collection of a quantity of fluid, all appropriately instrumented.

The Fluid Flow Group of the Process Measurements Division (part of the Chemical Science and Technology Laboratory) at the National Institute of Standards and Technology provides air flow rate calibration services over a range of 3.7×10^{-5} m^3/min to 78 m^3/min (0.001 scfm to 2740 scfm) [2]. Reference conditions of 20 °C and 101325 Pa are used throughout this document for standard volumetric flows. Three different measurement systems are used to cover this range, as shown in Table 1, with some overlap of capability between them. The lowest flows, from 3.7×10^{-5} m^3/min to 2.2×10^{-2} m^3/min (0.001 scfm to 0.79 scfm), are measured with piston provers, using a "dynamic" procedure [3-6]. A similar dynamic procedure is used to cover the intermediate range, from 2.2×10^{-2} m^3/min to 1.1 m^3/min (0.79 scfm to 38.2 scfm), using bell provers. The highest flow rates are calibrated by a "static" method [3-6], employing a timed diversion into a large constant volume tank that is initially at near vacuum pressure (PVTt). This report details the performance capability of the positive displacement provers at NIST: the piston and bell provers, used for low and intermediate flow rates. The system used for calibration at high flow rates is described in a separate report.

Three piston provers are used for flowmeter calibrations at NIST. The small, medium, and large piston provers have effective collection volumes of approximately 130 cm^3, 700 cm^3, and 7400 cm^3 (7.9 in^3, 43 in^3, and 450 in^3) respectively. The corresponding inside diameters of the prover tubes are 1.90 cm, 4.44 cm, and 14.38 cm (0.75 in, 1.75 in, and 5.66 in). The three provers are mounted together in

a console and connected by a manifold to a single air source and inflow line. This arrangement allows more than one prover to be used for calibration of a flowmeter if its range so requires, and facilitates intercomparison between piston provers. Three bell provers are available at NIST with usable displacement volumes of 0.056 m^3, 0.114 m^3, and 0.361 m^3 (2 ft^3, 4 ft^3, and 12.7 ft^3).

Table 1. Air flow rate calibration facilities and flow range capabilities in the NIST Fluid Flow Group. (Collection times of 15 s and 210 s were used for the pistons and bells, 1800 s to 100 kPa and 40 s to 200 kPa used for the PVTt system. Reference temperature and pressure are 293.15 K, 101.325 kPa.)

Facility	**Volume (m^3)**	**Min. Flow (m^3/min)**	**Max. Flow (m^3/min)**	**Volume (ft^3)**	**Min. Flow (ft^3/min)**	**Max. Flow (ft^3/min)**
Small Piston	1.304×10^{-4}	3.72×10^{-5}	5.22×10^{-4}	4.604×10^{-3}	1.31×10^{-3}	1.84×10^{-2}
Med. Piston	7.088×10^{-4}	2.03×10^{-4}	2.84×10^{-3}	2.503×10^{-2}	7.15×10^{-3}	0.100
Large Piston	7.417×10^{-3}	2.12×10^{-3}	2.97×10^{-2}	0.2619	7.48×10^{-2}	1.05
Small Bell	5.644×10^{-2}	1.61×10^{-2}	0.226	1.993	0.570	7.97
Med. Bell	0.1139	3.25×10^{-2}	0.455	4.021	1.15	16.1
Large Bell	0.3605	0.103	1.44	12.73	3.64	50.9
PVTt	25.867	0.862	77.6	913.5	30.5	2741

This report describes procedures for the use of the piston and bell provers and presents detailed uncertainty analyses for these two calibration systems. Note that this report documents the uncertainty of the flow measurements provided by the piston and bell provers only. The uncertainty of the calibration coefficients for a meter under test has components beyond those discussed herein due to the instrumentation associated with the meter. An example of a meter uncertainty analysis can be found in the sample report in Appendix A.

2. THEORETICAL BACKGROUND

A typical system for measuring bulk flow rate is composed of inlet piping to the meter under test, the meter under test, piping connecting the meter under test to a collection volume (the approach volume), and a diverter valve which is used to direct flow into the collection volume. A long straight run of piping upstream and downstream are needed to provide the appropriate velocity profile through the meter under test to prevent errors due to installation effects. The calibration of meters using such a system is based on the equation of conservation of mass:

$$0 = \frac{\partial}{\partial t}\int_V \rho \cdot \mathrm{d}V + \int_A \rho \cdot \vec{v} \cdot \mathrm{d}\vec{A} \qquad (1)$$

where r is the fluid density, $\partial / \partial t$ is the partial derivative with respect to time, and V is the stationary control volume for the mass balance. The quantity $\vec{v}$ is the velocity of the fluid, and $d\vec{A}$ is the unit vectorial surface area element with direction outward and normal to the control surface which surrounds the control volume. As applied to the piston prover or bell prover, a convenient form of the continuity equation gives the mass flow rate, $\dot{m}$, into the system as:

$$\dot{m} = \frac{r_c \cdot V_c}{\Delta t} + \frac{\Delta r_a \cdot V_a}{\Delta t} + \dot{m}_\ell \quad . \qquad (2)$$

Here V_c is the collection volume swept by the piston or bell displacement during the time interval, Δt. The quantity V_a is the remaining volume in the system; it includes the volume of the flowmeter being tested, the approach piping connecting the meter under test to the calibrator, the tare volume in the prover, and tubing for pressure transducer connections. The mean density of the gas in the collection volume, r_c, is calculated from pressure and temperature measurements made during the run, and Δr_a is the change in mean density of the gas in the approach piping between the start and end of the run.

The second term on the right hand side of (2) accounts for "storage effects" in the connecting volume, V_a: if the density of the gas in V_a increases above some initial value (due to decreasing temperature or increasing pressure), then gas is effectively "stored" in the connecting piping and the piston (or bell) velocity is reduced. For example, if the temperature profile within the connecting piping had not reached steady state, then the density of the gas in the connecting pipe could be increasing, leading to an increasing total mass of gas in the connecting piping, and further meaning that less gas is reaching the collection volume than is passing through the meter under test at that moment. It is important to recognize that the gas does not have to attain the same temperature *throughout* the prover and piping to make the storage effects term zero, rather the temperature distribution within the system must have reached steady state so that changes in density with respect to time are zero throughout the system. Steady state can be attained by allowing the temperatures to stabilize at a given flow condition before final flow collections are made. These effects are made smaller by working with gas temperatures that are close to room temperature. Further, these effects can be quantified by making repeated flow collections and observing trends in the temperature and flow rate measurements over the time during which steady state is reached.

There is no term in (2) to account for changes in the volume of the connecting piping or tare volume of the calibrator during a collection time as these effects are negligible (relative volume changes are $\approx 3\alpha\Delta T \approx 3$ x 10^{-7} for our case since $\alpha \approx 10^{-6}$ and $\Delta T \leq 0.1$ K). The term $\dot{m}_\ell$ is included to represent leakage flows into or out of the system. Leaks out of the system are kept small since leakage checks are performed and leaks minimized before calibrations are begun. The only significant source of mass flow into the system, (which did not flow through the meter under test) is from evaporation of the sealing liquid (mercury for the pistons, oil for the bells) during a gas collection.

3. POSITIVE DISPLACEMENT PROVERS

3.1. Measurement Principle

The use of the piston prover is based on the principle of measuring the time interval required to collect a known volume of gas at measured temperature and pressure. It is designed to operate at or near ambient pressure and temperature. The device, shown in figure 1, consists of a precision bore glass tube in which is placed a plastic piston slightly smaller in diameter. A horizontal groove retains mercury which forms a low friction seal between the piston and the tube. The system is termed a dynamic one because the fluid collection is initiated and terminated by the moving piston as it passes through a light beam used for timing purposes. As the piston rises by virtue of the small excess pressure (0.5 kPa) of the gas introduced at the bottom of the tube, it successively starts and stops a timer by blocking a pair of light beams, each beam passing through machined slits at the extremes of the measuring volume. Each light beam is produced by a 6 V miniature lamp. The light from these lamps passes through a collimating lens, and the light sensor is a silicon photo diode.

The bell prover system is based on the same principles described above. The bell prover (fig. 2) consists of a cylindrical tank open at the top and a central "dry well", which together form an annulus that is nearly filled with sealing oil. Into this annulus is placed an inverted cylindrical tank, i.e., the bell, open at the bottom and having a dome-shaped top. Its weight is nearly balanced by counterweights so that it can be raised or lowered by a small differential pressure (0.3 kPa) to collect and measure a volume of gas. A smaller counterweight is mounted on a cam so that it provides a correction for buoyancy effects as the bell immersion in the sealing liquid changes. Imperfection of the cam and counterweight system leads to variation in the pressure under the bell during the collection interval. This pressure variation contributes to the flow measurement uncertainty and is treated in a later section. Rollers and guide rods provide lateral stability in the bell position as it moves upwards.

3.2. Calibration Procedure

3.2.1. Piston Prover

Before performing a flowmeter calibration, the meter and associated instrumentation (for measuring temperatures and pressures) are installed upstream from the piston prover. To obtain a flow measurement, flow is established through the approach piping and the flowmeter and a bypass valve (fig. 1) is adjusted so that the piston is "floated" off its bottom support. Filtered dry air is available from an air compressor and other gases from pressurized cylinders may be used (nitrogen, carbon dioxide, and argon). The bypass valve is closed so that all of the flow is diverted into the prover causing the piston to rise, attain a constant speed, and pass into the collection volume. The piston is cycled up and down in the cylinder several times (by closing and opening the bypass valve) in order to attain better thermal equilibrium between the flowing gas and the prover system prior to a formal flow measurement.

After cycling the piston several times, the bypass valve is closed and the calibration process begins, during which data for the meter under test are recorded, and the piston passage between the start and stop photo-detectors is electronically timed. The gas temperature is measured with a temperature sensor inserted at the entrance of the collection volume, and the gas pressure is measured with an absolute pressure transducer connected to a tap at the location shown in figure 1. The temperature and pressure of the gas entering the piston cylinder are measured during the entire gas collection period and averaged. Strictly, the temperature and pressure upon *completion* of the collection interval is the quantity needed, but for the piston prover, the changes in temperature and pressure over time are small, and the averaging process serves to filter instrument noise. A normal calibration test consists of five piston strokes (runs) for each of five flow rates performed on each of two different days.

Details of the time, pressure and temperature measurements, along with the displaced volume determination, and their contributions to the total uncertainty of the system, are described in a later section.

3.2.2. Bell Prover

To make a calibration measurement using the bell prover**,** the meter and its instrumentation are installed and the piping system is tested for leaks. Flow is established through the flowmeter and the bypass valve (fig. 2) is adjusted so that the bell is "floated" off its bottom support. Then the valve is closed so that all the flow is diverted into the bell. Like the piston prover, the bell prover is exercised in this manner in order to attain a steady temperature condition prior to the commencement of the calibration process. The bypass valve is then adjusted to float the bell as before. The valve is closed and following a brief period of acceleration and pressure transients, the bell attains a constant velocity and steady state conditions, and the bell passes through the collection interval. The collection interval for the bell is measured using photodiode optical switches which designate the start and stop positions.

The temperature and pressure of the collected gas upon completion of the collection interval are needed. As for the piston prover, the pressure is measured and averaged over the entire period of the collection interval since the pressure variations due to the prover travel are small and the averaging serves to filter measurement noise. However, the *last* temperature measurement gathered before the completion of the collection is used. This is because at the highest flows, the temperature variations in the course of a collection are as large as 0.15 K (0.05 %). The collected gas temperature is measured with a sensor inserted into the upper part of the bell, the gas pressure under the bell is measured with an absolute pressure transducer connected to a tap in the inflow pipe (see fig. 2), and the collection is timed with an electronic timer. Details of these measurements and their contribution to the total uncertainty of the system are described in a later section.

4. SOURCES OF UNCERTAINTY

The sources of uncertainty arising in the application of (2) are discussed generally in this section, and the uncertainties are further developed and quantified in section 5.

The first term on the right hand side of (2) is the largest term and represents the basic, uncorrected collection rate in the cylindrical volume of either prover. It is subject to potential errors in the determination of the collection volume, V_c , the timing interval, Δt , and the density, r.

The uncertainty of the gas density determination is due to uncertainties in the measurement of the pressure and mean temperature of the collected gas, as well as the goodness of fit of the best fit function used to calculate the density, and the quality of the experimental data used to determine the function. The uncertainties in temperature and pressure measurements are related to calibration quality, sampling errors, and sensor drift over time.

For the piston prover, the uncertainty of the collection volume is due to uncertainties in measuring the mean diameter of the cylinder, in measuring the separation between the start and stop location (the collection length), and the effects of thermal expansion due to room temperature variations. For the bell prover, the collection volume uncertainty is derived from the uncertainties in the mean outside diameter of the bell, the wall thickness of the bell, the collection length, and the effects of thermal expansion. In addition, the effects of the oil seal on the volume must be accounted for as these can also contribute to the volume uncertainty. Therefore, the change in oil level due to the bell displacement, and the adherence of oil to the walls of the bell as it rises must be considered.

The uncertainty of the timing interval measurement is due to the accuracy of the timer calibration, the uncertainties of its actuation by the start and stop switches, and any rocking of the piston or bell as it passes the switches.

The second term on the right hand side of (2) (storage effects) becomes an error source if a change in density within the connecting piping is non-zero during the collection period due to changes in the temperature and pressure in the connecting piping. It should be noted that V_a can be a substantial term relative to V_c , so that amplification of the density change effect is possible.

Leakage out of the prover at significant rates is avoided by pressurizing the closed system with the piston or bell balanced in a raised position and monitoring the constancy of its vertical position for an extended period when corrected for temperature and atmospheric pressure variations in the environment.

Uncertainty due to extra mass flow into the prover is calculated from vapor pressure data for the mercury or oil sealants.

5. COMPONENTS OF UNCERTAINTY

The uncertainty analysis of this section uses the estimation approaches and the terminology suggested by "Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results" [7]. Uncertainty components have been quantified by statistical analysis of experimental data or by informed engineering estimates to obtain relative standard uncertainties with an approximate level of confidence of 67 %. These relative standard uncertainties are then used to calculate a relative *combined uncertainty* by taking the root-sum-squares (RSS) of the components. The combined uncertainty is multiplied by a *coverage factor* of 2 to obtain a relative *expanded uncertainty*, a two standard deviation or 95 % approximate level of confidence uncertainty.

The classification of the uncertainties into *type A* and *type B* components has been done based on the examples in [7] and the designation for each uncertainty component is given in brackets { }. Whenever a combined uncertainty is composed of subcomponents of both type A and type B, the combined uncertainty is taken to be type B. The instrument types used with the provers as well as their calibration records are available in the NIST Fluid Flow Group.

5.1. Uncertainty of the Piston Provers

5.1.1. Gas Density

The gas density used to convert the measured volumetric flow to a mass flow is obtained from best fit equations for a given temperature and pressure [8,9]. The relative uncertainty of the density value has components due to uncertainty in the gas temperature measurement, the gas pressure measurement, the best fit equation, and the experimental data used for the fit. Gas density uncertainty due to uncertainty in

molecular weight, the universal gas constant, and the purity of the gases used have been examined and found negligible when compared to the components covered below.

5.1.1.1. Temperature Calibration

The temperature sensors used in the piston prover system are calibrated with a transfer standard thermometer that is periodically calibrated by the NIST Thermometry Group [1]. Accounting for the uncertainties of the transfer standard, temperature nonuniformities in the temperature cell used during calibration, calibration drift, and the residuals after a best fit of the temperature sensor calibration gives a standard uncertainty of 0.06 K (relative standard uncertainty 0.020 %) or less {B}.

5.1.1.2. Temperature Sampling

The location of the temperature sensor for each prover tube just upstream of the glass tube inlet introduces additional uncertainty. Heat transfer from the room through the glass cylinder walls can lead to temperature differences between gas at the temperature measurement location and the mean temperature of the gas in the full collection volume. It has been found experimentally that the gas temperature at the sensor location decreases once gas is directed into the collection volume by closing the bypass valve. This is because the gas entering the prover is normally cooler than room temperature due to the expansion at the pressure regulator used to control flow to the meter under test. The gas continues to warm after it passes the sensor location via heat transfer from the glass cylinder walls. When the gas is exhausted from the cylinder past the temperature sensor by opening the bypass valve, the temperature sensor shows an increasing value. Therefore in experimental measurements, a roughly sinusoidal temperature trace (0.03 K peak to peak) is observed coincident with the gas entering and leaving the prover. This temperature difference will be used as part of the temperature sampling uncertainty.

Additional sampling uncertainty results when the following scenario is considered. Temperature stratification in the room housing the piston prover leads to temperature gradients in the vertical direction

within the cylinder walls. For long collection intervals, this temperature gradient is imposed on the gas collected within the cylinder as well. If the heat transfer coefficient is sufficiently large, the gas is cooled again by the lower, cooler portion of the cylinder as it is exhausted. Therefore the magnitude of the observed sinusoidal temperature trace described above is damped and the results of this experiment are an underestimation of the uncertainty due to temperature sampling. Measurements of the temperature stratification in the room leads to additional temperature sampling relative uncertainty of 0.06 K. Based on all of the temperature sampling experiments, a reasonable assumption for a standard uncertainty in the temperature due to sampling errors is 0.09 K (0.030 %) {B}.

Additional temperature uncertainties due to frictional heating by the flowing gas, conduction through the temperature sensor sheath, Joule-Thomson effects, and other effects of the environment in which the sensor is used have been considered and found negligible. The combined standard uncertainty of the collection volume gas temperature, due to calibration and sampling errors, calculated by taking the RSS of the uncertainty components is 0.108 K or 0.037 % {B}.

5.1.1.3. Pressure Calibration

The pressure of the gas collected is measured in the piping upstream from the collection volume with an absolute pressure transducer. The transducer is periodically calibrated within the Fluid Flow Group with a piston pressure gage (traceable to the NIST Pressure and Vacuum Group). Based on the standard deviation of the residuals between the fit and the calibration data for the pressure gage, calibration results for the piston pressure gage by the Pressure and Vacuum Group, and the difference between successive periodic calibrations (drift), the relative standard uncertainty of the pressure gage over the operating range of the piston prover is 0.022 % {B}.

5.1.1.4. Pressure Sampling

Between the pressure measurement location and the entrance to the prover tube there are approximately 91 cm (36 in) of 1.3 cm (1/2 in) copper tubing and several fittings, including several elbows.

For the highest flow rate (corresponding to a 15 s collection time) the pressure loss between the measurement point and the prover tube is estimated to be 6.0×10^{-4} kPa (based on standard handbook pressure loss calculations through elbows, etc.), resulting in a 0.001 % pressure relative standard uncertainty due to the sampling location {B}.

Additional pressure uncertainties due to the effects of flow across a pressure tap and other influences of the environment in which the sensor is used are considered negligible. The standard uncertainty of the collection volume gas pressure, due to calibration and sampling errors, calculated by taking the RSS of the uncertainty components is 0.022 kPa or 0.022 % {B}.

5.1.1.5. Gas Density Function Fit

The density of the gas in the collection volume is calculated from the temperature and pressure measurements and a best fit function (based on a second order virial equation of state) to the experimental data documented in references [8] and [9]. Comparisons between the tabulated experimental data and the output of the best fit function show that the differences are bounded by 0.05 % (for air, nitrogen, carbon dioxide, and argon). Assuming a uniform probability distribution, the relative standard uncertainty of the best fit function is 0.029 % {B}.

5.1.1.6. Gas Density Experimental Data

The plots of uncertainty data in reference [8] for the gases used in the provers and for pressures near one atmosphere show that the uncertainty of the experimental data is bounded by 0.02 %. Assuming a uniform distribution, the relative standard uncertainty of the experimental density data used is 0.012 % {B}.

Gathering all of the density uncertainty components by taking the RSS results in a relative combined standard uncertainty of 0.044 % {B}.

5.1.2. Collection Volume, V_c

5.1.2.1. Cylinder Diameter

Cylinder diameters were measured at 3.8 cm (1.5 in) intervals along the 91 cm (36 in) length of the glass tubes using bore gages that were calibrated with NIST traceable prover rings. Four diameters were measured at each cross-section and averaged.

The standard uncertainties of the average diameters of the cylinders are 0.0005 cm, (0.0002 in) for the small piston cylinder, 0.0002 cm (0.00008 in) for the medium piston, and 0.0023 cm (0.0009 in) for the large cylinder. These diameter uncertainties lead to collection volume relative standard uncertainties of 0.053 %, 0.009 %, and 0.032 % for the three piston provers {B}.

5.1.2.2. Collection Length

Three pairs of aluminum length plates with two 0.0127 cm (0.005 in) slits nominally 45.7 cm (18 in) apart, were machined in the NIST Fabrication Technology Division and the distances between the slit pairs were measured by the NIST Precision Engineering Division. The standard uncertainty of the length between the two slits in the plates is 0.0005 cm (0.0002 in), and this leads to a collection volume relative standard uncertainty of 0.001 % {B}.

5.1.2.3. Thermal Expansion

The laboratory in which the piston provers are located is temperature controlled at 296 ±1.5 K. Using the 1.5 K temperature uncertainty and a thermal expansion coefficient for glass of 9×10^{-6} cm/cm K, standard uncertainties in diameter due to thermal expansion are 0.000025 cm, 0.00006 cm, and 0.00020 cm, respectively, for the small, medium, and large cylinders {B}.

For the length measurement, the temperature difference of 1.5 K will be used again. Using a thermal expansion coefficient for the aluminum plates of 25×10^{-6} cm/cm K, the corresponding standard uncertainty in length is 0.0017 cm {B}.

The effects of thermal expansion on the collection volume should be combined arithmetically rather than by RSS because an increase in temperature will cause length and diameter to both increase (correlated) and these uncertainties will never cancel each other. The effects of thermal expansion due to the 1.5 K temperature change will cause collection volume relative standard uncertainties of 0.006 % for all three piston provers {B}.

Combining the effects of the diameter, length, and thermal expansion uncertainties by RSS gives estimates of the collection volume standard uncertainty of 0.0691 cm^3 (0.053 %), 0.0780 cm^3 (0.011 %), and 2.448 cm^3 (0.033 %) for the small, medium, and large cylinders respectively {B}.

5.1.3. Collection Time, Δt

5.1.3.1. Timer Calibration

The relative of the timers used to measure Δt is 0.0001 s. In a high flow rate run of 15 s duration, this introduces a relative standard uncertainty of at most 0.001 % {B}.

5.1.3.2. Timer Actuation

Additional uncertainty is introduced by the method of actuating the timer. As the leading edge of the piston passes the "start" light slit (fig. 1), the light source is blocked from the detector and the voltage output drops from a positive value to zero. A second voltage drop occurs when the piston passes the "stop" light slit. A period timer is triggered by the start and stop voltage step changes. The uncertainty in the collection time due to timer actuation is no greater than the time required for the voltage to change between the high and low values. The voltage traces input to the timer have been collected with a storage oscilloscope, and the time necessary for the voltage step change has been measured. Based on these experiments, the uncertainty due to timer actuation is 0.006 s or 0.040 % for the worst case (the shortest collection time, 15 s). Combining start and stop uncertainties by taking the RSS gives a timer actuation relative standard uncertainty of 0.057 % {B}.

5.1.3.3. Piston Rocking

There is room for a very slight "rocking" of the piston as shown in fig. 3 due to the slight differences in diameter between the cylinder and piston. This effect can be considered either a collection volume length uncertainty or a collection time measurement uncertainty. For the medium prover, the piston diameter is 4.437 cm (1.747 in), the piston length is 4.470 cm, (1.76 in) and the cylinder diameter is 4.444 cm (1.7496 in). Figure 3 shows that this effect can cause standard uncertainties in slit interception amounting to 0.0035 cm (0.0014 in), or 0.008 % at each end, for a RSS relative standard uncertainty of 0.012 % {B}. The small piston also has a relative uncertainty due to piston rocking of 0.008 % at each end leading to 0.012 % when quadratically summed, while for the large piston the figures are 0.016 % and 0.023 %.

Combining the uncertainties in timer actuation and piston rocking by taking the RSS leads to a combined time relative standard uncertainty of 0.058 %, 0.058 %, and 0.061 % for a 15 s collection in the small, medium, and large piston provers respectively {B}.

5.1.4 Storage Effects

Changes in the density of the gas in the meter under test and the connecting piping upstream of the collection volume during the collection time lead to the storage or release of mass flow to or from V_a . The change in density within V_a can be due to either pressure or temperature changes. Density changes due to pressure changes during the collection time are negligible since the resistance to flow, the piston mass, and the attractive forces between the piston or mercury seal and the glass cylinder wall remain constant during the collection time. Temperature changes within V_a are a more valid concern. To attain a stable temperature profile within the meter and connecting piping, it is necessary to establish the flow through the meter under test for several minutes and then to cycle the piston up and down the cylinder at least three times before collecting flow data. Experimental temperature measurements made at several tap locations in the connecting piping show that the gas temperature in V_a changes by less than 0.020 K during a collection interval, which corresponds to a density change of 0.007 %. The significance of the density change in V_a

can be amplified or reduced depending on the ratio of the approach volume to the collection volume. For the small, medium, and large piston provers, V_a / V_c is 1.5, 0.5, and 0.1 respectively, resulting in relative standard uncertainties in mass flow due to storage effects of 0.011 %, 0.004 %, and 0.001 % respectively {B}.

5.1.5 Leakage and Sealant Vapor Pressure

Tests to measure any leakage out of the piston prover are conducted periodically or after any change in the piping is made, such as if a temperature sensor is removed for calibration and then re-installed. If a leak any larger than 0.010 % of the minimum flow measured by that piston is detected, its source is ascertained and corrected. Reference data for the saturation vapor pressure of mercury at room temperature show that this source of mass flow is far less than 0.001 % and hence this source of uncertainty will be neglected. Therefore, a reasonable value for the relative standard uncertainty due to leakage effects is 0.010 % {B}.

5.1.6. Piston Prover Uncertainty Statement

The flow rate relative uncertainty can be obtained by combining the uncertainties for density, volume, time, and storage effects as listed in Table 2. The RSS of all uncertainties in Table 2 yields the relative combined standard uncertainties for the small, medium, and large piston provers respectively of 0.096 %, 0.080 %, and 0.088 %. Applying a coverage factor of 2 to provide a 95 % approximate level of confidence, gives a relative expanded uncertainty of 0.192 %, 0.160 %, and 0.176 % for the small, medium, and large piston provers respectively.

Table 2. A summary of the piston prover uncertainties. The quantities in the column labeled "value" are standard uncertainties for each uncertainty category. The column labeled "%" gives the relative standard uncertainties for each uncertainty category expressed as a percentage. A coverage factor of 2 has been used to convert the combined standard uncertainty to the relative expanded uncertainty with a 95 % approximate level of confidence.

Uncertainty Category	**Relative Standard Uncertainty**					
	Small Piston Prover		**Medium Piston Prover**		**Large Piston Prover**	
	Value	**%**	**Value**	**%**	**Value**	**%**
Gas Density		**0.053**		**0.053**		**0.053**
Temperature	0.108 K	0.037	0.108 K	0.037	0.108 K	0.037
Pressure	0.022 kPa	0.022	0.022 kPa	0.022	0.022 kPa	0.022
Fitting Function		0.029		0.029		0.029
Experimental Data		0.012		0.012		0.012
Collection Volume	0.0691 cm^3	**0.053**	0.0780 cm^3	**0.011**	2.448 cm^3	**0.033**
Cylinder Diameter	5.0×10^{-4} cm	0.053	2.0×10^{-4} cm	0.009	2.3×10^{-3} cm	0.032
Collection Length	5.0×10^{-4} cm	0.001	5.0×10^{-4} cm	0.001	5.0×10^{-4} cm	0.001
Thermal Expansion	8.0×10^{-3} cm^3	0.006	4.6×10^{-2} cm^3	0.006	4.8×10^{-1} cm^3	0.006
Collection Time	0.0102 s	**0.058**	0.0102 s	**0.058**	0.0102 s	**0.061**
Timer Calibration	1.0×10^{-4} s	0.001	1.0×10^{-4} s	0.001	1.0×10^{-4} s	0.001
Timer Actuation	8.5×10^{-3} s	0.057	8.5×10^{-3} s	0.057	8.5×10^{-3} s	0.057
Piston Rocking		0.012		0.012		0.023
Storage Effects		**0.011**		**0.007**		**0.001**
Leakage and Vapor Pressure		**0.010**		**0.010**		**0.010**
Flow Combined Uncertainty		**0.096**		**0.080**		**0.088**
Flow Expanded Uncertainty		**0.192**		**0.160**		**0.176**

5.2. Uncertainty of the Bell Provers

The uncertainty analysis for the bell prover is based primarily on experimental measurements from the smallest bell. It is expected that the medium and large bell provers will have similar or better uncertainties than the small bell. Therefore, in what follows, the small bell prover will be assessed as the worst case situation.

5.2.1. Gas Density

The uncertainty in the density of the gas collected in the bell provers has the same components as the piston provers. Uncertainty in density arises from: the temperature measurement, the pressure measurement, the density fitting function, and the density experimental data {B}.

5.2.1.1. Temperature

The temperature sensor calibration data for the bell provers has a standard deviation of 0.060 K or less. The sensor is installed near the center of the collection volume and due to the incoming flow there is ample mixing to ensure that the temperature measured is a good sample of the entire volume. Unlike the piston prover system, the bell prover temperature is taken as the last temperature measured before the completion of the collection interval. The response time of the pressure and temperature sensors is adequate to accurately reflect changes in temperature and pressure over time. Nevertheless, the bells are cycled up and down several times before a formal flow measurement is made to attain better temperature equilibration within the bell prover system. Based on experimental measurements, a reasonable temperature sampling standard uncertainty for the bells is 0.03 K (0.010 %).

Additional temperature uncertainties due to frictional heating by the flowing gas, conduction through the temperature sensor sheath, and other effects of the environment in which the sensor is used are considered negligible. Combining the temperature calibration and sampling uncertainties by RSS results in a combined temperature standard uncertainty of 0.067 K or a relative standard uncertainty of 0.022 % {B}.

5.2.1.2. Pressure Calibration

The same pressure transducers are used for both the piston and bell provers. The relative standard uncertainty of the pressure calibration is 0.022 % {A}.

5.2.1.3. Pressure Sampling

Between the pressure tap in the inflow pipe and the entrance to the small bell there are several fittings and approximately 120 cm (48 in) of pipe. For high flow rates the pressure losses between the measurement point and the bell can be estimated using the following relation:

$$P_{\text{loss}} = \left(\frac{\lambda \cdot \ell}{d} + k_1 + k_2 + \ldots \right) \cdot \frac{\rho \cdot U^2}{2} \qquad (3)$$

where λ is the friction coefficient, ℓ and d are the length and diameter of the pipe, respectively, the k terms are loss coefficients for the respective fittings, U is the average gas velocity in the pipe, and ρ is the gas density.

For a high flow rate of 1800 cm^3/s, the mean velocity in the 4.83 cm (1.9 in) diameter pipe is 98 cm/s (3.2 ft/s). The Reynolds number is 3000, for which λ = 0.02 from the Moody friction factor curves [10].

$\lambda \cdot \ell / d$ = (0.02)(120)/4.83	= 0.50
3 elbows at k = 0.75	= 2.25
1 enlargement	= 1.00
Total	3.75

Substituting the above loss coefficients into the pressure loss equation above results in a pressure loss of 0.002 kPa, or a 0.002 % reduction {B}.

In addition to the sampling location, there is an issue of changes in pressure in the collection volume during the course of a bell stroke. Unlike the piston prover where the force applied to the collected gas is for all practical purposes constant, the counterweight system of the bells is imperfect and leads to changes in the pressure during a run. The quantities of interest (for both the piston and bell provers) are the mean pressure and temperature of the gas at the same instant that the collection is completed. However, to reduce the effects of noise in sensor signals, average values over the entire collection interval are used instead. Experimental measurements (on all three bell provers) show that the difference between the final pressure and the average pressure over the collection interval is less than 0.007 kPa, leading to relative standard uncertainties in the density of 0.007 % {B}.

Additional pressure uncertainties due to flow across the pressure taps and other influences of the environment in which the sensor is used are considered negligible. Using quadrature to combine all of the pressure uncertainty components gives a relative standard uncertainty of 0.023 % or 0.023 kPa {B}.

5.2.1.4. Gas Density Function Fit and Experimental Data

These sources of uncertainty are the same as given in the piston prover section, 0.029 % and 0.012 %. Gathering all of the density uncertainty components by taking the RSS results in a combined density relative standard uncertainty of 0.045 % {B}.

5.2.2. Collection Volume, V_c

The bell volume has traditionally been determined either by direct transfer of known air volumes from calibrated bottles or by methods involving direct dimensional measurement of the bell [11-13]. The preferred procedure at NIST is to "strap" the bell to obtain the outside volume via an average diameter, and to obtain the inner volume by subtracting the volume of metal in the bell wall. The volume of the bell wall is

measured by comparing the rise in oil level due to bell immersion with the rise observed due to the immersion of a known volume.

5.2.2.1. Bell Area

The outside diameter measurements of the bell and dry well were made using strapping techniques with an NIST-calibrated pi tape at numerous vertical locations. Statistical techniques applied by the NIST Statistical Engineering Division provided the bell diameter as a function of vertical distance from top to bottom of the bell. The mean outside diameter is 39.1821 cm (15.426 in) over the bell length corresponding to the timing distance. The dry well diameter is 33.7985 cm (13.306 in).

It is noteworthy that errors in the strapping measurements may occur due to ellipticity of the strap resulting from tape misalignment on the bell perimeters. Such misalignment of the tape gives rise to the interpretation that data points with lower values should approximate the "true" value. This approach results in an estimate of 0.005 cm (0.002 in) standard uncertainty in the average diameter, or 0.026 % relative standard uncertainty in the cross sectional area of the bell {B}.

It is shown in Appendix B that ellipticity of the bell characterized by major and minor axes which differ from circular by $2e$ introduces a relative error of $8(e/D)^2$. In a survey, the bell diameter was checked with calipers and the average e was determined to be less than 0.02 cm (0.008 in), resulting in a negligibly small error of 0.0002 %.

5.2.2.2. Bell-wall Sectional Area

The details of the determination of average bell-wall sectional area by immersion of a known volume are given in Appendix C, in which the relative standard uncertainty of this determination is estimated as 0.95 %. Since the bell-wall volume is only 0.78 % of the total, the contribution of this uncertainty is only 0.007 % {B}.

5.2.2.3. Collection Length

The length of the bell travel is determined by measuring the distance between two photodiode switches attached to the dry well. These are used to generate a rising and falling voltage edge which triggers a counter to start and stop. The distance between the switches was measured with a transfer standard calibrated by the NIST Precision Engineering Division using a laser interferometer [1]. The travel length for the bell, in a start and stop mode is 47.0184 cm (18.511 in) with a standard uncertainty of ±0.013 cm (0.005 in), or a relative standard uncertainty less than 0.03 % {B}.

5.2.2.4. Thermal Expansion

The laboratory in which the bell provers are located is temperature controlled at 296 ±1.5 K. Therefore, the bell could be used at a temperature approximately 1.5 K different from that at which it was strapped. Using thermal expansion coefficients of 19 x 10^{-6} and 12 x 10^{-6} cm/cm - K, for the brass bell and steel support for the photodiode switches respectively, gives a relative standard uncertainty of 0.007 % {B}.

5.2.2.5 Oil Level

The 0.602 cm (0.237 in) change in oil level due to bell displacement over the collection distance is obtained from direct measurement. This oil level change results in a correction to the bell collection volume of 0.320 % (see fig. 2). The uncertainty in the measurement of the drop in oil level stems primarily from the 0.025 cm (0.010 in) standard uncertainty in setting the bell positions at the ends of the measurement stroke of 47.018 cm (18.511 in). This amounts to a 0.076 % relative standard uncertainty in a quantity that is 0.320 % of the collection volume, and therefore the oil level uncertainty is negligible.

5.2.2.6 Oil Film Adherence

The oil film adherence is essentially the same on the inside and the outside of the bell wall. However, if the inside and outside oil surface areas are sufficiently different, the two oil-level depletions may be inconsistent with the pressure difference between the bell and atmosphere. A transfer of oil occurs between the inside and outside of the bell to balance this inconsistency, and a systematic error is

introduced. A theory to quantify this bias was developed by Smith [14]. The volume of liquid adhering to the bell, V_L, is given as

$$V_L = \frac{2 \cdot \pi}{3} \cdot \left(\frac{u \cdot U}{g} \right)^{\frac{1}{2}} \cdot h_r \cdot D \qquad (4)$$

where u is the kinematic viscosity of the sealing oil, U is the bell rise velocity, g is the acceleration due to gravity, h_r is the height through which the bell is raised, and D is the inside diameter of the bell. Evaluating this equation with u = 0.047 cm^2/s, U = 1.65 cm/s (corresponding to a high flow rate), h_r = 47 cm and D = 39 cm, we obtain, V_L = 34.15 cm^3 .

It is also shown in [14] that the net change in gas volume, V_G, due to the oil films is

$$V_G = V_L \cdot \frac{S_i \cdot \left(1 + 2 \cdot b/D\right) - S_o}{S_i + S_o} \qquad (5)$$

where S_i is the inner surface area of the oil, i.e., between the inside of the bell and the dry well, S_o is the outer surface area of the oil, i.e., between the outside of the bell and the tank, and b is the thickness of the bell wall. For S_i = 299.3 cm^2 , S_o = 487.2 cm^2, and b = 0.76 mm, V_G = 7.4 cm^3 , giving a bias relative to the bell collection volume of -0.013 % {B}; i.e., there is a smaller volume of gas collected than is accounted for by computations which include only the dimensions of the bell and the change in oil level due to bell immersion. This quantity will be considered the relative standard uncertainty due to oil film adherence.

Using quadrature to total the previously listed uncertainty components for the bell collection volume, the relative combined standard uncertainty for the volume is 0.043 % {B}.

5.2.3. Collection Time, Δt

5.2.3.1 Timer Calibration

The same timer is used for both the bells and the pistons, and its calibration standard uncertainty is 1×10^{-4} s, or less than 0.001 % for the shortest collection time (15 s) {B}.

5.2.3.2. Timer Actuation

The collection time is measured via photodiode switches as described in section 3.2.2. Tests of the switches by the same methods previously described for the piston prover indicate a reasonable estimate of the timer actuation standard uncertainty is 0.006 s, or 0.040 % {B}.

5.2.3.3. Bell Rocking

The bell provers have a source of uncertainty which is analogous to the piston rocking issue and can be considered a collection time measurement uncertainty. Tipping and lateral motion of the bell is restrained by sets of sheaved wheels which roll on three brass rods, but slight lateral displacements are still possible. Measurements with a dial gage of the bell position during its travel show lateral displacements of 0.05 cm (0.02 in) or less over the 45.7 cm (18 in) collection length.

For a tipped position that generates 0.05 cm lateral displacement, geometry shows that the vertical displacement can be as large as 0.133 mm (0.0052 in), or 0.028 % of the bell collection length. If we allow for opposite displacements at the ends of the collection period by addition in quadrature, the total timing relative standard uncertainty due to bell rocking is 0.040 % {B}. Combining all timing uncertainties by RSS gives a relative standard uncertainty of 0.057 % {B}.

5.2.4. Storage Effects

As for the piston prover, temperature and pressure changes in the gas within the approach volume result in V_a being a source or sink of mass flow. Experimental measurements on all three bells show that the pressure within the approach volume increases by no more than 0.014 kPa during the course of a collection which results in a density relative standard uncertainty in V_a of 0.014 %. Temperature changes within V_a are minimized by running the flow through the meter under test and connecting piping for several minutes and by cycling the bell up and down at least three times before a formal flow measurement is made. In this way, temperature changes in V_a during a collection are kept less than 0.020 K, for a density relative standard uncertainty of 0.007 %. Combining the uncertainties due to pressure and temperature by quadrature leads to a possible density change of 0.016 %. As for the piston provers, this density uncertainty must be weighted by the ratio of the approach volume to the collection volume. The ratio V_a / V_c for the three bell provers is 0.1, 0.5, and 0.7 for the small, medium, and large bells respectively. Using the largest ratio of 0.7 leads to a storage effect relative standard uncertainty of 0.011 % {B}.

5.2.5. Leakage and Sealant Vapor Pressure

Tests to measure the leakage of gas from the bell prover are conducted periodically or after any piping change, such as the removal and re-installation of a temperature sensor. If leaks larger than 0.010 % of the minimum flow measured by the prover are detected, the source of the leak is ascertained and corrected. The saturation vapor pressure of the oil used to seal the gas within the bell is less than 3 Pa. Therefore, if the rate of mass transfer from the oil surfaces under the bell to the gas collected is sufficient to achieve saturation, the percentage of the collected gas that is oil vapor would be less than (3 Pa / 101325 Pa) x 100 or 0.003 % (using one atmosphere for the total pressure under the bell). Noting that these two uncertainties must be opposite in sign, 0.010 % will be taken as the relative standard uncertainty due to leakage and the sealant vapor pressure together {B}.

5.2.6. Bell Prover Uncertainty Statement

For the bell prover the flow rate uncertainty can be obtained by taking the root sum of the squares of the uncertainties for density, volume, time and storage effects as listed in Table 3. The relative combined standard uncertainty of the bell prover system is 0.086 %. Applying a coverage factor of 2 to provide a 95 % approximate level of confidence, gives a relative expanded uncertainty of 0.172 % for the bell prover system.

Table 3. Summary of uncertainties for the small bell prover. The quantities in the column labeled "value" are standard uncertainties for each uncertainty category. The column labeled "%" gives standard uncertainties for each uncertainty category expressed as a percentage. A coverage factor of 2 has been used to convert the combined uncertainty to the relative expanded uncertainty with a 95 % approximate level of confidence.

Uncertainty Category	**Relative Standard Uncertainty**	
	Value	**%**
Collection Volume Density		**0.045 %**
Temperature	0.067 K	0.022 %
Pressure	0.023 kPa	0.023 %
Fitting Function		0.029 %
Experimental Data		0.012 %
Collection Volume		**0.043 %**
Bell Area		0.026 %
Bell Wall Section Area		0.007 %
Collection Length	1.25×10^{-2} cm	0.030 %
Thermal Exp. of Bell		0.007 %
Oil Film Adherence		0.013 %
Collection Time		**0.057 %**
Timer Calibration	1.0×10^{-4} s	0.001 %
Timer Actuation	6.0×10^{-3} s	0.040 %
Bell Rocking	6.0×10^{-3} s	0.040 %
Storage Effects		**0.011%**
Leakage and Vapor Pressure		**0.010 %**
Combined Uncertainty		**0.086%**
Expanded Uncertainty		**0.172%**

6. SUMMARY

The foregoing analysis presents a description of the system performance of the small air flow calibration facilities in the NIST Fluid Flow Group. Meters for measuring gas flow can be calibrated over the range from 3.7 x 10^{-5} m^3/min to 1.4 m^3/min (0.001 scfm to 51 scfm, reference temperature and pressure are 293.15 K and 101325 Pa) using positive displacement techniques as needed for Test Nos. 18010C-18040C and 18050S in NIST Special Publication 250. The flow rate of gas passing through the meter under test is determined from pressure, temperature, volume, and transit time measurements of a displaced volume of gas. The relative expanded uncertainties for this type of measurement are ±0.192 % for piston provers and ±0.172 % for bell provers.

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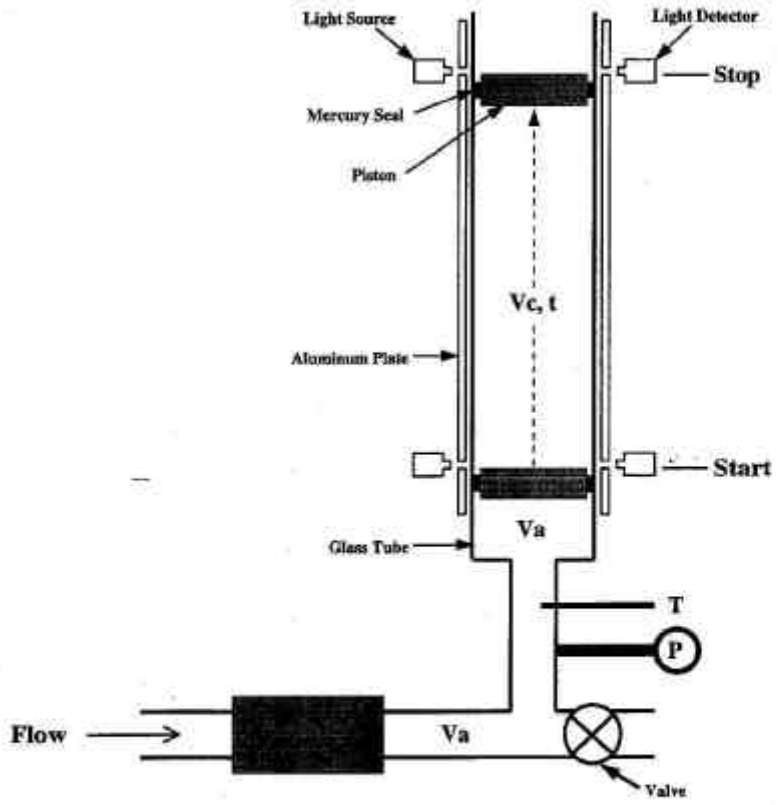

Figure 1. The piston prover apparatus showing the collection volume (V_c), the approach volume (V_a), the location of temperature (T) and pressure sensors (P), and the sensors used to measure the collection time (Δt).

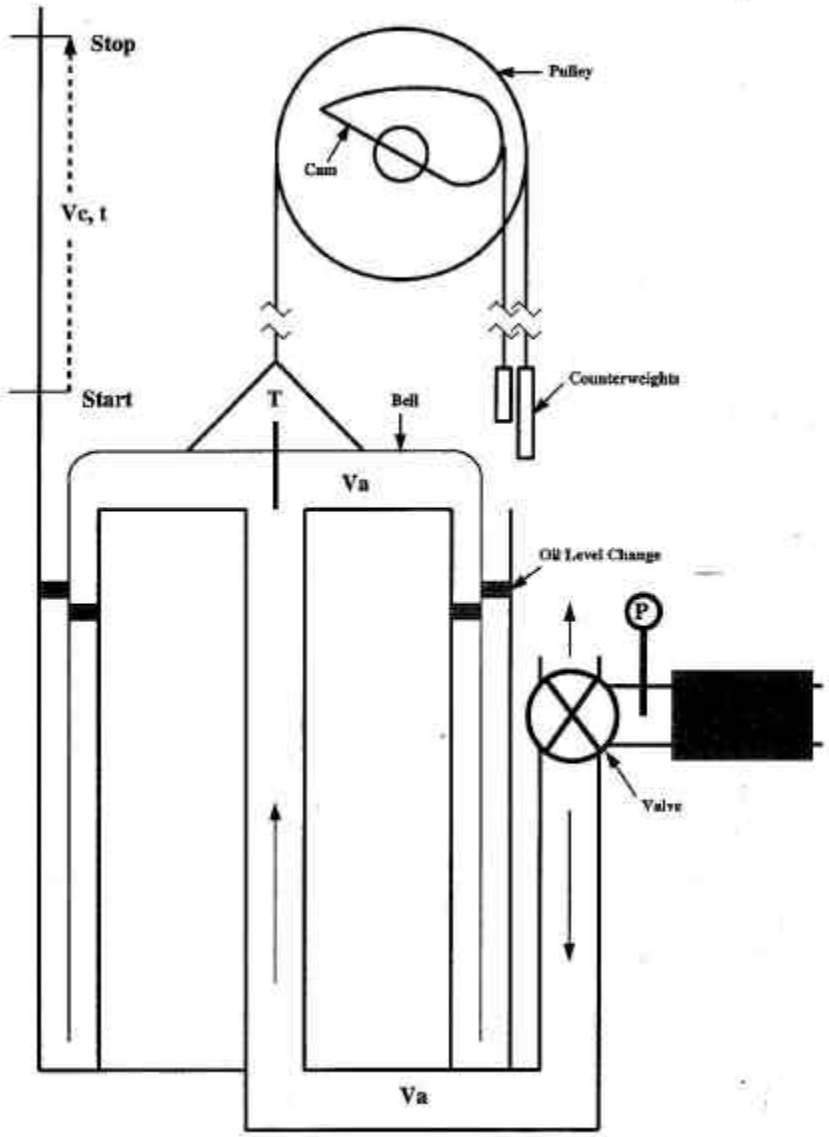

Figure 2. The bell prover apparatus showing the collection volume (V_c), the approach volume (V_a), the location of temperature (T) and pressure sensors (P), and the sensors used to measure the collection time (Δt).

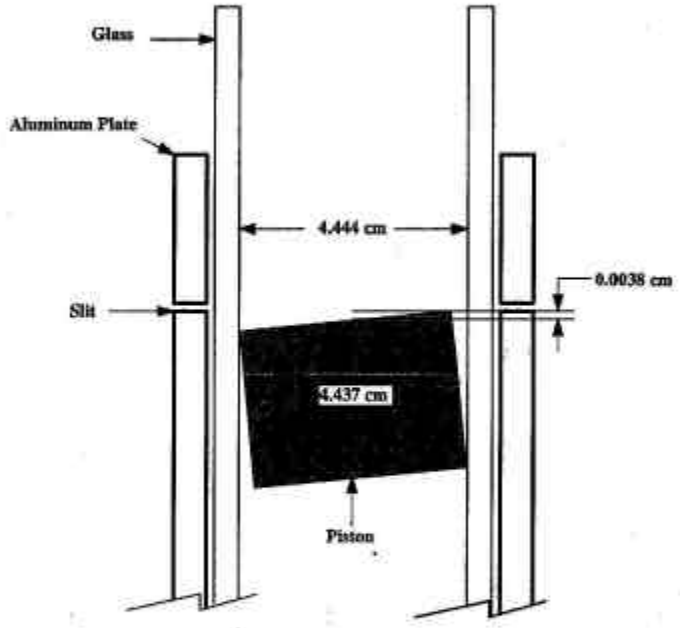

Figure 3. Diagram of the piston rocking phenomenon and the resulting collection length (or time) uncertainty. The light slit is not drawn to scale.

APPENDIX A: Sample Calibration Report

U.S. DEPARTMENT OF COMMERCE

NATIONAL INSTITUTE OF STANDARDS AND TECHNOLOGY
Gaithersburg, MD 20899

REPORT OF CALIBRATION

OF

CRITICAL FLOW METERS

May 4, 1998

Mfg: Meter Builders, Inc
Serial No: 1234
Throat Diameter: 0.813 mm
Capacity: 0.27 - 0.83 gm/sec

submitted by

Flowmasters, Inc
Metertown, MD

(Purchase Order No. A123, dated April 16, 1998)

A critical venturis with nominal throat diameter of 0.813 mm (0.032 in) has been calibrated by flowing filtered dry air at a constant rate through the venturi and then into a volumetric prover. The prover measures volumetric flow by collecting gas in a known volume over a measured period of time. The meter was run at five flow set points and five (or more) averages were gathered at each of these flows on two different occasions. As a result, the tabulated data points for these runs are averages of ten or more individual calibration measurements.

The venturi was installed using ISO standard[1] approach and exit tubes with two wall taps for the measurement of temperature (T_1) and pressure (P_1) upstream from the venturi. The upstream pipe diameter was 2.1 cm. The temperature and pressure were measured with NIST sensors[2]

[1] ISO 9300, *Measurement of Gas Flow by Means of Critical Flow Venturi Nozzles*, Geneva, Switzerland, 1990.

[2] The instrument make and model is stated for completeness of the calibration record and to establish the chain of calibration traceability and is not an endorsement of the product.

(Tmeter SN 30032, thermistor #7 and Pmeter SN 0387/0055). A recovery factor, *r*, of 0.70 was used to convert measured temperature to stagnation temperature, T_0. Stagnation temperature was calculated from the equation:

$$T_0 = T_1 \cdot \left[1 + \frac{g-1}{2} \cdot M^2 \cdot (1-r)\right] \quad (1)$$

and the stagnation pressure, P_0 , from the equation:

$$P_0 = P_1 \cdot \left[1 + \frac{g-1}{2} \cdot M^2\right]^{\frac{g}{g-1}} \quad (2)$$

where g is the specific heat ratio and *M* is the Mach number in the approach pipe, both based on P_1 and T_1. However, both of these corrections are less than 0.05 % in this case since the upstream Mach number for the venturi assemblies never exceeds 0.03.

The Reynolds number is included in the data tables and it was calculated with the following expression:

$$Re = \frac{4 \cdot \dot{m}}{\pi \cdot d \cdot \mu} \quad (3)$$

where $\dot{m}$ is the mass flow of gas, *d* is the nominal throat diameter, and m is the gas viscosity, all with consistent units so that *Re* is dimensionless. The viscosity of air was calculated via:[3]

$$m = \left(\frac{145.8 \cdot T_0^{1.5}}{110.4 + T_0}\right) \cdot 10^{-7} \quad (4)$$

where m has units g / (cm s), and T_0 is in K. The discharge coefficient C_d was calculated from the expression:

$$C_d = \frac{4 \cdot \dot{m} \cdot \sqrt{R \cdot T_0}}{\pi \cdot d^2 \cdot P_0 \cdot C^*} \quad (5)$$

where *R* is the gas constant (the universal gas constant, 8.314471 J / (mol K), divided by the gas molecular weight, 28.966 g/mol), and C^* is the critical flow factor calculated using:

[3] Hilsenrath, J., Beckett, C. W., Benedict, W. S., Fano, L., Hoge, H. J., Masi, J. F., Nuttall, R. L., Touloukian, Y. S., and Woolley, H. W., *Tables of Thermal Properties of Gases*, NBS Circular 564, 1955.

$$C^* = 0.68309 + 1.42025 \cdot 10^{-5} \cdot T_0 - 2.80046 \cdot 10^{-8} \cdot T_0^2 + 3.47447 \cdot 10^{-5} \cdot P_0 + \ldots$$
$$\ldots - 1.80997 \cdot 10^{-7} \cdot P_0 \cdot T_0 + 2.46278 \cdot 10^{-10} \cdot P_0 \cdot T_0^2 \qquad (6)$$

with C^* dimensionless, T_0 in K, and P_0 in kPa. Equation (6) is based on a fit to reference data.[4] The calibration results are presented in the following Table and Figure.

Table 1. Calibration results for 0.813 mm venturi.

T_0 (K)	P_0 (kPa)	$\dot{m}$ (g/s)	C^*	Re	C_d	U_r
296.40	208.33	0.2747	0.68541	23525	1.0813	0.21
296.44	311.84	0.4120	0.68569	35288	1.0833	0.20
296.54	414.79	0.5487	0.68597	46979	1.0843	0.20
296.63	518.31	0.6864	0.68625	58755	1.0852	0.20
296.81	626.49	0.8302	0.68654	71034	1.0859	0.20

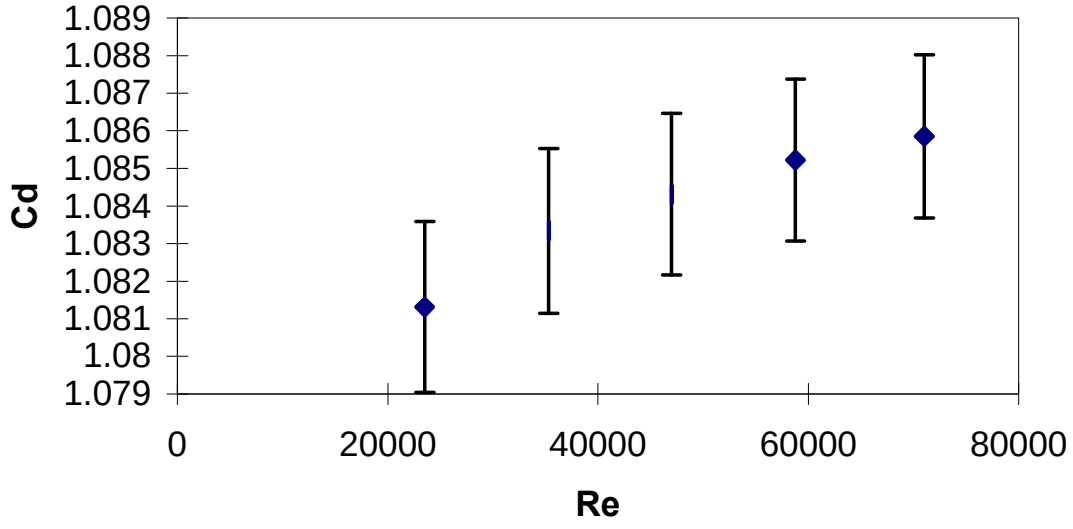

Figure 2. Calibration results for 0.813 mm venturi.

[4] Johnson, Robert C., *Real Gas Effects in Critical Flow through Nozzles and Tabulated Thermodynamic Properties*, NASA Technical Note D-2565, January, 1965.

An analysis was performed to assess the uncertainty of the discharge coefficients obtained for the meter under test.[5, 6] This process involves identifying all of the significant uncertainty components and obtaining standard uncertainty values (67 % level of confidence) for each component. It is also necessary to obtain the sensitivity coefficients for each component by partial differentiation of eq. (5). Then all of the uncertainty terms can be combined by the root-sum-squares method to obtain the combined standard uncertainty, u_c, and this value can be multiplied by a coverage factor of k = 2.0 to give the relative expanded uncertainty, U_r. Uncertainty components are denoted as type A or type B by letters within brackets.

For the venturi discharge coefficients, uncertainty components include the relative standard uncertainty of the mass flow measured by the volumetric prover ($u_{\dot{m}}$), the relative standard uncertainty of the meter pressure measurement (u_P), the relative standard uncertainty of the meter temperature measurement (u_T), and the reproducibility[7] of the meter under test (u_R). It will be assumed that the uncertainties in the gas constant, R, the venturi throat diameter, d, and in C^* are negligible: this assumes that the meter user will utilize the same values and correlations given above and used in the determination of the discharge coefficients (or will re-calculate the discharge coefficients with their own, preferred values). Note that if the venturis were subsequently used in a gas other than dry air, additional uncertainty due to C^* and the viscosity used to calculate Re would be an issue.

$$U_r = k \cdot u_c = k \cdot \sqrt{\left(\frac{\partial C_d}{\partial \dot{m}} \cdot u_{\dot{m}}\right)^2 + \left(\frac{\partial C_d}{\partial P} \cdot u_P\right)^2 + \left(\frac{\partial C_d}{\partial T} \cdot u_T\right)^2 + \left(u_R\right)^2} \qquad (7)$$

The NIST flow facilities used to measure the standard flows in this calibration have a relative expanded uncertainty of 0.19% (approximate 95% level of confidence) and hence a relative combined standard uncertainty of 0.095% {B}. This uncertainty specification has been calculated by using the root-sum-square method to combine all of the uncertainty components which arise in the use of the facility (such as collection volume temperature and pressure, the magnitude of the collection volume, and the collection time). The sensitivity coefficient for mass flow ($\partial C_d / \partial \dot{m}$) is 1.0.

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[5] Taylor, B. N. and Kuyatt, C. E., *Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results*, NIST TN 1297, 1994 edition.

[6] Coleman, H. W. and Steele, W. G., *Experimentation and Uncertainty Analysis for Engineers*, John Wiley and Sons, 1989.

[7] Reproducibility is herein defined as closeness of agreement between measurements with the flow changed and then returned to the same nominal value.

The relative standard uncertainty of the meter pressure measurement is 0.02% {B} based on pressure calibration data, and the sensitivity coefficient is 1.0. The relative standard uncertainty of the meter temperature measurement is 0.03% {B} (0.1 K) based on temperature calibration data, and the sensitivity coefficient is 0.5.

To measure the meter reproducibility, the standard deviation of the discharge coefficients was used to calculate the relative standard uncertainty (the standard deviation divided by the mean) at each of the five nominal flow set points {A}. Using the values given above and eq. (7) results in the relative expanded uncertainties for the discharge coefficients listed in the data Table and shown as error bars in the Figure. The largest relative expanded uncertainty value for the venturi discharge coefficient was 0.21 %.

For the Director,
National Institute of Standards and Technology

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Leader, Fluid Flow Group
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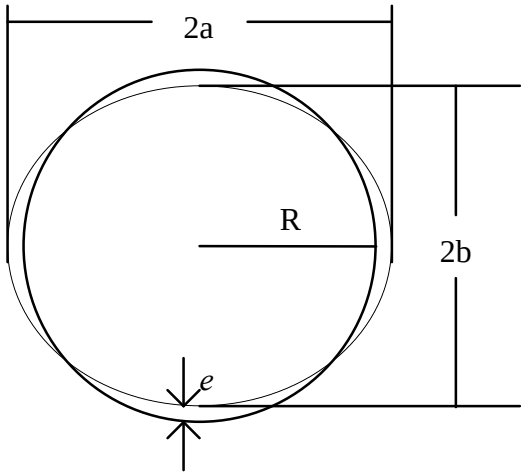
APPENDIX B: Estimate of the Effect of Ellipticity of Bell

Assume that the departure from circularity can be expressed in terms of *e* as shown below:



The perimeter of the ellipse, *P*, is:

$$P = 2 \cdot \pi \cdot \frac{a^2 + b^2}{2}. \qquad \text{(B1)}$$

In terms of *R* and *e*, this becomes:

$$P = 2 \cdot \pi \cdot R \cdot \sqrt{1 + \left(\frac{e}{R}\right)^2} \approx 2 \cdot \pi \cdot R \cdot \left[1 + \frac{1}{2} \cdot \left(\frac{e}{R}\right)^2\right]. \qquad \text{(B2)}$$

When we measure this *P* with a pi tape, we are actually getting an equivalent diameter,

$$D = 2 \cdot R \cdot \left[1 + \frac{1}{2} \cdot \left(\frac{e}{R}\right)^2\right] \qquad \text{(B3)}$$

for which the corresponding area is, approximately,

$$A_0 = p \cdot R^2 \cdot \left[1 + \left(\frac{e}{R}\right)^2\right]. \qquad \text{(B4)}$$

But the actual area of the ellipse is:

$$A = \pi \cdot a \cdot b = \pi \cdot R^2 \cdot \left[1 - \left(\frac{e}{R}\right)^2\right], \qquad \text{(B5)}$$

so that the fractional error in the area is:

$$\frac{A_0 - A}{A} \approx 2 \cdot \left(\frac{e}{R}\right)^2 \qquad \text{(B6)}$$

This area difference is relatively small. For example, with e = 1 mm and R = 196 mm, the area error would be 0.005 %.

APPENDIX C: Determination of Bell Wall Volume by Immersion

The immersed volume, V_r, is a cylindrical rod 77.8 cm (30.625 in) long and 2.54 cm (1 in) in diameter. When it is suspended by a wire and fully immersed in the oil (from which the bell has been removed) of surface area A_o, the surface rises a distance H_r.

$$H_r = \frac{V_r}{A_o} \tag{C1}$$

Immersion of the bell over its measurement length, L, causes an oil rise, H_b,

$$H_b = \frac{\left(A_b + A_s\right) \cdot L}{A_o - \left(A_b + A_s\right)} \tag{C2}$$

where A_b is average cross sectional area of the bell metal and A_s is the cross sectional area of the scale (plus screwheads) attached to the bell.

Combining these equations gives:

$$A_b = \frac{H_b \cdot V_r}{H_r \cdot \left(L + H_b\right)} - A_s \tag{C3}$$

which can be subtracted from the average outside area obtained from the strapping.

H_b and H_r are each determined from measurement of oil surface elevations with a micrometer point gage. Assigning 0.0025 cm (0.001 in) uncertainty to each level determination, the total uncertainty is 0.0036 cm (0.0014 in) for H_b and H_r, which have approximate values of 0.61 cm (0.24 in) and 0.48 cm (0.19 in) respectively; the corresponding relative standard uncertainties are 0.58 % and 0.74 %. The relative standard uncertainty in V_r, based on 0.0013 cm (0.0005 in) and 0.0127 cm (0.005 in) standard uncertainties in measurement of diameter and length

respectively, is 0.052 %. Because the bell position is set manually for these measurements, a 0.025 cm (0.01 in) standard uncertainty is assigned at each end, giving a relative standard uncertainty in L of 0.076 %, the standard uncertainty in H_b being negligible in comparison with the uncertainty in L. The relative combined standard uncertainty for the first term on the right hand side is 0.95 %.

The scale cross section is 3.175 cm x 0.318 cm (1.25 in x 0.125 in). These dimensions can be determined within 0.0013 cm (0.0005 in), resulting in an A_s relative standard uncertainty (ignoring screwheads) of 0.40 %. Because A_s is approximately 10 % of the first term, its relative standard uncertainty contribution is 0.040 %, giving a relative combined standard uncertainty in A_b of 0.95 %.