

THE LONG TERM CALIBRATION STABILITY OF CRITICAL FLOW NOZZLES AND LAMINAR FLOWMETERS

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Abstract

The National Institute of Standards and Technology (NIST) has performed calibrations of gas flowmeters using piston and bell provers for three decades. Most of the meters calibrated have been either critical flow nozzles or laminar flowmeters since these meter types are generally chosen as the working standards or transfer standards by other flow calibration laboratories. Often the same meters have been returned to NIST numerous times over a decade or more for periodic calibration. The data from these repeated calibrations have been examined to gain insight on the long term calibration stability of these widely used generic meter types. The calibration curves normally are stable to 0.2 % of reading or better, on the same order as the relative standard uncertainty of the piston and bell provers themselves.

Introduction

The Fluid Flow Group at NIST has calibrated gas flowmeters for customers for about thirty years using piston and bell provers. Generally the calibration customers are secondary laboratories in the utility and aerospace industries that use these flowmeters as transfer standards or working standards to calibrate many other flowmeters within their company. Nearly all of the gas flow transfer standards calibrated by NIST are either critical flow venturi nozzles or laminar flowmeters. The gas flow metering community

has found that these devices are relatively inexpensive, easy to use, and they maintain their calibration well over time.

Some gas flowmeters have been returned to NIST for calibration on a periodic basis over extended time intervals. Examination of the archived calibration reports turned up 16 laminar flowmeters that have had repeat calibrations, and within this set of meters, 60 calibrations have been performed. Twenty-three critical nozzles have had repeat calibrations and the total number of calibrations within the nozzle set is 78. Hence the archival calibration data gives one the opportunity to assess how well these two generic meter types maintain their calibration stability over time.

In this paper, we will discuss the uncertainty of the NIST Fluid Flow Group's piston and bell provers, and the uncertainty of the flowmeter discharge coefficients determined there, the makeup of the flowmeter population examined, and the stability of their discharge coefficients. We will also examine the calibration histories of some particular flowmeters.

Calibration Facilities

The NIST Fluid Flow Group operates a set of three piston provers and three bell provers (Figure 1) which cover a flow range from 0.05 g/min to 1950 g/min (3.7×10^{-2} L/min[⊗] to 1440 L/min).¹ The three piston provers are mounted together in a console and connected by a manifold to a single inflow line. The collection volumes of the three prover cylinders are nominally 130 cm³, 710 cm³, and 7400 cm³. In the piston prover system, the metered gas is diverted (using a valve) into a glass cylinder to raise a mercury-sealed piston. As the piston rises through the cylinder, it successively starts and stops a timer by blocking the light passing through machined slits at the ends of the collection volume. The temperature and pressure of the gas entering the collection volume are measured with a temperature sensor and a pressure gage installed in the inlet pipework. These temperatures and pressures are used to calculate the density of the collected gas, and the density is used to convert the measured volumetric flow rate into a mass flow rate. The bell provers have instrumentation requirements and operational procedures analogous to the piston provers but they utilize an oil seal and an inverted bell. The collection timer is triggered by spring metal contacts. The collection volumes of the three bells are nominally 0.056 m³, 0.114 m³, and 0.361 m³.

The Fluid Flow Group has used the same piston and bell provers since about 1970. Over the years, various improvements in the instrumentation, sensor locations, and collection volume measurements have reduced the flow measurement relative standard uncertainty from 0.29 %* to 0.19 % or better in 1998.^{1, 2, 3, 4} In the 1970's, prover temperatures were

⊗ All volumetric flows are stated for conditions of 293 K and 101.325 kPa.

* All uncertainties are 95 % confidence level values, coverage factor $k = 2$, see Taylor and Kuyatt, 1994.

measured with thermocouples, prover pressure was measured via water manometer along with the barometric pressure measured using a Bourdon tube gage. Starting in 1986, an upgrade of the piston and bell provers was undertaken. Improvements in the uncertainty of the collection volumes were made. The prover pressure was measured with an absolute pressure transducer instead of the Bourdon tube / water manometer combination. The prover instrumentation readings and most of the flowmeter instrumentation data was collected with a personal computer. These improvements reduced the prover flow relative uncertainties to 0.22 %. In the 1990's, the temperature sensors were changed from thermocouples to thermistors and some improvements were made in the location of the temperature sensors. A recent uncertainty analysis¹ gives a relative standard flow uncertainty of 0.19 % or less and several intercomparisons with other laboratories and crossover checks between the six piston and bell provers confirm this uncertainty.⁵

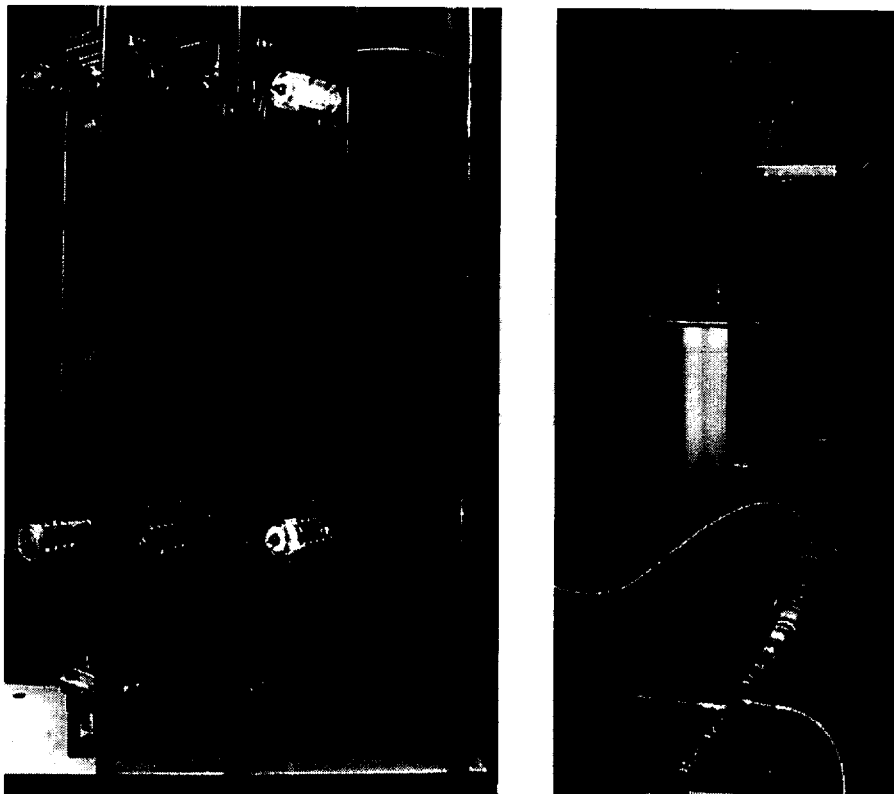


Figure 1. Piston provers and medium bell prover.

Customer calibrations are generally conducted at five evenly incremented flows of dry air, with five prover collections made at each flow to quantify meter repeatability, and this set of tests is replicated on two different days to assess meter reproducibility.* Meters are installed with upstream straight pipe runs of thirty pipe diameters or more which reduces installation effects on the meter to a negligible level.

* Definitions of terms are given in Taylor and Kuyatt, Appendix D, 1994.

Uncertainty of the Meter Discharge Coefficients

Reynolds number (Re) and discharge coefficient (C_d) were used to examine the calibration data for both the critical nozzles and the laminar flowmeters. Details about these quantities and the methods of their calculation are given in Appendices A and B. The uncertainty of the discharge coefficients used to evaluate flowmeter stability will now be considered. The discharge coefficient uncertainty depends on the uncertainty of the standard flow (from the piston and bell provers), the uncertainty of the instrumentation used to measure temperatures and pressures at the meter under test, and the reproducibility of the meter. For all of the flowmeters in this analysis, NIST instrumentation (not the customer's) was used to measure the flowmeter temperatures and pressures. Reproducibility was assessed by calculating the standard deviation of the mean for the ten repeat C_d determinations made at nominally the same flow.

Since the interest of the present paper is to assess the stability of flowmeter calibrations over time, systematic uncertainties that were consistent throughout the period of testing could be dropped from the uncertainty analysis. For instance, an error in the value of the bell prover collection volume would not lead to drift in flowmeter discharge coefficients if the same systematic error was present each time a calibration was performed. Unfortunately, these reductions in uncertainty cannot be exploited because of changes made in the calibration facilities over time. Using the collection volume example, at certain times in the history of the bell provers, the start and stop locations have been changed and the collection volume redetermined. Hence this "systematic" uncertainty may have been positive for one calibration and negative for another and must be maintained at the full level of uncertainty.

Laminar Flowmeters

An analysis was performed to assess the uncertainty of the discharge coefficients obtained for the meters under test.^{4, 6} This process involves identifying all of the significant uncertainty components and obtaining 67 % confidence level values for each component. It is also necessary to obtain the sensitivity coefficients for each component by partial differentiation of the discharge coefficient equation. Then all of the uncertainty terms can be combined by the root-sum-squares method to obtain the combined relative standard uncertainty, u_c , and this value can be multiplied by a coverage factor of $k = 2.0$ to give the relative expanded uncertainty, U_r . In this paper, the symbol u denotes relative standard uncertainty.

For the laminar flowmeter flowing dry air, the uncertainty components include the relative standard uncertainty of the actual flow rate at the meter inlet (due to uncertainties in the standard flow and in the upstream pressure and temperature) ($u_{V_{up}}$), the relative standard uncertainty of the differential pressure measurement ($u_{\Delta P}$), the uncertainty of the calculated viscosity (due to uncertainties in the upstream pressure and temperature) (u_{μ}), and the reproducibility of the meter under test (u_R). The sensitivity coefficients for the

laminar flowmeter in Equation 1 are all equal to 1.0.

$$U_r = k \cdot u_c = k \cdot \sqrt{\left(\frac{\partial C_d}{\partial \dot{V}_{up}} \cdot u_{\dot{V}_{up}}\right)^2 + \left(\frac{\partial C_d}{\partial \Delta P} \cdot u_{\Delta P}\right)^2 + \left(\frac{\partial C_d}{\partial \mu} \cdot u_{\mu}\right)^2 + (u_R)^2} \quad (1)$$

The uncertainty analysis does not include uncertainty in the experimental measurements of viscosity found in references which can amount to 1% or more. To prevent errors due to viscosity, the meter user must use the same gas and expression used by NIST to calculate viscosity when using the discharge coefficients. There are small uncertainties due to uncertainty in viscosity which arise when two calibrations are not performed at exactly the same temperature, but given the good temperature control in the calibration laboratory, this uncertainty is negligible. Also, uncertainty in the values of the laminar tube length, hydraulic radius, and the number of tubes are neglected since the same values were used each time the discharge coefficients were calculated.

For the data collected in the 1970's, the instrumentation associated with a laminar flowmeter included: a water manometer for measuring the pressure drop across the meter, a thermocouple to measure the upstream gas temperature, another thermocouple to measure the room temperature, and a second water manometer to measure the pressure drop between the upstream side of the flowmeter and atmospheric pressure. The absolute pressure upstream of the meter was calculated by summing the second water manometer reading and the atmospheric pressure measured with a Bourdon tube. The atmospheric pressure and room temperature were used to arrive at water density and air buoyancy corrections necessary to convert the water manometer readings to pressure units. The uncertainty of the upstream pressure measurement was improved in the late 1980's by new transducers. In the 1990's, the uncertainty of the differential pressure measurement was reduced by replacing the water manometer by a resonant silicon diaphragm gage. Temperature and absolute pressure uncertainties were also improved by sensor and calibration method upgrades. As a result of the calibration improvements over time, the typical value for the relative standard uncertainty of a laminar flowmeter discharge coefficient measured by the NIST Fluid Flow Group was reduced from 0.42 % in 1970 to 0.24 % in 1998 (see Table 1).

Table 1. Typical values of relative standard uncertainty for the provers, laminar flowmeter, and critical nozzle discharge coefficients from the NIST Fluid Flow Group from 1970 to 1998.

Year	Piston and Bell Prover Uncertainty (%)	Laminar Flowmeter C_d Uncertainty (%)	Critical Nozzle C_d Uncertainty (%)
1970-1986	0.29	0.42	0.37
1986-1995	0.22	0.32	0.26
1995-1998	0.19	0.24	0.22

Critical Flow Nozzles

For the nozzle discharge coefficients, uncertainty components include the relative standard uncertainty of the mass flow measured by the volumetric prover (u_m), the relative standard uncertainty of the meter pressure measurement (u_p), the relative standard uncertainty of the meter temperature measurement (u_T), and the reproducibility of the meter under test (u_R). It will be assumed that the uncertainties in the gas constant, R , the nozzle throat diameter, d , and in the critical flow factor, C^* , are negligible: this assumes that the same values and correlations are used in each determination of the discharge coefficients. Note that if the nozzles were subsequently used in a gas other than dry air, additional uncertainty due to C^* and the viscosity used to calculate Re would be an issue. The sensitivity coefficients in Equation 2 are equal to 1.0 except for the temperature coefficient, which equals 0.5.

$$U_r = k \cdot u_c = k \cdot \sqrt{\left(\frac{\partial C_d}{\partial \dot{m}} \cdot u_m\right)^2 + \left(\frac{\partial C_d}{\partial P} \cdot u_p\right)^2 + \left(\frac{\partial C_d}{\partial T} \cdot u_T\right)^2 + (u_R)^2} \quad (2)$$

Typical relative standard uncertainties for critical nozzle discharge coefficients range from 0.37 % in 1970 to 0.22 % in 1998, as shown in Table 1. The uncertainties are less for the nozzle than for the laminar flowmeter because: 1) the small differential pressure measurement for the laminar flowmeter historically had a relatively high uncertainty, and 2) an extra pressure and temperature measurement are necessary in order to obtain the actual volumetric flow at the laminar flowmeter.

The Flowmeters

The full scale flows and the years of calibration for the 16 laminar flowmeters studied in this analysis are listed in Table 2. The same data as well as throat diameters for the 23 critical flow nozzles are given in Table 3. The laminar flowmeters were produced by two manufacturers and the critical flow nozzles were made by two other manufacturers. The meter manufacturers are represented by the letters A, B, C, and D in Tables 2 and 3. The wide range of flows covered by the laminar flowmeters tested (70 000 to 1), means that some of the laminar flowmeters are of the rolled sheet, sine channel design, while others are several circular tubes in parallel. The throat diameters of the nozzles range from 0.28 mm to 9.0 mm and they cover a 1000 to 1 flow range.

Table 2. Full scale flows, manufacturer, and years calibrated for the laminar flowmeters.

Meter Number	Full Scale Flow (L/min)	Manufacturer	Years Calibrated
1	0.1	A	83, 87, 91, 94
2	0.5	A	78, 84
3	0.9	A	70, 74, 78, 81, 85, 89, 92
4	2	A	78, 82, 91
5	14	A	81, 87
6	17	A	70, 74, 78, 81, 85, 89, 92
7	85	A	79, 81, 91
8	85	A	71, 77
9	280	B	93, 94, 96, 97
10	280	B	93, 95, 97
11	340	A	70, 74, 78, 81, 85, 89, 92
12	650	B	94, 98
13	1130	B	88, 92, 95
14	1980	A	71, 73, 78
15	2832	B	88, 93
16	7080	A	70, 74, 81, 85, 89, 93

Some of the flowmeters were calibrated only over a portion of their full flow range due to the flow limitations of the largest bell. (The NIST Fluid Flow Group has a Pressure-Volume-Temperature-Time facility for flows larger than the bell provers can accommodate, but normally customers have opted for calibration over part of the meter range rather than incur the extra expense and turn around time of a calibration spanning two facilities). In cases where the dimensionless quantities in the final calibration report did not agree with prior calibration results within the relative standard uncertainty of the piston and bell provers, the dimensionless quantities were recomputed from the raw calibration data. This was particularly important for the laminar flowmeters since there have been inconsistencies over the years in what pressure location was used to calculate the gas density and volumetric flow at the meter (see Appendix B). In a few cases, old data sets were culled from the analysis, usually because of uncertainty about the arrangement of instrumentation on the meter or missing dimensions on instrument readings. Estimated values of length, hydraulic radius, and number of tubes in the laminar flowmeter were used. Nozzle throat diameters were nominal values. The data for laminar flowmeters Number 1, 3, 6, 11, and 16 are particularly valuable since they hold results from as many as 7 calibrations per meter over a 22 year period. For the critical flow nozzles, meters Number 2, 5, 12, and 17 are of great interest because their data span 26 years with 5 calibrations each.

Table 3. Nominal throat diameters, full scale flows, manufacturer, and years calibrated for the critical flow nozzles.

Meter Number	Throat Diam. (mm)	Full Scale Flow (L/min)	Manufacturer	Years Calibrated
1	0.28	4.35	C	77, 83, 89
2	0.33	6.1	D	71, 74, 79, 92, 97
3	0.38	8.1	C	77, 83, 89
4	0.64	22.5	C	77, 83, 89
5	0.79	34.6	D	71, 74, 79, 92, 97
6	1.12	70.0	C	77, 83, 84, 89
7	1.12	70.0	C	89, 92, 98
8	1.12	70.0	C	78, 81, 83, 88
9	1.58	139	C	77, 83, 89
10	1.58	139	C	86, 85, 97
11	1.58	139	C	92, 98
12	1.78	177	D	71, 74, 79, 92, 97
13	2.24	279	C	77, 83, 89
14	2.24	279	C	89, 98
15	3.18	564	C	77, 83, 84, 89
16	3.18	564	C	78, 81, 83, 88
17	4.32	1040	D	71, 74, 79, 92, 97
18	4.50	1130	C	77, 83, 89
19	4.50	1130	C	89, 98
20	6.35	2250	C	77, 83, 89
21	6.35	2250	C	83, 95, 97
22	9.00	4520	C	77, 83, 89
23	9.00	4520	C	85, 91, 98

Results and Discussion

Sample plots of calibration data for laminar flowmeter Number 6 and critical nozzle Number 12 are shown in Figures 2 and 3. A good number of data sets show discharge coefficient plots similar to Figures 2 and 3, that are stable within the facility uncertainty specifications over long portions of the three decade period considered. This indicates that the flows measured with the piston and bell provers have been highly consistent from 1970 to 1998 and that the uncertainty specifications for the facilities have been met. Further, these and other calibration history plots demonstrate that laminar flowmeters and critical flow nozzles can be remarkably stable flowmeters.

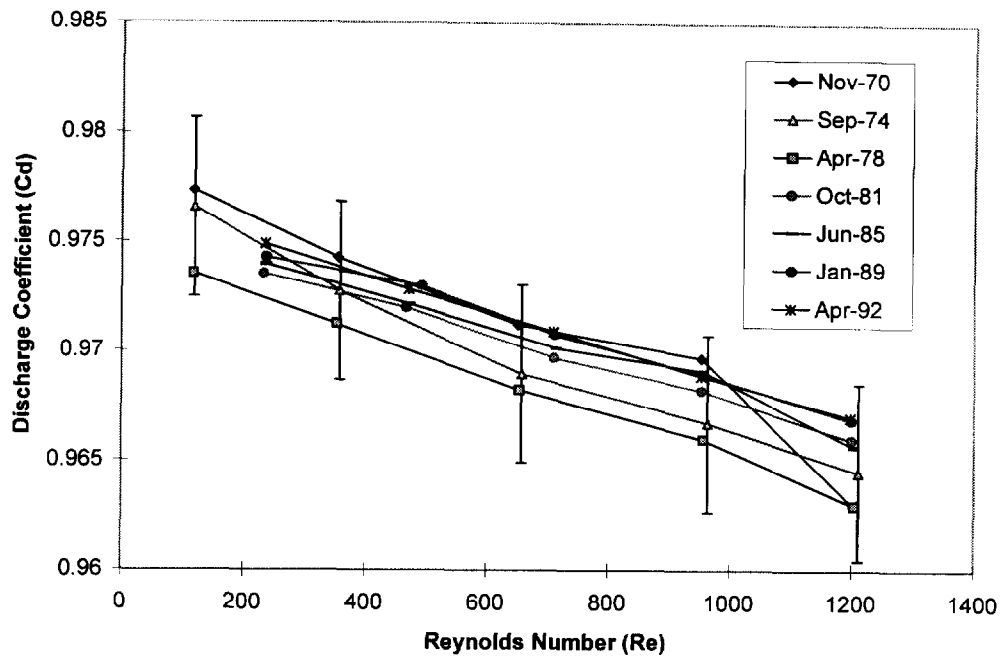


Figure 2. Calibration history for laminar flowmeter Number 6. The error bars show the 1970's uncertainty level for the discharge coefficient.

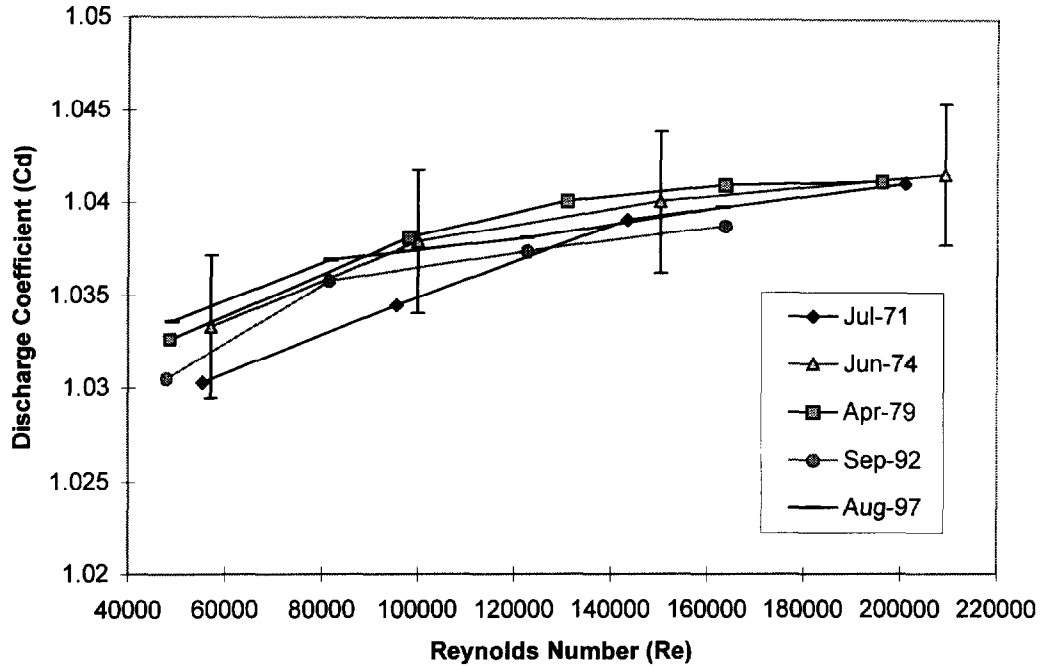


Figure 3. Calibration history for critical nozzle Number 12. The error bars show the 1970's uncertainty level for the discharge coefficient.

The percent drift between flowmeter calibrations was calculated by averaging the percent change in the discharge coefficient at each of the five nominal flows tested. Histograms of the percent drift per year for the two flowmeter types are shown in Figures 4 and 5. For the laminar flowmeters, the standard deviation was 0.19 % / year while the critical nozzles had a standard deviation of 0.07 % / year. The standard deviation of the laminar flowmeters was driven up by a few meters which had large shifts between calibrations, and some of these cases will be examined more closely later. About 82 % of the laminar flowmeter calibrations showed calibration drift of less than 0.2 % / year while 93 % of the critical nozzles showed drifts of less than 0.2 % / year. The mean for the critical nozzle histogram is +0.007 %, while the mean for the laminar flowmeter histogram is +0.017 %, perhaps indicating a tendency for the discharge coefficient to drift upwards for both meter types.

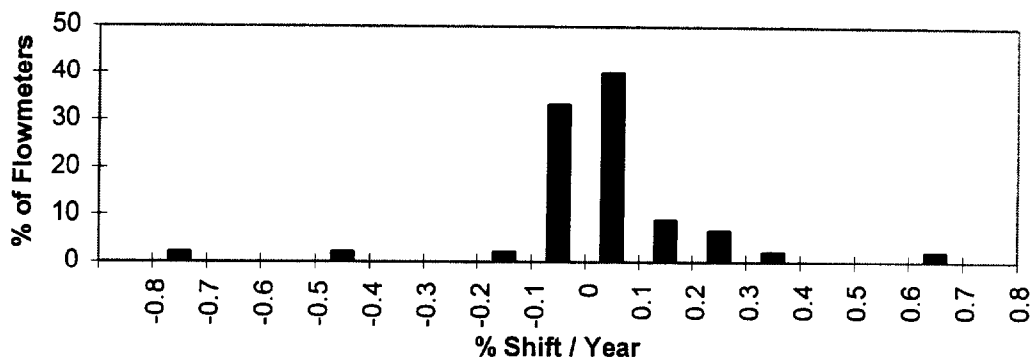


Figure 4. Histogram of percent drift per year for the laminar flowmeter population.

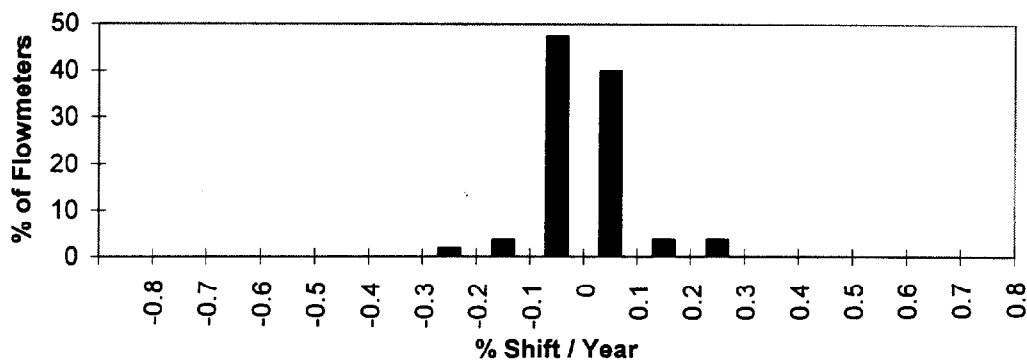


Figure 5. Histograms of percent drift per year for the critical nozzle population.

The change in discharge coefficient between flowmeter calibrations is often less than the 0.22 % to 0.42 % uncertainty of the C_d measurement. Only when the drift between calibrations is greater than the C_d measurement uncertainty can we state with confidence that the change was a drift in the flowmeter. The calculated calibration differences are not necessarily due to drift alone, but are also due to the measurement uncertainty.

Therefore the drift / year figures presented place an *upper bound* the flowmeter drift, but the drift *may be better* than the values calculated herein.

Data for particular meters with long calibration histories will now be considered. Figure 6 presents the percent drift of the discharge coefficient relative to the first calibration versus the year of calibration for laminar flowmeters Number 1, 3, 4, 6, 11, and 16. Some of the laminar flowmeters, such as Numbers 11 and 16, show C_d shifts of more than 1 %. These shifts could be caused by damage during handling or by dirt obstructing some of the laminar flow tubes. The shifts are not inherent to laminar flowmeters, as evidenced by 0.3 % or better stability for flowmeters Number 1, 4, and 6 over periods of up to 22 years.

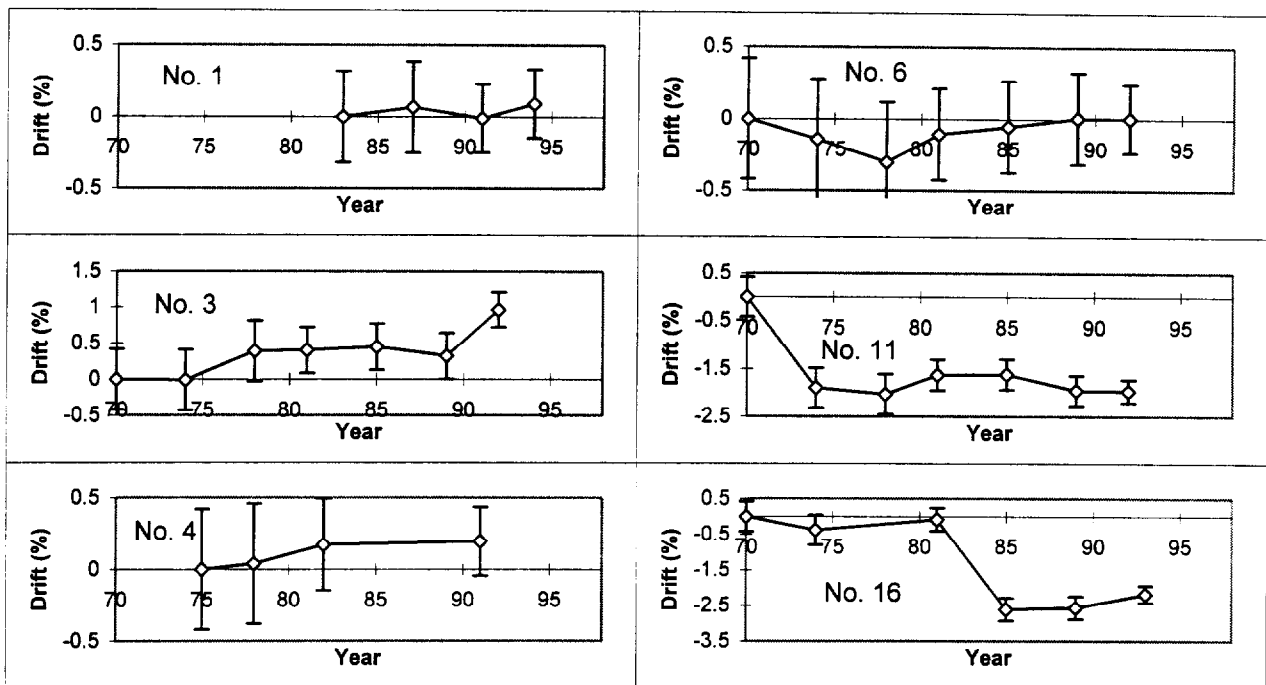


Figure 6. Laminar flowmeter percent drift relative to the first calibration versus the year of calibration for six selected meters with long calibration histories. Note that the Y-axis scaling differs between some of the plots. The C_d error bars are from Table 1.

Calibration histories for critical nozzles Number 2, 5, 12, 15, 16, and 17 are presented in Figure 7. A set of four of these nozzles have been calibrated 5 times over a 26 year period. For these four nozzles, the calibration data from 1971 departs from the remaining calibrations, but still agree with the other C_d values within the relative standard uncertainty of the discharge coefficient measurements. If the 1971 data were ignored, the discharge coefficients for these four nozzles remained stable within 0.3 % over a 23 year period. Critical nozzle Number 16 yielded the same C_d within 0.04 % four times over a 10 year period.

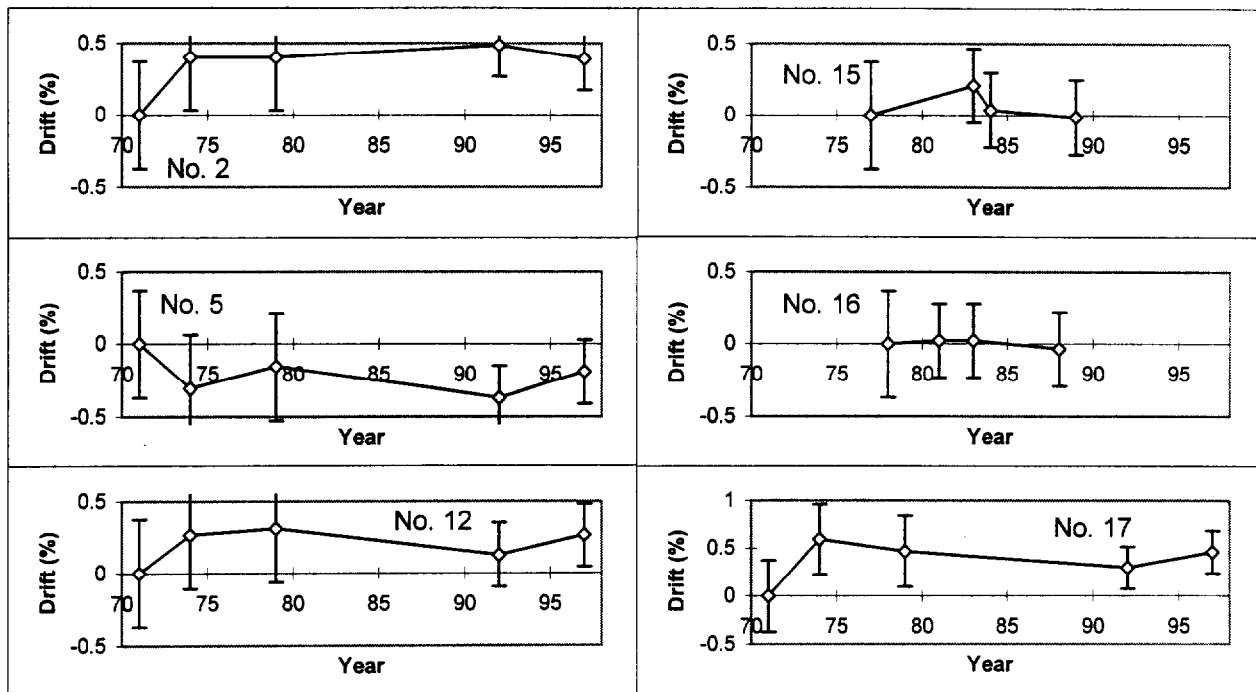


Figure 7. Critical nozzle percent drift relative to the first calibration versus the year of calibration for six selected meters with long calibration histories. Note that the Y-axis scaling differs between some of the plots. The C_d error bars are from Table 1.

A set of four nozzles included in the analysis has a clear trend of increasing C_d 's over time (see Figure 8). Communication with the owner of the nozzles reveals a plausible explanation: the compressed air used through the nozzles is not filtered and is known to contain grit. Presumably, using the unfiltered gas has eroded the throat diameters of these nozzles, leading to discharge coefficients which monotonically increase with the passage of time.

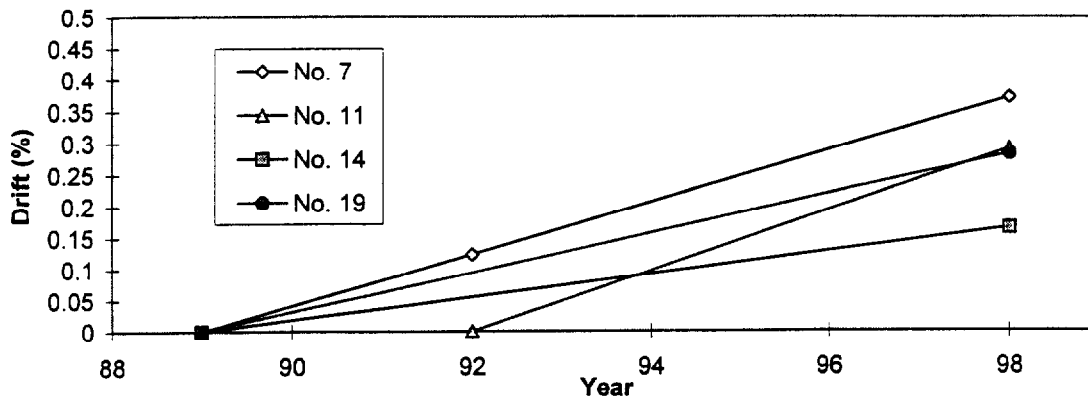


Figure 8. Calibration drift for four critical nozzles of the same set, trending upwards probably due to erosion of the nozzle throats by grit in the flow.

Discussion and Conclusions

The calibration histories of 16 laminar flowmeters and 23 critical nozzles that have been calibrated periodically over the past three decades by the NIST Fluid Flow Group have been examined. These two generic meter types represent the vast majority of the flowmeters sent to NIST for calibration for use in other labs as working or transfer standards. The calibration histories support the choice of these meter types as transfer standards, with numerous meters showing less drift than can be resolved with the piston and bell provers used as the flow standards in the calibrations. Also, the drift / year histograms and the calibration histories for particular meters show that the piston and bell provers have remained within their uncertainty specifications over the studied time interval (1970 to 1998).

The piston and bell prover flow standards as well as the instrumentation used with the meter under test have been improved over the last three decades so that discharge coefficients were measured with relative standard uncertainties of about 0.42 % in the 1970's, while in the 1990's they were measured with about 0.24 % relative standard uncertainty.

The standard deviations of the drift / year of the discharge coefficients for the two flowmeter populations were 0.07 % and 0.19 % for nozzles and laminar flowmeters respectively. The larger standard deviation for the laminar flowmeters can be largely traced to a few meters which suffered shifts of more than 1 %, perhaps due to shock damage or unnoticed dirt blocking flow tubes.

The mean drift / year of the discharge coefficients for the critical nozzle population was +0.007 % and the mean drift / year for the laminar flowmeter population was +0.017 %, i. e., both meter types showed slightly increasing discharge coefficients over time. For the critical nozzle population, some of the increase can be attributed to a subset of four nozzles which probably have increasing real throat diameters due to erosion by dirty flow. But even with this set of four nozzles removed from consideration, the critical nozzle mean drift / year is +0.003 %. If the laminar flowmeters with large calibration drifts (more than 0.3 % per year for any interval) are removed from the analysis, the mean drift / year is still positive (+0.025 %). It is not clear why the laminar flowmeter discharge coefficients increased over time: flow tube blockage would cause the opposite effect.

Transfer standards which are set up with redundancies (two flowmeters, two pressure transducers, etc.) give the user valuable tools for evaluating the calibration stability of the transfer standard. The flows delivered by the two redundant systems can be compared, and the transfer standard (and its instrumentation) can be calibrated externally when the differences are considered unacceptably large. However, even with a redundant transfer standard that agrees acceptably during internal checks, some periodic calibration by an outside laboratory is still called for. For instance, two critical nozzles could have throat erosion occurring simultaneously so that they agreed with each other reasonably well, but differed from the true flow by an unacceptable margin. Sets of calibrated flowmeters

with overlapping flow ranges can be used to achieve the desired redundancy. For these flowmeter sets, it is important that internal cross-over checks during which the same flow is measured by two meters simultaneously be conducted on a regular basis. In this way, calibration problems can be caught before they proliferate, without incurring too often the expense and downtime of sending the transfer standards for calibration by another laboratory. The average calibration interval for the laminar flowmeter population was 3.8 years while for the critical nozzles the average interval was 5.6 years. Such long intervals are reasonable as long as redundancies are used to carry out internal checks, but without the internal checks, large unsuspected errors may be present.

Acknowledgments

The quality of the flowmeter calibration results in this paper is a tribute to the people who have designed improvements, maintained the instrument calibrations, and operated the piston and bell provers in the NIST Fluid Flow Group over the past 30 years, including: Charles Collett, M. Shafer, Harley Allion, Filmer Ruegg, Ken Benson, Jim Melvin, Bill Cleveland, James Whetstone, Paul Baumgarten, George Mattingly, Mike Hall, John Houser, and Don Ward.

Appendix A: Dimensionless Quantities for Critical Nozzle Calibration

Reynolds number (Re) and discharge coefficient (C_d) are used by NIST for the presentation of calibration data for critical flow nozzles.⁷ The calculation process generally follows the methods described in ASME and ISO standards for critical nozzles.^{8, 9} The correlations used to calculate necessary properties for dry air are also given here along with uncertainties for the correlation outputs over certain input ranges.

The inputs to the process are the upstream gas temperature and pressure, T_1 and P_1 , the approach pipe diameter, D , the nozzle throat diameter, d , and the mass flow from the flow standard, \dot{m}_{std} . A recovery factor, r , of 0.75 is used to convert measured absolute temperature to the stagnation temperature, T_0 , via the equation:

$$T_0 = T_1 \cdot \left(1 + \frac{\gamma - 1}{2} \cdot M^2 \cdot (1 - r) \right), \quad (A1)$$

and the stagnation pressure, P_0 , is calculated from the equation:

$$P_0 = P_1 \cdot \left(1 + \frac{\gamma - 1}{2} \cdot M^2 \right)^{\frac{\gamma}{\gamma - 1}}, \quad (A2)$$

where γ is the specific heat ratio and M is the Mach number in the approach pipe, both based on P_1 and T_1 . The errors caused by using P_1 and T_1 instead of P_0 and T_0 to calculate M and γ are negligible for nozzle designs which follow published standards (i.e., with $D/d > 4$). To obtain the Mach number, first the specific heat ratio is calculated using the following relationship:

$$\gamma = 1.39263 + 7.915 \cdot 10^{-5} \cdot T - 1.822 \cdot 10^{-7} \cdot T^2 + \left(\frac{20.2}{T}\right)^{2.36} \cdot \left(\frac{P}{101.325}\right)^{1.015}, \quad (\text{A3})$$

where T is in K, P is in kPa, and γ is dimensionless. The standard deviation of the residuals of this correlation when compared to the specific heat ratio values tabulated in Hilsenrath *et al.*¹⁰ is 0.005 % for T between 240 K and 340 K and P between 100 kPa and 1000 kPa. The uncertainty of the experimental values given in Hilsenrath *et al.* is less than 0.02 %. Therefore the expanded uncertainty for the specific heat ratio calculated by Equation A3 is 0.041 %[#] over the specified range of T and P .

The approach pipe Mach number is calculated using the equation:

$$M = \frac{4 \cdot \dot{m}_{std}}{\pi \cdot D^2 \cdot \rho \cdot \sqrt{\gamma \cdot R \cdot T}} \quad (\text{A4})$$

In Equation A4, R is the gas constant (the universal gas constant, $R_u = 8.314471$ J / (mol · K),¹¹ divided by the gas molecular weight, 28.966 g/mol). All of the quantities in Equation A4 must have consistent units so that M is dimensionless. The gas density, ρ , can be calculated for dry air via the following correlation:

$$\rho = \frac{1}{1.23838 + 287.04 \cdot \frac{T}{P} - 3012.2 \cdot T^{-1.334} - 7.3049 \cdot 10^{-4} \cdot \frac{P}{T} + 2.5304 \cdot 10^{-2} \cdot \frac{P}{T^{1.25}}}, \quad (\text{A5})$$

where P is in kPa, T is in K, and ρ is in g/cm³. Equation A5 is based on a power function fit to second and third virial coefficient data. The standard deviation of the residuals when compared to tabulated data in Hilsenrath *et al.* is 0.007 % for T between 240 K and 340 K and P between 100 kPa and 1000 kPa. Allowing for experimental uncertainty in the tabulated data of 0.012 % gives an expanded uncertainty for the density from Equation A5 of 0.028 %.

The discharge coefficient, C_d is calculated from the expression:

[#] This and subsequent property correlation uncertainties are calculated as follows. One standard deviation (67 % confidence level) values for the experimental uncertainty and the fit equation uncertainty are combined by root-sum-square, and then multiplied by a coverage factor of 2 to attain a 95 % confidence interval uncertainty for the correlation. See Taylor and Kuyatt, 1994.

$$C_d = \frac{4 \cdot \dot{m}_{std} \cdot \sqrt{R \cdot T_0}}{\pi \cdot d^2 \cdot P_0 \cdot C^*} \quad , \quad (A6)$$

where C^* is the critical flow factor calculated using:

$$C^* = 0.68309 + 1.42025 \cdot 10^{-5} \cdot T_0 - 2.80046 \cdot 10^{-8} \cdot T_0^2 + 3.47447 \cdot 10^{-5} \cdot P_0 + \dots \\ \dots - 1.80997 \cdot 10^{-7} \cdot P_0 \cdot T_0 + 2.46278 \cdot 10^{-10} \cdot P_0 \cdot T_0^2, \quad (A7)$$

with C^* dimensionless, T_0 in K, and P_0 in kPa. Equation A7 is based on a fit to Johnson's tabulated data.¹² The residuals between Equation A7 and Johnson's tabulated values have a standard deviation of 0.006 % for T_0 between 240 K and 340 K and P_0 between 0 kPa and 1000 kPa. The residuals between Equation A7 and C^* calculated via Hilsenrath *et al.*'s specific heat ratio data have a standard deviation of 0.026 % over approximately the same P and T range and this value will be used as the uncertainty for the experimental data, giving an expanded uncertainty for Equation A7 of 0.053 %.

The Reynolds number can be calculated with the following expression:

$$Re = \frac{4 \cdot \dot{m}_{std}}{\pi \cdot d \cdot \mu} \quad , \quad (A8)$$

where μ is the gas viscosity, and with all quantities in consistent units so that Re is dimensionless. The viscosity of air can be calculated via an equation used by Hilsenrath *et al.*:

$$\mu = \left(\frac{145.8 \cdot T_0^{1.5}}{110.4 + T_0} \right) \cdot 10^{-7} \quad , \quad (A9)$$

where μ has units of Poise or $g/(cm \cdot s)$, T_0 is in K, and P_0 is in kPa. Since Equation A9 is the same function used by Hilsenrath *et al.*, the uncertainty of the fit to the tabular data is zero. The experimental uncertainty and the relative standard uncertainty of the viscosity correlation for T between 220 K and 400 K is 1.0 %. The viscosity used in the calculation of Reynolds number is based on the stagnation temperature and pressure, not the conditions at the throat (which can be calculated if adiabatic conditions are assumed). Therefore this Reynolds number has the unfortunate quality that it is based on the length scale at one location and the viscosity at another, but it is conventional to use this form due to its convenience of calculation.

The Reynolds number and discharge coefficients listed in NIST calibration reports are those given in Equations A6 and A8. However, a theoretical Reynolds number, Re_{th} is

more convenient at the time of meter usage. The theoretical mass flow, \dot{m}_{th} , can be calculated with a rearranged version of Equation A6 with C_d assumed equal to 1.0. Using the definition of the discharge coefficient

$$\dot{m}_{std} = C_d \cdot \dot{m}_{th} , \quad (A10)$$

one can calculate that,

$$\text{Re}_{th} = \frac{4 \cdot \dot{m}_{th}}{\pi \cdot d \cdot \mu} = \frac{\text{Re}}{C_d} . \quad (A11)$$

The advantage to casting the calibration data in the form of discharge coefficient versus theoretical Reynolds number is that Re_{th} can be calculated directly from known quantities, eliminating the need for an iterative process to determine C_d .

Appendix B: Dimensionless Quantities for Laminar Flowmeter Calibration

Dimensional analysis as developed by Buckingham has been performed previously for the laminar flowmeter by Allion,¹³ Baker and Schaefer,¹⁴ Ruegg and Allion,¹⁵ and by Todd.¹⁶ Assuming that the pertinent variables are density, ρ , pressure drop across the meter, ΔP , viscosity, μ , the length of the flow tubes, L , the hydraulic radius of the flow tubes, a_h , and the volumetric flow through the laminar flowmeter, \dot{V} , the following dimensionless parameters result:

$$\Pi_1 = \frac{a_h}{L} , \quad (B1)$$

$$\Pi_2 = \frac{\rho \cdot \Delta P \cdot \ell^2}{\mu^2} , \quad (B2)$$

and,

$$\Pi_3 = \frac{\dot{V} \cdot \mu}{\Delta P \cdot \ell^3} , \quad (B3)$$

where ℓ represents a length scale (a_h or L are the obvious candidates). The parameters Π_2 and the inverse of Π_3 (with L used as the length scale, and $\dot{V} = \dot{V}_{std}$, the standard flow from a calibration facility) have been previously called the *viscosity coefficient* and the *flow coefficient* respectively. Plots and tables of the flow coefficient versus viscosity coefficient are normally used in NIST calibration reports to present laminar flowmeter

calibration data. When calculating dry air density and viscosity, the previously given Equations A5 and A9 are used.

The viscosity and flow coefficients have the significant advantage that they use the length of the tubes, L , as the length scale, which is relatively easy to measure. However, other choices for the dimensionless quantities are physically more meaningful. Using the Hagan-Poiseuille equation for the theoretical volumetric flow, \dot{V}_{th} , through n_t flow tubes with circular cross section,

$$\dot{V}_{th} = \frac{\Delta P \cdot \pi \cdot a_h^4 \cdot n_t}{8 \cdot \mu \cdot L} \quad , \quad (B4)$$

and the resulting relationship for the mean velocity in each flow tube, \bar{u} ,

$$\bar{u} = \frac{\dot{V}_{th}}{\pi \cdot a_h^2} = \frac{\Delta P \cdot a_h^2 \cdot n_t}{8 \cdot \mu \cdot L} \quad , \quad (B5)$$

one can derive the following dimensionless quantities which utilize the dimensional qualifications of Π_1 , Π_2 , and Π_3 ,

$$Re_{th} = \frac{\rho \cdot (2 \cdot a_h) \cdot \bar{u}}{\mu} = \frac{\rho \cdot \Delta P \cdot a_h^3}{4 \cdot \mu^2 \cdot L} = \frac{1}{4} \cdot \Pi_1 \cdot \Pi_2 \quad , \quad (B6)$$

$$C_d = \frac{\dot{V}_{std}}{\dot{V}_{th}} = \frac{\dot{V}_{std} \cdot 8 \cdot \mu \cdot L}{\Delta P \cdot \pi \cdot a_h^4 \cdot n_t} = \frac{8}{\pi} \cdot \Pi_3 \cdot \frac{1}{\Pi_1} \quad , \quad (B7)$$

in which Re_{th} is the theoretical Reynolds number through an individual flow tube and C_d is the flowmeter discharge coefficient.

The Reynolds number and discharge coefficient are not as convenient to calculate as the viscosity and flow coefficients since obtaining the hydraulic radius and the number of tubes is difficult. But if reasonable values for these quantities can be obtained, then having the Reynolds number as the abscissa in a calibration data plot allows the meter user to see how close the calibrated flow range of the meter approaches the laminar to turbulent transition. Also, the C_d values will be nominally 1.0, allowing small departures from non-ideal behavior to be readily apparent.

The drop in pressure from the upstream to downstream side of a laminar flowmeter leads to density changes, and hence volumetric flow changes, from the upstream to downstream side of the meter. The location of the pressure measurement therefore affects the values of the laminar flowmeter dimensionless quantities through the density and volumetric flow (see equations B2 and B3). Therefore it is necessary to specify the location for

which the dimensionless quantities have been calculated. Candidate pressure locations are: 1) the upstream side of the flowmeter, 2) the downstream side, and 3) the average of these two pressures (the pressure at the middle of the laminar flowmeter). It is current practice at NIST to use the upstream pressure unless otherwise requested by the calibration customer.

References

- ¹ Wright, J. D. and Mattingly, G. E., *NIST Calibration Services for Gas Flowmeters: Small Gas Flow Facilities, NIST SP250 Supplement*, in press.
- ² Ruegg, F. W. and Shafer, M. R., *Flow Measurement: Procedures and Facilities at National Bureau of Standards*, ASHRAE, San Francisco, Ca., Jan. 19-22, 1970.
- ³ Mattingly, G. E., *Gas Flow Measurement: Calibration Facilities and Fluid Meter Traceability at the National Bureau of Standards*, Proc. Inst. Gas Technol. Conf. on Gas Flow Meas., Chicago, IL, 1986.
- ⁴ Taylor, B. N. and Kuyatt, C. E., *Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results*, NIST Technical Note 1297, 1994.
- ⁵ Wright, J. D., Mattingly, G. E., Nakao, S., Yokoi, Y., and Takamoto, M., *Intercomparison Tests of a NIST Primary Standard with a NRLM Transfer Standard for Small Mass Flow Rates of Nitrogen Gas*, Metrologia, in press.
- ⁶ Coleman, H. W. and Steele, W. G., *Experimentation and Uncertainty Analysis for Engineers*, John Wiley and Sons, 1989.
- ⁷ Benson, K. R., Cleveland, W. G., *NBS Computations for Sonic Nozzles to Determine Mass Flowrate and Discharge Coefficient*, internal National Bureau of Standards document, April, 1977, revised 1986.
- ⁸ *Measurement of Gas Flow by Means of Critical Flow Venturi Nozzles*, ASME/ANSI MFC-7M-1987, American Society of Mechanical Engineers, New York, NY, 1987.
- ⁹ *Measurement of Gas Flow by Means of Critical Flow Venturi Nozzles*, ISO 9300 : 1990 (E), International Organization for Standardization, Geneva, Switzerland, 1990.
- ¹⁰ Hilsenrath, J., Beckett, C. W., Benedict, W. S., Fano, L., Hoge, H. J., Masi, J. F., Nuttall, R. L., Touloukian, Y. S., and Woolley, H. W., *Tables of Thermal Properties of Gases*, NBS Circular 564, 1955.

¹¹ Moldover, M. R., Trusler, J. P. M., Edwards, T. J., Mehl, J. B., and Davis, R. S., *Measurement of the Universal Gas Constant R Using a Spherical Acoustic Resonator*, J of Res NBS, Vol. 93, NO. 2, March-April, 1988, pp. 85 - 143.

¹² Johnson, Robert C., *Real Gas Effects in Critical Flow through Nozzles and Tabulated Thermodynamic Properties*, NASA Technical Note D-2565, January, 1965.

¹³ Allion, H. H., *Calibration and Use of Laminar Flowmeters*, Proceedings 1965 Appalachian Gas Measurement Short Course, Univ. W. Va., 1965.

¹⁴ Baker, D. W. and Shafer, M. R., *Preliminary Notes on Laminar Flowmeters Flowing Gases*, internal National Bureau of Standards document, April, 1967.

¹⁵ Ruegg, F. W. and Allion, H. H., *An Examination of the Effects of Heat Transfer and Compressible Flow on the Performance of Laminar Flowmeters*, ASME Flow Measurement Symposium, Pittsburgh, Pa., 1966.

¹⁶ Todd, D. A., *A Universal Calibration Curve for Laminar Flowmeters*, Proceedings of the 2nd International Symposium on Flow, Instrument Society of America, St. Louis, Mo., March 23 - 26, 1981.