INTRODUCTION

An object being returned from orbit will exhibit a dispersion pattern at the landing area that is caused by uncertainties in the orbital parameters, retro-maneuver variations and other factors. Failures of the reentry systems can shift the impact area either up-range or down-range of the intended landing area. This primer provides two methods that may be used in computing impact probability and casualty expectancy for such failures.

CASUALTY EXPECTANCY

Casualty expectancy (Ec_i) for an area (A_i) is defined by the following equation as the expected number of casualties occurring during the event:

$$Ec_i = P_i\left(\frac{A_{ci}}{A_i}\right)N_i \qquad (1)$$

The total casualty expectancy (Ec_t) for all areas is:

$$Ec_t = \sum_i^n Ec_i \qquad (2)$$

Where:

The populated area exposed is A_i

The object's impact probability in A_i is P_i

The casualty area of the object to people in A_i is A_{ci}

The number of people in A_i is N_i

Casualty expectancy is highly dependent on the probability of failure, computation of the probability of impact (P_i) and the numbers of people exposed in a given area. In general, use of the variables cited above requires judgment as to when approximations can be used without over or underestimating the risks that can occur.

PROBABILITY OF IMPACT

Most impact dispersion areas exhibit a bivariate normal distribution. The general equation for this distribution is shown below.

$$P = \frac{1}{2\pi\sigma_x\sigma_y}\int\int[-1/2(\frac{x^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2})]d_x d_y ....(3)$$

Two methods used for computing P_i are illustrated in Figures 1 and 2. The first method uses the basic equation for a bivariate distribution and the second illustrates a method used when the exposure is time dependent. This computation can take many forms other than the examples shown, however, the general equations are applicable.

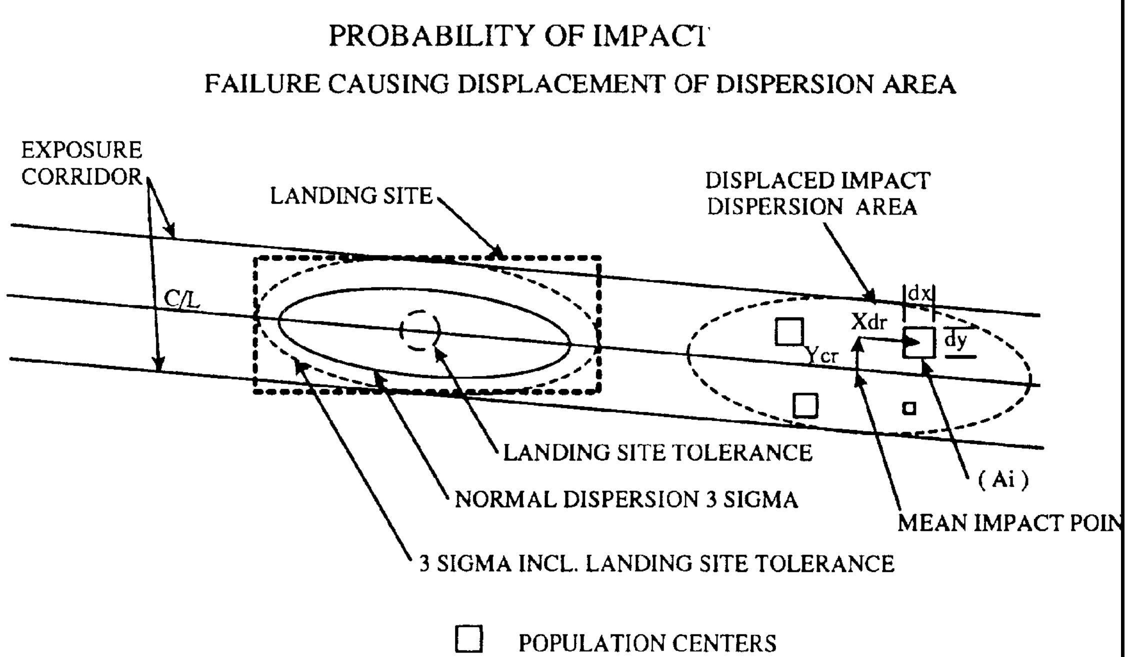
Figure 1 gives an illustration of a failure that produces a finite displacement of the impact dispersion area in the down-range direction and Figure 2 illustrates a time variant displacement based on the time of failure. A finite displacement of the impact dispersion can be caused by various systems failures such as timing system failure. A time dependent shift can be caused if the retro-motor, for example, should fail at various times during its burn period.

A problem always addressed by risk analysts is when approximations, such as averaging of the numbers of people exposed, can be applied without biasing the results. If the dispersion area of exposure is very large but contains highly concentrated populated areas well away from the mean point of impact, then use of average population densities may tend to overestimate the risk. If the dispersion area of exposure is small and does not contain any significant populated areas, using average population density for a larger region may tend to underestimate the risks. If there are significant population centers near the mean point of impact, averaging will usually underestimate the risks. Hence, one must examine the exposed areas and make reasoned judgments on when approximations can be used.

Shown in Figure 1 is a planned landing site showing the normal 3 Sigma dispersion area of the reentry vehicle. Also shown is the landing zones tolerance to variations of the mean point of impact that the site can contain. This tolerance is necessary to accommodate the variations in de-orbit opportunities which occur. The 3 Sigma dispersion area is increased by including the landing site tolerance which also defines an exposure corridor both up and downrange of the landing site.

The methodology depicted in Figure 1 can be used for determining the Probability of Impact (P_i) when the shifted dispersion area contains significant population centers. For other rural areas exposed, the population density can be approximated by removing the city populations and computing the average population density for the remaining population in the region. If the dispersion area does not contain any major population centers, the average population density can be derived as described above or the region's average population density including cities can be used as a conservative

estimate for computing E_c.



$$P_i = \frac{P_f * dx * dy}{2 \pi \sigma_x \sigma_y} \quad EXP \left[-1/2 \frac{(Xdr)^2}{(\sigma_x)^2} + \frac{(Ycr)^2}{(\sigma_y)^2} \right]$$

WHERE:

C/L IS THE CENTERLINE OF THE EXPOSURE CORRIDOR

Pi IS THE PROBABILITY OF IMPACT ON AREA Ai FOR A SINGLE OBJECT

P (IMPACT OF AT LEAST ONE OF N OBJECTS) = $N [P_i - (N - 1)/2 * P_i^2]$ USED IF MORE THAN ONE OBJECT

Pf IS THE PROBABILITY OF FAILURE CAUSING THE IMPACT DISPLACEMENT

Ai IS THE AREA DEFINED BY dx * dy

Xdr AND Ycr ARE CROSS AND DOWNRANGE DISTANCES FROM THE MEAN TO THE CENTER OF Ai

dy AND dx SHOULD BE LESS THAN σ_y AND σ_x

Figure 1. DISPLACED DISPERSION PROBABILITY OF IMPACT

EXAMPLE 1

This example will use the methodology depicted in Figure 1 and Equations (1) and (2). The following conditions are established for the example:

1. **The probability of a failure causing a down-range shift is = 0.001**
2. **The dispersion area is defined as a bivariate normal distribution centered on the mean point of impact with σx = 10 s.mi. and σy = 4 s.mi.(s.mi. is statute miles)**
3. **There are three cities in the dispersion area as follows:**
   a. **City 1:**
      Population N1 = 200,000
      City Area A1 = dx (4 s.mi.) x dy (3 s.mi.) = 12 square miles.
      Distance to centroid of A1 is Xdr (+ 4 s.mi.) and Ycr (+ 5 s.mi.)
   b. **City 2:**
      Population N2 = 50,000
      City Area A2 = dx (2 s.mi.) x dy (2 s.mi.) = 4 sq.mi.
      Distance to centroid of A2 is Xdr(-5 s.mi.) and Ycr (-7 s.mi.)
   c. **City 3:**
      Population N3 = 30,000
      City Area A3 = dx (2.5 s.mi.) x dy (1 s.mi.) = 2.5 sq.mi.
      Distance to centroid of A3 is Xdr (+ 15 s.mi.) and Ycr (- 1 s.mi.)

4. **The total 3 σ area exposed is 60 x 24 miles (1,440 sq. mi.) and contains 14,400 people in addition to the three cities above.**

5. **The casualty area of two objects is 30 sq. ft. for each object.**

The results for Example 1 are shown below in Table 1.

	P_i x (10^{-5})	E_c x (10^{-7})
CITY 1	4.0	7.2
CITY 2	0.61	0.82
CITY 3	0.63	0.81
REMAINING AREA	194.8	0.21

	P_i x (10^{-5})	E_c x (10^{-7})
TOTAL	200	9.1
AVERAGE	200	4.4

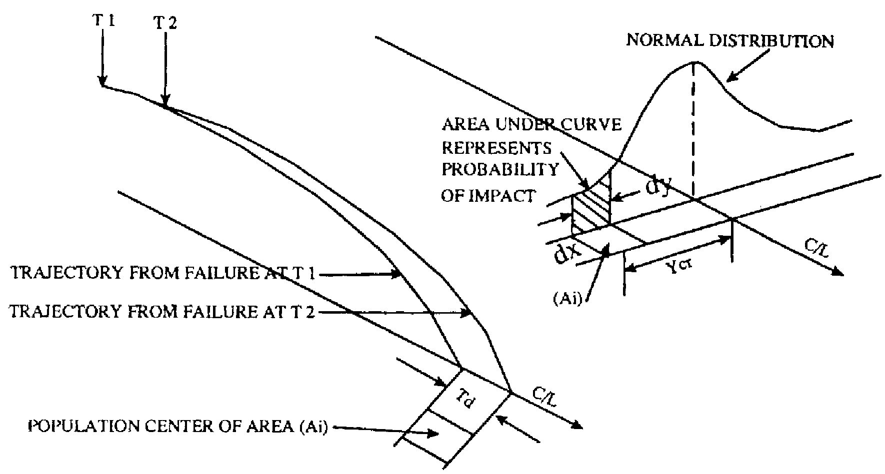
TABLE 1. EXAMPLE 1 PROBABILITY OF IMPACT AND CASUALTY EXPECTANCY

Shown in Table 1 is the total casualty expectancy determined using the equations cited. The casualty expectancy was also computed using the total population averaged over the entire exposed area (AVERAGE). This illustrates the tendency to underestimate the risk by averaging the population over the area exposed.

The methodology depicted in Figure 2 can be used when the time dependent exposure area sweeps over various populated regions. When the distance exposed as a function of time is small (50-200 miles/sec.) and the exposure corridor narrow, any significant population centers exposed should be considered independently and the average populations of other areas included as described above. If the distance exposed as a function of failure time is large and no major cities or metropolitan areas are near mean of the exposure corridor, average population densities may be acceptable. In general, the analyst should perform sample calculations to determine the effect of cities before concluding that use of average population densities is appropriate.

PROBABILITY OF IMPACT

FAILURE CAUSING TIME DEPENDENT DISPLACEMENT OF DISPERSION AREA



$$P_i = \frac{\dot{P}_f * T_d * dy}{\sqrt{2\pi}\,\sigma_y} \; EXP\left[-1/2 \; \frac{(Y_{cr})^2}{(\sigma_y)^2}\right]$$

WHERE:

P_i IS THE PROBABILITY OF IMPACT IN AREA A_i

$\dot{P}_f$ IS THE FAILURE RATE PROBABILITY PER SECOND

A_i IS THE AREA OF THE POPULATION CENTER CONTAINED IN AREA $dx * dy$

Y_{cr} IS THE CROSS RANGE DISTANCE TO THE CENTER OF dy

T_d IS THE DWELL TIME OF EXPOSURE TO A_i

dy SHOULD BE LESS THAN σ_y

Figure 2. TIME DEPENDENT PROBABILITY OF IMPACT

EXAMPLE 2

This example will use the methodology depicted in Figure 2 and by Equations (1) and (2). This methodology is applicable to the failure of a retro-motor during its thrusting period. The following conditions are established for this example.

1. The probability of failure of the retro-motor is 0.01 and the burn time of the motor is 16 seconds. The failure rate probability (Pf) assuming a uniform failure distribution is then:

   0.01 x 1/16 = 6.25 x 10^{-4}/sec.

2. The acceleration caused by the retro-motor during burn is 40 ft./sec.^{2} and the range shift sensitivity near the landing site is 6 s.miles/ft/sec. If the retro-motor failed during the last second of burn, the area exposed would be (1sec. x 40ft./sec.^{2} x 6s.mi./ft./sec. = 240 miles) long.

3. The cross range dispersion is a normal distribution that is centered on the ground track center line (C/L) with σy = 4 s.miles.

4. There are three cities contained in the exposure area as follows:

   a. City 1:
      Population = 50,000
      dy = 2, dx = 2 and Ycr = 1 s.miles

   b. City 2:
      Population = 200,000
      dy = 3, dx = 3 and Ycr = -10 s.miles

   c. City 3:
      Population = 800,000
      dy = 3, dx = 4 and Ycr = 3 s.miles

   d. The total area exposed for this 1-second period is 240 s.miles x 24 s.miles = 5,760 sq. s.miles. This area contains a population of 100,000 people in addition to the cities.

e. The casualty area of the re-entry object is 30 sq.ft.

The results from this example are shown in Table 2 below. Shown in Table 2 is the total casualty expectancy determined using the equations cited. The casualty expectancy was also computed using the total population averaged over the entire exposed area (AVERAGE). This example also illustrates the tendency to underestimate the risk by averaging the population over the area exposed.

	P_i x (10^{-6})	E_c x (10^{-7})
CITY 1	1.0	0.13
CITY 2	0.1	0.24
CITY 3	2.4	1.7
REMAINING AREA	621.5	0.12
TOTAL	625	3.1
AVERAGE	625	1.3

TABLE 2. EXAMPLE 2 PROBABILITY AND CASUALTY EXPECTANCY

This type of failure during the retro-motor burn will cause the re-entry vehicle (RV) to remain in orbit for failures early in the burn period and cause impacts down range of the landing site for failures later during the motor burn period. When the RV has been decelerated sufficiently to re-enter and impact, the initial impact distance will be approximately half way around the world down range from the landing site. As deceleration continues, the impact distance down range moves rapidly back toward the landing site until burn-out of the motor occurs. To determine the total risk from such a failure requires the process above be completed for each time interval and the risks for each summed.

SUMMARY

Presented above are two methods that may be used in computing risks for reentry vehicles. Obviously, there may be a number of failures which can also produce risk to areas outside the landing area. Once all the failures and their risks have been determined, they can be summed to arrive at the total mission risk for all areas.

The effects on risk of averaging various population data over large or small areas can lead to over or underestimating the risks as described and illustrated in the examples provided. The exposure area should be carefully examined and the risk sensitivity from major population centers tested before concluding that average population densities can be used to estimate the risk.