Privatizing Versus Prefunding Social Security in a Stochastic Economy

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Abstract

It is well known that, absent considerations about progressivity, privatizing Social Security (i.e., moving to a defined-contribution system) and prefunding Social Security's existing defined-benefit structure will lead to an identical reduction in unfunded liabilities within a deterministic economy with perfect capital markets. This paper shows that this equivalence no longer holds in a stochastic OLG production economy with guaranteed benefits because the guarantee alters the intergenerational risk-sharing properties of the two policies. At the current contribution rate, prefunding the existing defined benefit would reduce ex-ante risk-adjusted unfunded liabilities by 130 percent. By contrast, moving to a privatized system that guarantees a minimum benefit equal to what people would have received under Social Security would reduce ex-ante unfunded liabilities by only 36 percent. This is despite the fact the expected average benefit in the privatized system is almost a full order of magnitude larger than the minimum benefit. The stark difference between the two policies is robust to a wide range of alternative policy designs including different transition mechanisms, guarantee levels and contribution rates.

The critical difference between privatization and prefunding is that, while both policies insure against downside risk, prefunding also passes the upside potential to future generations. Ex ante, this feature is very effective in compensating future generations for the downside risk they face, provided that the contribution rate is set high enough. By contrast, the same high contribution rate in a privatized system with guarantees is not very effective in reducing risk faced by future generations. While a higher contribution rate means that each dollar invested in the new private accounts requires much less insurance from future generations in order to satisfy the minimum benefit, the sheer number of dollars being insured is larger. The net impact on unfunded liabilities is quite small because a higher contribution rate is an ineffective hedging device. Although privatization leads to a higher expected retirement benefit than prefunding, this comes at a zero-sum cost to future generations in the form of higher risk-adjusted unfunded liabilities.

The comparative advantage of prefunding the existing defined benefit in reducing unfunded liabilities, however, critically assumes that the policymakers can credibly "lock in" the associated trust fund, making it unavailable to finance new spending (including increasing future benefits or other spending) or tax cuts. It also assumes that the government earns market rates on its investments and does not use political criteria in selecting stocks and bonds for its portfolio. Considerable caution should, therefore, be exercised in interpreting these mathematical results; alternative assumptions about the political economy could change the results.

Key words: Social Security Privatization, Risk and Insurance, Performance Guarantees
JEL Codes: D10, G10, G11, E60, E20

I. INTRODUCTION

Privatizing Social Security would subject a greater amount of retirement saving to market risk by moving to a system of personal accounts invested in the capital market. Indeed, there have been 15 years in this century alone in which the real value of the US stock market fell over 40 percent in the succeeding decade (Figure 1).[1] Moreover, some economists believe that the market might fall another 40 percent during the next 10 years.[2]

But many observers have concluded that the US government would never force individuals in a privatized system to bear the risk themselves (see, e.g., Aaron [1997]; Bosworth and Burtless [1997]; Diamond [1998]). At a minimum, a tacit performance guarantee on these new private accounts would exist since the government cannot credibly commit to allow people to retire with little means of survival. The basic question, therefore, is not whether a privatization plan should incorporate a safety net but whether policymakers recognize that it would already be there.

In every major privatization plan implemented, a minimum guarantee has, in fact, been made explicit. The modern pension systems in Chile and El Salvador, reflecting the most ambitious privatizations to date, promise to "top up" under-performing private accounts so that they produce a rather generous minimum retirement benefit.[3] Australian retirees are guaranteed to receive at least as much as they would have under the previous system. The World Bank's (1994) privatization plan also recommends a generous guarantee. In each of these scenarios, the minimum benefit level is

[1] These include 1908-'12, 1937, 1939, 1965-'66 and 1968-'73. Not all drops, however, have been sudden. The stock market declined by about 50 percent in real value between 1973 and 1975. The market did not return to its pre-1973 level for almost 10 years. Shiller (1996) provides an overview of market risk.

[2] Campbell and Shiller (1998) argue that, contrary to the efficient market hypothesis, the price-dividend ratio is a powerful predictor of future prices. They predict a 38 percent loss in the real value of stocks over the next decade.

[3] These governments address certain obvious moral hazard concerns by penalizing fund managers that under-perform relative to the mean. This, in turn, generates near-identical competing broad-based funds (Diamond and Valdés-Prieto [1994]). Future taxpayers are still exposed to considerable risk, however, if the mean return underperforms.

financed on a pay-as-you-go basis.

Despite the importance of performance guarantees, no study has systematically explored their economic implications. To carry out that analysis, the actuarial value of a minimum benefit would have to be taken into account. In addition, one should compare how privatizing Social Security with a minimum benefit would perform relative to the popular alternative policy proposal of prefunding the existing defined benefit. In a deterministic world, it is well known that the two policies are equivalent. This is no longer true in an uncertain world. While both policies insure against downside risk on a pay-as-you-go basis, prefunding the Social Security's current "fixed" (wage-indexed) benefit passes the upside potential associated with equities to future generations.

This paper shows that realistic minimum performance guarantees are quite costly and incorporating them explicitly into the analysis fundamentally alters the conventional wisdom that privatizing Social Security will significantly reduce unfunded liabilities. Privatization may reduce unfunded liabilities only a modest amount and it might even increase liabilities. This is true even when the contribution rate in the new system is set at a high enough level such that the expected benefit level in the privatized system is a full order of magnitude larger than the minimum benefit.

By contrast, prefunding Social Security's existing defined benefit can substantially reduce unfunded liabilities, especially at higher level contribution levels. For example, at a 10 percent contribution rate in the new system, I estimate that, while privatization leads to only a 35 percent reduction in unfunded liabilities under benchmark parameters, prefunding the current defined benefit reduces liabilities by 68 percent. With a 15½ percent contribution rate, privatization reduces unfunded liabilities by only 36 percent while prefunding the current defined benefit reduces unfunded liabilities by almost 110 percent. The fundamental difference comes from the fact that while both policies insure against downside risk on a pay-as-you-go basis, prefunding the existing defined benefit passes the potential upside to future generations as compensation. This compensation is particularly valuable at high contribution levels.

Section II introduces a stochastic two-period OLG model with production. Individuals face stochastic returns to both equities and wage-indexed Social Security along with a risk-free interest rate. The use of a formal model allows all of the calculations to be utility based and it recognizes that the historic experience of equity returns represent draws from an underlying distribution whose tails include extreme events beyond those found in the historic series. Evaluating the associated risks in a utility context, as well as properly characterizing the *moments* of the underlying distribution of stock returns, is very important. This is because the empirical rarity of extreme downturns in stock prices does not imply that equities have little risk. For example, Jagannathan and Kocherlakota (1996) have shown that, even for a 40-year holding period in which stocks outperform bonds with *almost* 100 percent certainty, the optimal portfolio consists of 60 percent bonds and 40 percent stocks for a person with a reasonable aversion to risk.

Sections III and IV investigate the impact that prefunding Social Security's current defined benefit as well as privatization, respectively, have on unfunded liabilities. These sections focus on the relatively straightforward case in which the risky assets in the new system are expected to exactly fund benefits equal to those that would be received under Social Security. This allows for a fair comparison between pay-as-you-go Social Security and privatization versus prefunding the current defined benefit. It also helps develop the basic intuition behind the costly nature of performance guarantees. A stochastic version of the well-known Modigliani-Miller equivalence between debt and pay-as-you-go financing is derived for the case of prefunding the existing defined benefit. This result demonstrates the costly nature of performance guarantees when the contribution rate in the new system is low. It is also demonstrates how privatization can actually *increase* unfunded liabilities when the new accounts are designed to exactly replace Social Security benefits. Several policy designs are considered—none of which lead to significant reductions in unfunded liabilities.

Section V generalizes the model to allow for a broad set of policy options including a range of guarantee levels and contribution rates. Guaranteeing a benefit level smaller than that provided by Social Security reduces the liabilities placed on future generations. Mandating a high contribution level, so that the new accounts are expected to more-than-replace Social Security benefits, is an indirect form of means testing. At first blush, one might think that this would also reduce the risk placed on future workers. This section demonstrates that higher contribution rates are indeed very effective in reducing unfunded liabilities for prefunding the existing defined benefit but not for privatization. This result holds even when the contribution rate is set at a high level—one in which the expected benefit level in the privatized system is a full order of magnitude larger than the guaranteed benefit level. To be sure, a higher contribution level means that each dollar invested in the new private accounts requires much less insurance from future generations in order to satisfy the minimum benefit. But the sheer number of dollars being insured is larger. This latter effect nearly offsets the former. And these calculations assume that policymakers do not guarantee a level of benefits in excess of Social Security. This might be unrealistic in a privatized system with a high expected benefit. If policymakers choose to guarantee a benefit equal 150% of Social Security's average benefit, privatization would *increase* the ex-ante value of unfunded liabilities by 3 percent at a 10 percent contribution rate. By contrast, high contribution levels are effective in reducing unfunded liabilities for prefunding the existing defined benefit level because the high expected value of assets effectively pays future generations for the insurance they provide against downturns.

Section VI concludes.

II. A TWO-PERIOD OLG MODEL

The framework is a standard two-period OLG (Diamond) model with production subject to various shocks. There are *m* possible sources of exogenous shocks, $s_t \in S \subset \mathbb{R}^m$, in each period where the set *S* is assumed to be compact. Without loss in generality, these shocks are assumed to be i.i.d. with a stationary distribution function $Z\colon\ S \rightarrow [0,1]$ such that $Z(s') \equiv Pr\,(s_{t+1} \leq s')$.[4] For any bounded, continuous function, $h\colon\ S \rightarrow \mathbb{R}$, the function $\int_S h(s')Z(ds')$ is assumed to be continuous.

Consumers

Consumers live for two periods and a consumer born at time *t* is assumed to have the following preference over a sequence of consumption, $\{C_{j,\,t+j-1}\}_{j=1}^{2}$:

(1)
$$E_t\left\{\sum_{j=1}^{2} \lambda^j U(c_{j,t+j-1})\right\}$$

where the utility function $U\colon\ \mathbb{R}_+ \rightarrow \mathbb{R}$ is strictly concave, strictly increasing, and continuously differentiable. The term $\lambda \in (0\,,1)$ is the subjective discount factor which is assumed to be state and time invariant. Purely for illustrative simplicity, preferences are modeled as additively separable. However, the analytical results herein do not require additively separable preferences. The expression $E_t(\cdot)$ is expectations operator and is defined as $E_t[h(s')] = \int_S h(s')Z(ds')$.

The agent earns a labor income equal to $w_t \equiv w(s_t)$ at age 1 known at time *t* and retires in period 2. The agent can invest in equities that pays a state-contingent rate of return equal to $e_{t+1} \equiv e(s_{t+1})$ at the beginning of period *t*+1. The agent can also invest in bonds that pay a constant

[4] Allowing for a more general Markov process is straightforward by defining *Z* over the product space: $Z\colon\ S \times S \rightarrow [0,1]$ such that $Z(s,s') \equiv Pr\,(s_{t+1} \leq s' \,|\, s_t = s)$.

riskless rate of return equal to r known at time t. The individual faces a tax on labor earnings equal to $\tau_t \equiv \tau^{ss} + \tau^g + \tau_t^c$ where τ^{ss} is the Social Security payroll tax, τ^g is rest-of-government taxes and $\tau_t^c \equiv \tau^c(s_t)$ is a state-contingent labor tax described in more detail below (currently set to zero). The Social Security payroll tax, τ^{ss}, is invested into a wage-indexed pay-as-you-go Social Security asset that pays a state-contingent rate of return equal to the growth rate of wages, $g_{t+1} \equiv w(s_{t+1}) / w(s_t) - 1$. Second-period consumption, therefore, equals,

(2)
$$c_{2,t+1} = \left[(1-\tau_t)w_t - c_{1,t}\right]\cdot\left[(1+r)\alpha + (1+e_{t+1})(1-\alpha)\right] + b_{t+1}$$
$$b_{t+1} = \tau^{ss}w_t(1+g_{t+1})$$

where α ($0 \le \alpha \le 1$) is the share of private assets invested into bonds while (1-α) is the share of assets invested in equities. Time subscripts for these portfolio share variables are omitted. Social Security benefits, b_{t+1}, equal the expression shown in equation (2) with pay-as-you-go financing.

The consumer's first-order conditions are given by

$$E_t\left[M_t(1+r)\right] = E_t\left[M_t(1+e_{t+1})\right] = 1$$

where

(4)
$$M_t \equiv \frac{\lambda U'(c_{2,t+1})}{U'(c_{1,t})} \quad .$$

This paper focuses on an Arrow-Debreu economy with complete and perfect markets.[5] This same assumption is implicitly made in most analysis of proposals to privatize or prefund Social Security. This assumption implies that government involvement in financial markets neither directly

[5] However, it is only assumed that the generation born at time t knows its own initial endowment (w_t) and those of generations born before time t.

improves nor harms the efficiency of these markets. It also implies a lack of arbitrage opportunities, either across the population or across time, since all agents place the same premium at the margin on a particular gamble. More formally, for any bounded, continuous function, $h\colon\ S \rightarrow \mathbb{R}$,

$$E_t\left[M_t \cdot h(s_{t+1})\right] = E_t\left[M_{t+j} \cdot h(s_{t+1})\right] \qquad j \geq 1 \tag{5}$$

In reality, of course, markets are imperfect and incomplete in many ways. Market failure arguments in favor of Social Security include the possible absence of real fair annuities, problems associated with insuring the risk associated with a varying working life, and non-tradeable human capital (Diamond [1977]; Merton [1983]). Some people also face borrowing constraints (Zeldes, 1989). And future generations might be over-exposed or under-exposed to contemporaneous shocks under the baseline policy.[6] The correlation between demographics and asset prices is also likely to be important over the next few decades (Brooks, 1996) as are many other sources of non-technological risks in equity prices. Including all of these market failures and risks would require an empirical examination over all households over all time. It would also require using a large-scale simulation model since it is unlikely that the two-period model developed herein—in which both lending and risk- sharing markets between living agents are closed—is able to adequately simulate these market imperfections. But introducing more periods (much beyond three) runs into the well-known "curse of dimensionality" associated with solving large-scale models with aggregate uncertainty and stochastic steady states.

It is unlikely, however, that introducing more complexities would undermine the stark

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[6] The government's baseline policy (including Social Security, Medicare, progressive taxation and counter-cyclical deficits, etc.) already exposes future workers to a tremendous amount of risk that is both correlated and orthogonal to the risks associated with performance guarantees. Both types of risks—not just correlated risks—are important in assessing both the willingness to accept additional correlated risk and the amount of additional orthogonal risk that is desirable.

differences in unfunded liabilities generated by privatization versus prefunding the current defined benefit. This is because these differences are traced to fundamental discrepancies in how these two policy initiatives share risk between generations, a distinction that is likely to be robust to considerable model enhancements. Nonetheless, the exact numerical computations should be interpreted with some caution.

Production

Net output (national income) is given by

(6) $$Y_t \equiv F(K_t, L_t, s_t) \equiv L_t f(k_t, s_t) \quad .$$

Market clearing requires $k_t = \left[(1-\tau)w_{t-1} - c_{1,t-1}\right]\cdot(1-\alpha)$ where, without loss in generality, all debt is assumed to help finance government consumption . In the special case of Cobb Douglas, factor incomes are given by,

(7) $$w_t = (1-\theta)A_t k_t^{\theta}$$

(8) $$e_t = \theta A_t k_t^{\theta-1} - \delta_t$$

where θ is the capital share, A_t is the state of technology and δ is the contingent depreciation rate.[7] If only A is random (as in, e.g., Stokey and Lucas, 1989) then wages and equity returns are perfectly correlated. If only δ is random (as in, e.g., Gordon and Wilson, 1989) then wages are uncorrelated with equity returns. Allowing for both (as in Bohn, 1998) generates an intermediate scenario.

Social Security and Rest of Government

[7] As emphasized below, the exact choice of preferences and technology is not important since all of the calculations herein focus on privatizing or prefunding the benefit associated with the marginal dollar in the wage tax base. These calculations are compatible with any choice of preferences and technology that generate balanced growth with observable market prices and covariances.

Social Security's pay-as-you-go design requires that contemporaneous benefits equal receipts:

(9) $$b_{t+1} = \tau^{ss} w_t (1 + g_{t+1}) = \tau^{ss} w_{t+1}$$

Government debt evolves according to

(10) $$D_{t+1} - D_t = G_t + r D_t - \tau^g w_t$$

where G is government consumption. Government debt and consumption is adjusted to defend the risk-free rate (as in, e.g., Bohn, 1998).

III. A FIXED BENEFIT GUARANTEE: PREFUNDING SOCIAL SECURITY'S DEFINED BENEFIT

Some have advocated prefunding Social Security, either partially or fully, as an alternative to privatization so that benefits remain defined by law instead of being subject to market risk as in privatization (Social Security Advisory Council, 1996). Equivalently, the government could privatize Social Security but fix benefits at their wage-indexed levels by subsidizing/taxing all returns to equities that produce a benefit below/above the fixed level. A confiscatory tax on excess returns above that necessary to finance the fixed benefit level is what fundamentally distinguishes prefunding the current defined-benefit system from privatization. For comparative purposes, it is convenient to model prefunding in this second way. This section explicitly models this tax and so we can conceptually think of the assets residing in "individualized accounts," similar to a privatized system, with this tax in place.

As Feldstein (1997) notes, there are really only two basic methods of transition from a pay-as-you-go public pension system to a prefunded system: (1) transition with some recognition of

Social Security taxes paid ("recognition bonds"[8]), or, (2) substitution from pay-as-you-go financing to prefunding with an add-on tax used to finance the new funded system. The first approach was essentially the method employed in several Latin American countries. This approach has also been simulated for the U.S. economy by Kotlikoff (1996) and Kotlikoff, Smetters and Walliser (1997) using different choices of tax bases to service and pay off the bonds. The second approach, the "substitution" method, has been analyzed in Feldstein and Samwick (1997) who demonstrate that a payroll tax of only about 2.0 percent, if invested in the private capital market, would, in expectation, substitute for Social Security by rendering the same retirement income that the pay-as-you-go OASDI payroll tax (currently 12.4 percent but eventually increasing to 18.75 percent under current law-benefits) provides.[9] This is because the social marginal product of capital has historically exceeded the growth rate of the pay-as-you-go tax base by a large margin.

Both transition methods lead to the same new long-run economy if the principal of the bonds are paid off, or if the value of the bonds are originally set to zero. They are different, however, if, in the former case, the bonds are serviced but the principal is never paid off. I consider this case in order to derive the stochastic version of the well-known Modigliani-Miller equivalence between debt and pay-as-you-go financing. I then consider the substitution method (or, equivalently, when the bonds are set equal to zero in value).

<u>A Stylized Prefunding Proposal Using Recognition Bonds</u>

The experiment begins at time t = 1. Generation-0 agents, the current elderly, receive

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[8] In a standard defined-benefit system (i.e., not taking the "individualized accounts" interpretation), recognition bonds would be replaced with regular bonds.

[9] New workers face a 14.4 percent combined payroll tax in the first year. The tax rate decreases over time as Social Security recipients die. The payroll tax rate becomes 14.20 percent after five years, 13.81 percent after 10 years, 10.71 percent after 25 years, 4.69 percent after 55 years and eventually 2 percent (Feldstein and Samwick, Table 2).

benefits under the current pay-as-you-go Social Security system. These benefits are paid for by generation-1 agents—new workers—who, in turn, receive a recognition bond, B_t, in place of future pay-as-you-go benefits. The exact choice for B_t will not matter in the presence of a benefit fixed at the wage-indexed level. Because, however, many plans to prefund Social Security—whether it be expanding the trust fund or privatization—attempt to exploit the equity premium, it is convenient to calculate each recognition bond implicitly as $E_t[B_t \cdot (1+e_{t+1})] = E_t[\tau^{ss} w_t \cdot (1+g_{t+1})]$, or, in explicit terms,

(11)
$$B_t = \frac{(1+\bar{g}_{t+1}) \cdot \tau^{ss} w_t}{1+\bar{e}_{t+1}}$$

where $\bar{g}_{t+1} = E_t(g_{t+1})$ and $\bar{e}_{t+1} = E_t(e_{t+1})$. In words, the recognition bond equals the benefits that an agent expects to receive under Social Security, discounted using the expected rate of return to equities. Conceptually, we can think of this bond as being deposited into a new individual retirement account invested in the capital market.[10] The bond is *expected* to yield the same retirement benefits as the agent would receive under Social Security. However, the risk-adjusted value of the recognition bond is only about 13½ of the value of Social Security's unfunded liability under our benchmark parameters (Section V). Hence, we have not simply made "implicit debt explicit" in the usual sense.

The generation-2 cohort is the first one that does not face the payroll tax. Instead, they make growth-adjusted interest payments on the recognition bonds. Generation-2 agents also insure that generation-1's recognition bonds, invested in the capital market, perform exactly as expected—i.e.,

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[10] If generation-1 agents instead invested their recognition bond in the risk-free asset then generation-2 agents would have to *deterministically* top up generation-1 agents benefits by exactly the difference in the value of Social Security's unfunded liability less the value of the recognition bond; or, equivalently, the bond would have to be calculated using the riskless rate with the same effects.

Social Security benefits are exactly replaced. This fixed guarantee is accomplished with the government's ability to subsidize/tax equity returns faced by generation *t* by taxing/subsidizing the subsequent generation, *t*+1: returns below expectation are topped up while returns above expectation are taxed at a confiscatory rate. This *contingent* payroll tax, τ^c, levied on a growth-adjusted tax base, therefore, equals, at time *t*,

$$\tau_{t+1}^{c} = \frac{\tau_t^{ss} w_t (1+g_{t+1}) - B_t (1+e_{t+1})}{w_{t+1}} \tag{12}$$

Note that $E_t(\tau_{t+1}^{c} w_{t+1}) = 0$, i.e., equities will replace wage-indexed Social Security benefits on *average* without the need for more revenue. This may give the guarantee an appearance of being low cost.

Future generation-(t>2) agents continue to make growth-adjusted interest payments on the initial recognition bonds. They also insure that the previous generation receives the same guarantee on the same level of investment. In this way, each generation retires with a value of income that exactly replaces Social Security.

Denote the change in the remaining lifetime expected-value budget constraint for generation-*t* agents as Δ_t. Prefunding Social Security's defined benefit with recognition bonds and a fixed (wage-indexed) rate-of-return guarantee changes each generation's budget-constraint as follows:

$$(13) \qquad \Delta_{t=0} = 0$$

$$\begin{aligned}
\Delta_{t=1} &= -E_t\left[M_t \cdot \tau^{ss} w_t (1+g_{t+1})\right] + B_t + E_t(M_t \cdot w_{t+1}\tau^c_{t+1}) \\
&= -E_t\left[M_t \cdot \tau^{ss} w_t (1+g_{t+1})\right] + B_t + E_t\left[M_t \cdot \tau^{ss} w_t (1+g_{t+1})\right] - B_t \cdot E_t\left[M_t \cdot (1+e_{t+1})\right] \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\Delta_{t\geq 2} &= \left[B_t - (1+r)B_{t-1}\right] + E_t\left[\tau^{ss} w_t \left\{1 - M_t \cdot (1+g_{t+1})\right\}\right] + \left\{E_t\left[M_t \cdot w_{t+1}\tau^c_{t+1}\right] - E_{t-1}\left[(1+r)M_{t-1} \cdot w_t \tau^c_t\right]\right\} \\
&= \left[B_t - (1+r)B_{t-1}\right] + E_t\left[\tau^{ss} w_t \left\{1 - M_t \cdot (1+g_{t+1})\right\}\right] + E_t\left[M_t \cdot \tau^{ss} w_t (1+g_{t+1})\right] \\
&\qquad - E_t\left[M_t \cdot B_t (1+e_{t+1})\right] + (1+r)E_{t-1}\left[M_{t-1} \cdot \tau^{ss} w_{t-1} (1+g_t)\right] \\
&\qquad + (1+r)E_{t-1}\left[M_{t-1} \cdot B_{t-1}(1+e_t)\right] \\
&= \left[B_t - (1+r)B_{t-1}\right] + E_t\left[\tau^{ss} w_t \left\{1 - M_t \cdot (1+g_{t+1})\right\}\right] + E_t\left[M_t \cdot \tau^{ss} w_t (1+g_{t+1})\right] \\
&\qquad - B_t E_t\left[M_t \cdot (1+e_{t+1})\right] + E_t(\tau^{ss} w_t)\, E_{t-1}\left[M_{t-1} \cdot (1+r)\right] \\
&\qquad + (1+r)B_{t-1}\, E_{t-1}\left[M_{t-1} \cdot (1+e_t)\right] \\
&= 0
\end{aligned}$$

The change in the expected budget-constraint for the initial elderly, $\Delta_{t=0}$, equals zero since, by construction, the initial elderly continue to receive full benefits under the current system.

The change in the expected budget-constraint for the initial workers, $\Delta_{t=1}$, is more involved. The first term in the first equality reflects the present value of the loss of benefits resulting from terminating the pay-as-you-go financed benefit after these workers paid into the system. The second term in the first equality reflects the recognition bond that generation-1 workers receive as partial compensation for the loss of future benefits. The third term in the first equality equals the value of the guarantee that an investment of the size of the recognition bond invested in equities will perform exactly as expected. This term is equal to the risk premium associated with the zero-mean gamble posed by the contingent tax given in equation (12). This contract has a zero price only under risk neutrality ($r = \bar{e}$). The second equality in $\Delta_{t=1}$ comes from inserting the equation (12) for the contingent tax, τ^c, applying the rule of independent expectations where appropriate and rearranging terms. The third equality comes from the first-order conditions, (3).

The change in the expected value budget-constraint for future workers, $\Delta_{t\geq 2}$, is a little more complex. The first term in the first equality reflects the fact that generation-($t\geq 2$) agents are required to make growth-adjusted interest payments on the recognition bonds; the per-capita debt itself remains constant. The second term in the first equality equals the *net* value of eliminating Social Security. Finally, the third term in the first equality equals the *net* value of the rate-of-return guarantee. This is equal to the value of the guarantee received from the following generation less the value of the guarantee that generation-($t\geq 2$) agents must provide to the preceding generation (shown in equation $\Delta_{t=1}$). The subsequent equalities come from direct substitution of the contingent tax, τ^c; applying the rule of independent expectations; and inserting the first-order conditions.

Notice that each value of Δ equals zero. In other words, prefunding with a guarantee is completely neutral and induces no price changes. To see how the fixed guarantee has neutralized the effect of prefunding, notice that $\Delta_{t=1} < 0$ and $\Delta_{t\geq 2} > 0$ without the guarantee—or, equivalently if each generation bore the cost of its own guarantee. The result is intuitive: future generations can benefit from prefunding only if some initial generation is made worse off.[11] The addition of the guarantee completely eliminates, ex ante, both the short-run cost and the long-run benefit coming from prefunding. Ex ante, all agents receive the same benefits under the new system as under Social

[11] Changing the tax base can result in pareto improvements. But these improvements stem solely from tax reform, not prefunding.

Security. Even though the value of the recognition bond is worth only about 13½ percent of Social Security's net liability, the fixed guarantee accounts for the other 86½ percent (see calculations in Section V). Any seemingly "free lunch" that comes from attempting to get extra resources from investing the recognition bond in risky equities instead of into safe debt comes at a zero-sum cost to future generations in the form of greater risk. Notice that neutrality holds even if agents are risk neutral ($r = \bar{e}$). In this case, the value of the guarantee equals zero but now the value of the recognition bond B is more expensive and the total unfunded liabilities are unchanged.

Prefunding With An Add-On Tax

The substitution method is a special case of the recognition bond approach, with $B = 0$. As with recognition bonds, agents in each generation continue to provide the same performance guarantee on the same amount of investment made by the previous generation. What is different is that the initial young do not receive a recognition bond despite making payments to support the initial elderly. Instead, the initial young must come up on their own with an amount of assets equal in value to the recognition bond in equation (11) to put in their new private account. This is achieved with an add-on tax.

The change in the remaining lifetime resources in the fixed guarantee case becomes,

(14)
$$\Delta_{t=0} = 0$$

$$\Delta_{t=1} = -E_t\left[M_t \cdot \tau^{ss} w_t (1+g_{t+1})\right] + E_t(M_t \cdot w_{t+1} \tau^c_{t+1}) < 0$$

$$\Delta_{t\geq 2} = E_t\left[\tau^{ss} w_t \left\{1 - M_t \cdot (1+g_{t+1})\right\}\right] + \left\{E_t\left[M_t \cdot w_{t+1} \tau^c_{t+1}\right] - E_{t-1}\left[(1+r) M_{t-1} \cdot w_t \tau^c_t\right]\right\} > 0$$

The delta signs in equation (14) are straightforward. The value of $\Delta_{t=0}$ remains zero by the fact that privatization is designed not to impact the initial elderly. The value of $\Delta_{t=1}$ is negative due to the absence of the recognition bond; recall that $\Delta_{t=1} = 0$ with the recognition bond. The value of $\Delta_{t\geq 2}$ becomes positive due to absence of the growth-adjusted interest payments on the debt; recall that $\Delta_{t\geq 2} = 0$ with the interest payments. Notice that prefunding has increased the resources of future generations, $t \geq 2$, at the cost of the transitional generation, $t = 1$. This is because prefunding has now indeed led to a reduction in unfunded liabilities. By how much? Since prefunding with recognition bonds was exactly neutral, the reduction in liabilities using an add-on tax is precisely equal to the value of the recognition bond that is now *not* issued, or about 13½ percent of Social Security's unfunded liabilities.

IV. A MINIMUM BENEFIT GUARANTEE: PRIVATIZING SOCIAL SECURITY

The previous section analyzed the case of a fixed benefit. We now turn to the case of a minimum publicly-provided benefit as would be the case under most privatization plans. Specifically, it is now assumed that any shortfall in the value of the personalized accounts of generation-*t* agents below expectation is paid by generation-(*t*+1) taxpayers in the form of higher payroll taxes (as before). However, generation-(*t*-1) agents are now allowed to keep any return in excess of its expected value. The contingent payroll tax, τ^c, therefore gets redefined as:

$$\tau^c_{t+1} = \frac{\max\left[0, \tau^{ss}_t w_t(1+g_{t+1}) - B_t(1+e_{t+1})\right]}{w_{t+1}} \qquad (15)$$

Notice that, like the fixed benefit guarantee, the minimum guarantee is also waged indexed because Social Security benefits are waged indexed.

The first subsection considers using recognition bonds followed by the substitution method.

Recognition Bonds

Assuming that the recognition bond is computed as in equation (11), the change in the remaining lifetime expected value budget constraints now become:

(16)
$$\Delta_{t=0} = 0$$

$$\begin{aligned}
\Delta_{t=1} &= -E_t\left[M_t \cdot \tau^{ss} w_t (1+g_{t+1})\right] + B_t + E_t(M_t \cdot w_{t+1}\tau^c_{t+1}) \\
&= -E_t\left[M_t \cdot \tau^{ss} w_t (1+g_{t+1})\right] + B_t + E_t\left\{M_t \cdot \max\left[0, \tau^{ss} w_t (1+g_{t+1}) - B_t(1+e_{t+1})\right]\right\} \\
&= -E_t\left[M_t \cdot \tau^{ss} w_t (1+g_{t+1})\right] + B_t + E_t\left\{M_t \cdot \left[\tau^{ss} w_t (1+g_{t+1}) - B_t(1+e_{t+1})\right]\right\} \\
&\qquad + E_t\left\{M_t \cdot \max\left[0, B_t(1+e_{t+1}) - \tau^{ss} w_t (1+g_{t+1})\right]\right\} \\
&= E_t\left\{M_t \cdot \max\left[0, B_t(1+e_{t+1}) - \tau^{ss} w_t (1+g_{t+1})\right]\right\} \\
&> 0
\end{aligned}$$

$$\begin{aligned}
\Delta_{t\geq 2} &= \left[B_t - (1+r)B_{t-1}\right] + E_t\left[\tau^{ss} w_t \left\{1 - M_t \cdot (1+g_{t+1})\right\}\right] + \left\{E_t\left[M_t \cdot w_{t+1}\tau^c_{t+1}\right] - E_{t-1}\left[(1+r)M_{t-1} \cdot w_t \tau^c_t\right]\right\} \\
&= \left[B_t - (1+r)B_{t-1}\right] + E_t\left[\tau^{ss} w_t \left\{1 - M_t \cdot (1+g_{t+1})\right\}\right] \\
&\qquad + E_t\left\{M_t \cdot \max\left[0, \tau^{ss} w_t (1+g_{t+1}) - B_t(1+e_{t+1})\right]\right\} \\
&\qquad - (1+r)E_{t-1}\left\{M_{t-1} \cdot \max\left[0, \tau^{ss} w_{t-1}(1+g_t) - B_{t-1}(1+e_t)\right]\right\} \\
&= \left[B_t - (1+r)B_{t-1}\right] + E_t\left[\tau^{ss} w_t \left\{1 - M_t \cdot (1+g_{t+1})\right\}\right] \\
&\qquad + E_t\left\{M_t \cdot \left[\tau^{ss} w_t (1+g_{t+1}) - B_t(1+e_{t+1})\right]\right\} \\
&\qquad + E_t\left\{M_t \cdot \max\left[0, B_t(1+e_{t+1}) - \tau^{ss} w_t (1+g_{t+1})\right]\right\} \\
&\qquad - (1+r)E_{t-1}\left\{M_{t-1} \cdot \left[\tau^{ss} w_{t-1}(1+g_t) - B_{t-1}(1+e_t)\right]\right\} \\
&\qquad - (1+r)E_{t-1}\left\{M_{t-1} \cdot \max\left[0, B_{t-1}(1+e_t) - \tau^{ss} w_{t-1}(1+g_t)\right]\right\} \\
&= E_t\left\{M_t \cdot \max\left[0, B_t(1+e_{t+1}) - \tau^{ss} w_t (1+g_{t+1})\right]\right\} \\
&\qquad - (1+r)E_{t-1}\left\{M_{t-1} \cdot \max\left[0, B_{t-1}(1+e_t) - \tau^{ss} w_{t-1}(1+g_t)\right]\right\} \\
&< 0
\end{aligned}$$

The equations in (16) show that privatization now expands the resources of initial workers at the cost of reducing the resources of future workers. This reflects the *increase* in unfunded liabilities following privatization (by about 1 percent). Intuitively, privatization implies that first-period workers receive *at least* what they would have received under Social Security. While future workers provide a minimum guarantee, they no longer receive in return the benefits of any stock market performance above expectation in the form of a lower payroll tax. In other words, the government must confiscate any returns above expectation in order for privatization to simply be neutral: allowing agents to keep unusually high returns implies that privatization creates a windfall for current workers.

Substitution Using an Add-On Tax

Turning now to the substitution method, we get the following changes in the remaining lifetime expected value budget constraints:

$$\Delta_{t=0} = 0 \tag{17}$$

$$\begin{aligned}
\Delta_{t=1} &= -E_t\Big[M_t \cdot \tau^{ss} w_t (1+g_{t+1})\Big] + E_t(M_t \cdot w_{t+1}\,\tau^{c}_{t+1}) \\
&= -E_t\Big[M_t \cdot \tau^{ss} w_t (1+g_{t+1})\Big] + E_t\Big\{M_t \cdot \max\big[0, \tau^{ss} w_t (1+g_{t+1}) - B_t\,(1+e_{t+1})\big]\Big\} \\
&= -E_t\Big[M_t \cdot \tau^{ss} w_t (1+g_{t+1})\Big] + E_t\Big\{M_t \cdot \big[\tau^{ss} w_t (1+g_{t+1}) - B_t\,(1+e_{t+1})\big]\Big\} \\
&\qquad + E_t\Big\{M_t \cdot \max\big[0, B_t\,(1+e_{t+1}) - \tau^{ss} w_t (1+g_{t+1})\big]\Big\} \\
&= E_t\Big\{M_t \cdot \max\big[0, B_t\,(1+e_{t+1}) - \tau^{ss} w_t (1+g_{t+1})\big]\Big\} - B_t \\
&< 0
\end{aligned}$$

$$\begin{aligned}
\Delta_{t\geq 2} &= E_t\Big[\tau^{ss} w_t \big\{1 - M_t \cdot (1+g_{t+1})\big\}\Big] + \Big\{E_t\Big[M_t \cdot w_{t+1}\,\tau^{c}_{t+1}\Big] - (1+r)E_{t-1}\Big[M_{t-1} \cdot w_t \tau^{c}_t\Big]\Big\} \\
&= E_t\Big[\tau^{ss} w_t \big\{1 - M_t \cdot (1+g_{t+1})\big\}\Big] + E_t\Big\{M_t \cdot \max\big[0, \tau^{ss} w_t\,(1+g_{t+1}) - B_t\,(1+e_{t+1})\big]\Big\} \\
&\qquad - (1+r)E_{t-1}\Big\{M_{t-1} \cdot \max\big[0, \tau^{ss} w_{t-1}\,(1+g_t) - B_{t-1}\,(1+e_t)\big]\Big\} \\
&= E_t\Big[\tau^{ss} w_t \big\{1 - M_t \cdot (1+g_{t+1})\big\}\Big] + E_t\Big\{M_t \cdot \big[\tau^{ss} w_t\,(1+g_{t+1}) - B_t\,(1+e_{t+1})\big]\Big\} \\
&\qquad + E_t\Big\{M_t \cdot \max\big[0, B_t\,(1+e_{t+1}) - \tau^{ss} w_t\,(1+g_{t+1})\big]\Big\} \\
&\qquad - (1+r)E_{t-1}\Big\{M_{t-1} \cdot \big[\tau^{ss} w_{t-1}\,(1+g_t) - B_{t-1}\,(1+e_t)\big]\Big\} \\
&\qquad - (1+r)E_{t-1}\Big\{M_{t-1} \cdot \max\big[0, B_{t-1}\,(1+e_t) - \tau^{ss} w_{t-1}\,(1+g_t)\big]\Big\} \\
&= E_t\Big\{M_t \cdot \max\big[0, B_t\,(1+e_{t+1}) - \tau^{ss} w_t\,(1+g_{t+1})\big]\Big\} \\
&\qquad - (1+r)E_{t-1}\Big\{M_{t-1} \cdot \max\big[0, B_{t-1}\,(1+e_t) - \tau^{ss} w_{t-1}\,(1+g_t)\big]\Big\} - \big[B_t - (1+r)B_{t-1}\big] \\
&> 0
\end{aligned}$$

Notice that now privatization leads to an decrease in unfunded liabilities. This is indicated by $\Delta_{t=1} < 0$ and $\Delta_{t\geq 2} > 0$.

The delta signs for this case requires a little more explanation. The reason for the added complication is that previously, with recognition bonds, privatization actually increased unfunded liabilities as indicated by $\Delta_{t=1} > 0$ and $\Delta_{t\geq 2} < 0$ in equation set (16). The equations in (17) shows just the opposite, that generation-1 agents lose and generation-($t \geq 2$) agents gain: $\Delta_{t=1} < 0$ and $\Delta_{t\geq 2} > 0$. The intuition is as follows. The guarantee insures that generation-1 agents will have a minimum level of retirement income. But generation-1 agents were already going to receive this amount under Social Security and they must now come up with the initial investment, B, themselves. This amount is in addition to the payroll tax that they face to support the generation-0 retirees. While it is true that generation-1 agents keep any returns in excess of expectation (the source of non-neutrality under the recognition bond approach), they already had this opportunity to invest in the market with their own money. These agents, therefore, are strictly worse off under privatization. Generation-($t \geq 2$) agents, however, gain. This is because these agents do not have to participate in Social Security. It is true that these agents are providing inter-generational insurance and, therefore, face an uncertain payroll tax. However, these agents will only pay *at most* what they would have paid under Social Security, and less in probability. These agents, therefore, are strictly better off under privatization.

However, it is easy to show that the magnitude of the deltas in equation set (17) are smaller than those in the case of prefunding the fixed Social Security benefit level using the substitution method, equation set (14) (a proof in the more general case is given below). This is because, for a given add-on tax rate and guaranteed benefit level, privatizing Social Security leads to a smaller decrease in unfunded liabilities than prefunding the fixed waged-indexed defined-benefit level. The reason is that both privatization and prefunding provide the same guarantee against the downside risk but prefunding effectively commits the upside potential to future generations.

The key results for Sections III and IV are summarized in Table 1.

V. Alternative Guarantee and Contribution Levels

Until now, this paper considered only the case of exactly replacing Social Security. This assumption was useful in making a fair comparison between Social Security and a prefunded defined-benefit system versus a privatized system.

In reality, policymakers might consider plans that aim to more-than-replace Social Security. Feldstein and Samwick (1997), for example, suggest that oversaving in the form of higher contribution levels might be used to minimize the government's exposure to the downside risk. This section considers this option. I find that Feldstein and Samwick's insight is particularly important for the case of prefunding Social Security's defined benefit. In general, higher contribution levels lead to substantial declines in unfunded liabilities under prefunding but have only a tiny impact under privatization.

In reality, policymakers might also consider alternative guarantee levels as well. This is important because plans with high contribution levels might also be accompanied with higher guarantee levels. The computations in this section show that greatly increasing the contribution level while increasing the guaranteed benefit level just a little will generally mean that privatization is predicted to *increase* unfunded liabilities.

In order to give both prefunding a defined benefit and privatization the best chance at reducing liabilities, I now consider only the substitution method. As shown earlier, using recognition bonds (without paying off the principal) would lead to a smaller decrease in liabilities.

The formal analysis earlier assumed that the fixed or minimum benefit was wage indexed in order to provide a clean comparison with Social Security. Wage indexation makes sense in a pay-as-you-go system in which wages, not capital investments, are the source of internal rates of return. In practice, however, minimum benefits are generally inflation indexed but not wage indexed in order to provide pensioners with a guaranteed certain minimum benefit amount.[12] In theory, promising pensioners a non-risky minimum benefit amount would slightly increase the cost of privatization relative to prefunding the current waged-indexed Social Security system if wages and stock returns are highly correlated.[13] To keep the comparison fair, therefore, I also fix the benefit in new prefunded system at the non-wage-indexed amount and I compare prefunding and privatization relative to Social Security paying a guaranteed rate of return equal to $\bar{g}_{t+1}$.[14]

Modified State-Contingent Tax Rates

Recall that first-period agents in Sections III and IV contributed B_t at time t (equation (11)) into the new private account that replaced their Social Security benefit. With recognition bonds considered in Section III, agents get B_t from the government; under the substitution method considered in Section IV and in this section, agents have to come up with B_t themselves.

To generalize, suppose, instead, that $\psi \cdot B_t$ is contributed during the first period of life where ψ is a scaling factor. The expected retirement benefit, therefore, is ψ *times* the current Social Security benefit. Moreover, let the guaranteed benefit level equal $\chi \cdot \tau^{ss} w_t (1 + \bar{g}_{t+1})$ for both prefunding (where this amount is the fixed benefit) and privatization (where this amount is the minimum benefit) where χ is a scaling factor. In other words, the guaranteed benefit level equals χ *times* the benefit that a person is expected to receive under Social Security. (In the case of

[12] The Chilean minimum pension, for example, is adjusted for inflation once per year, or every time that inflation accumulates more than 10% since the last adjustment, whichever comes first (Vial Ruiz-Tagle and Castro, 1997). Policymakers in Chile have avoided tying the minimum benefit to the growth rate of wages during a pensioner's working years. They instead have chosen to look at longer trends in economic growth. Sustained economic growth in the past has often brought about ad-hoc increases in the guaranteed amount (*Ibid*).

[13] Numerically, the extra cost, though, is small since, even at very low frequency in which average wages and stocks tend to be more highly correlated, stocks have empirically been much more risky than average wage growth.

[14] Both prefunding and privatization lead to even (slightly) smaller reductions in unfunded liabilities when using g_{t+1} instead of $\bar{g}_{t+1}$ for Social Security due to the additional insurance value associated with the benefit in the new system being non-indexed. The key results herein were unaffected.

prefunding, a value of $\chi > 1$ means that we have increased the value of the defined benefit above its current level.) Note that ψ and χ can be chosen independently because the state-contingent tax that guarantees the performance of the new accounts is modeled separately.

The state-contingent tax rate in the case of prefunding a defined benefit now becomes

$$\tau^c_{t+1} = \frac{\chi \cdot \tau^{ss}_t w_t (1+\bar{g}_{t+1}) - \psi \cdot B_t (1+e_{t+1})}{w_{t+1}} \tag{18}$$

whereas, in the case of privatization, the contingent tax rate equals

$$\tau^c_{t+1} = \frac{\max\left[0, \chi \cdot \tau^{ss}_t w_t (1+\bar{g}_{t+1}) - \psi \cdot B_t (1+e_{t+1})\right]}{w_{t+1}} \tag{19}$$

Pricing the Change in Unfunded Liabilities

The exact policy initiative that should be analyzed is controversial since many proposals aim to only partially privatize or prefund Social Security. Moreover, in light of the equity premium puzzle debate, any choice of preferences and technology is also bound to be controversial (Kocherlakota, 1996). I sidestep these issues by comparing the reduction in unfunded liabilities associated with an incremental move toward privatization relative to prefunding. As in the calculations by Feldstein and Samwick (1997), the analysis herein derives the reduction in unfunded liabilities at today's prices. This is consistent with prefunding or privatizing the Social Security benefit associated with the marginal dollar in the wage tax base.[15] The focus on an incremental policy change means that new liabilities, in the form of guarantees, incurred after a policy change can be priced *exactly*, without linearizing, via arbitrage techniques which, in turn, requires the use

[15] Calculations in which the marginal product of capital was allowed to drop by 25 percent did not alter any of the stark differences between prefunding and privatization shown in the paper.

of only observable prices. This is because all the information concerning risk is already captured in the market prices and so preference parameters do not enter these equations directly.[16] Any set of preferences and technology compatible with balanced growth and observable prices and covariances must generate the same marginal value on a given liability.

The percent change in unfunded liabilities at the margin equals

$$
\text{(20)} \qquad \begin{aligned}\%\Delta_F &= \left\{1 - \frac{E_t(M_t \cdot w_{t+1}\tau^c_{t+1})}{E_t\left[M_t \cdot \tau^{ss} w_t(1+\bar{g}_{t+1})\right]}\right\}\cdot 100\% \\ &= \left\{1 + \psi \cdot \frac{1+r}{1+\bar{e}_{t+1}} - \chi\right\}\cdot 100\%\end{aligned}
$$

while the marginal percent change in unfunded liabilities resulting from privatization equals,

$$
\text{(21)} \qquad \begin{aligned}\%\Delta_M &= \left\{1 - \frac{E_t(M_t \cdot w_{t+1}\tau^c_{t+1})}{E_t\left[M_t \cdot \tau^{ss} w_t(1+\bar{g}_{t+1})\right]}\right\}\cdot 100\% \\ &= \left\{1 - \frac{1+r}{1+\bar{e}_{t+1}} \cdot \psi \cdot E_t\left(M_t \cdot \max\left[0, \left(\frac{\chi(1+\bar{e}_{t+1})}{\psi} - 1\right) - e_{t+1}\right]\right)\right\}\cdot 100\%\end{aligned}
$$

Computing the reduction in unfunded liabilities associated with prefunding Social Security, shown in equation (20), is quite easy and only requires observable values for the risk-free rate (r), the first moment of the stock price evolution ($\bar{e}_{t+1}$) and choices for the exogenous policy parameters, ψ and χ. Notice that, when the new system uses equity investment to exactly replace previous benefits ($\psi = \chi = 1.0$), equation (20) shows that prefunding Social Security with an add-on tax

[16] This insight, one of the key results from modern arbitrage pricing theory (Ingersoll, 1989), was formalized by Black and Scholes (1973) and Merton (1973) who demonstrated the absence of preference parameters in one-sided bets.

eliminates all unfunded liabilities only in the case of risk neutrality ($r = \overline{e}_{t+1}$). When agents are risk averse ($r < \overline{e}_{t+1}$), prefunding still leaves a positive liability. This is because every dollar that the government attempts to raise from exploiting the equity premium places additional risk on future generations with an actuarial value equal to one dollar. Eliminating all unfunded liabilities requires either a higher contribution rate ($\psi > 1$) or requires guaranteeing a benefit below the Social Security level ($\chi < 1$).

Computing the reduction in unfunded liabilities associated with privatizing Social Security, as shown in equation (21), is only slightly more complicated. This equation proves that using a contingent payroll tax to support the minimum benefit is mathematically equivalent to giving workers European put options on the underlying stocks. Specifically, the expression $E_t(\cdot)$ is the formula for a one-period put stock option on a dollars worth of equities with a strike price of $\$1\left[\frac{\chi}{\psi}(1+\overline{e})\right]$ next period. Intuitively, a higher contribution rate lowers the implicit strike price because each dollar does not have to perform as well for the minimum benefit guarantee to be satisfied; similarly, a higher guarantee level increases the implicit strike price. Pricing these one-sided bets requires that an additional assumption be made of the stock price's evolution, which, in a model with production, requires similar assumptions be made for the efficiency and depreciation processes. Assuming that these processes can be described by an Itô-type stochastic differential equation, the value of this option can be priced *exactly* in an economy with production—without any additional assumptions about the preferences beyond nonsatiation—using the popular Black-Scholes (1973) option pricing theorem. Because of our focus on the marginal dollar, nothing is lost by using the Black-Scholes pricing theorem instead of a calibrated two-period simulation model. In particular, the resulting marginal calculations using the Black-Scholes theorem are uniquely consistent with any choice of utility function and production function that generates the observed prices in the economy.

It is easy to prove that $\%\Delta_M < \%\Delta_F$ for identical parameters. That is, <u>for identical parameter values, prefunding Social Security reduces unfunded liabilities by more than privatization.</u> This reflects the fact that while both the fixed and minimum guarantees equally insure against equity returns below expectation, the fixed guarantee passes all excessive returns to future generations. In the language of options pricing, current workers are required to give future workers, in exchange for the put option, a call option with the exercise price noted above in the fixed guarantee case. To see this inequality formally, rewrite equation (20) as,

(20')

$$\%\Delta_F = \left\{1 + \psi \cdot \frac{1+r}{1+\bar{e}_{t+1}} - \chi\right\} \cdot 100\%$$

$$= \left\{1 - \frac{1+r}{1+\bar{e}_{t+1}} \cdot \psi \cdot E_t\left(M_t \cdot \max\left[0, \left(\frac{\chi(1+\bar{e}_{t+1})}{\psi} - 1\right) - e_{t+1}\right] - M_t \cdot \max\left[0, e_{t+1} - \left(\frac{\chi(1+\bar{e}_{t+1})}{\psi} - 1\right)\right]\right)\right\} \cdot 100\%$$

This equation shows that the fixed benefit guarantee can be decomposed into a one-period put option on a dollars worth of equities less a one-period call option on a dollars worth of equities, both with a strike price of $\$1\left[\frac{\chi}{\psi}(1+\bar{e})\right]$ next period.

Figure 2 decomposes the value of the tacit put and call options as a function of the tacit strike price $\$1\left[\frac{\chi}{\psi}(1+\bar{e})\right]$, which, in turn, is governed by the choices for χ, ψ and $\bar{e}$. The shapes of both the put and call curves shown in Figure 2 are due to risk aversion captured by a positive equity premium. In particular, as the value of the implicit strike price declines, the implicit put option becomes less valuable but the decline in value begins to lessen for a given decrease in the strike price. Risk-averse agents place more value on downside risk than upside potential and so even a put option with a relatively low strike price can have a fair amount of value. In comparison, the value of a call option is relatively more flat across the strike prices, since, e.g., a call option issued at the money can be exercised for a profit only if stocks already increase in value. The value of the option is bounded above zero and converges (rather quickly) to zero as the implicit strike price increase, or, conversely, as expected payoff of the option decreases.

Numerical Calculations

For the purpose of numerical calculations, we need to make some choices for the parameter values. Following Feldstein and Samwick, the annual average the risk-free rate (r) is set equal to 2 percent and the economy's expected growth rate ($\bar{g}$) is set equal to 1.1 percent per year. Bonds, therefore, are assumed to stochastically dominate Social Security. Privatization and prefunding are compared against Social Security's current mostly pay-as-you-go financing method in which the tax rate will eventually have to increase to 18.75 percent over the next 75 years in order to maintain current-law benefits. I also consider Feldstein and Samwick's choice of the average annual return to equities equal to 9 percent as one of possible parameter values for $\bar{e}$. The standard deviation of the first differences of logged real returns on the S&P500 since 1928 equals 0.20 and 0.164 since 1949. I conservatively chose 0.16 in all calculations.[17] Each period is assumed to represent 30 years and so the above annual rates are converted to their 30-year equivalents in all calculations herein.[18]

Overall, each of these parameter choices, although quite favorable to prefunding and privatization, seem fairly reasonable. The choice of $\bar{e}$ equal to 9 percent corresponds roughly to the long-run marginal social (before corporate taxes) marginal rate of return to capital (Poterba, 1997).

[17] Calculations using g_{t+1} instead of $\bar{g}_{t+1}$ for Social Security, referenced above in an earlier footnote, also depend on the covariance between equity returns and the growth rate in wages. It is easy to show that the underlying derivative product pays a rate of return that is a function of both the return to equities and the growth rate in wages. The second moment of this product is then derived using the historical series for equity returns and wage growth. These calculations add analytical complexity but, as footnoted above, only slightly strengthen the key results herein.

[18] The choice of 30 years follows the two-period illustrative calculations presented in Feldstein and Samwick.

It also happens to be close to the geometric mean rate of return to the S&P500 since 1926 before corporate taxes (Roger Ibbotson and Gary Brinson, 1993). This choice for $\bar{e}$, therefore, requires that corporate taxes paid by the new accounts are fully and costlessly rebated. The long-run after-tax rate of return to equities is closer to 7 percent (Figure 1). A real risk-free rate of 2 percent is fairly close to the 1.7 percent historic geometric mean rate of return earned by intermediate government debt (Figure 1). (Using 1 percent leads to even smaller declines in unfunded liabilities.) An economic growth rate of 1.1 percent is well below the historic geometric mean growth rate of about 3.3 percent (Figure 1). However, the smaller value of 1.1 percent is in line with the Social Security Administration's estimate for the next 75 years.

Table 2 reports the marginal reduction in unfunded liabilities from prefunding and privatizing Social Security for various choices of ψ, χ and $\bar{e}$. Also shown in Table 2 is the new tax rate, τ^N, that is necessary to produce an expected level of retirement income that is ψ times the expected coverage offered by Social Security:

$$\tau^N = \frac{\psi \cdot \tau(1+\bar{g})}{(1+\bar{e})} \qquad (22)$$

Consider first the policy parameter choices considered in Sections III and IV: $(\psi,\chi) = (1.0,1.0)$. Table 2 shows that the corresponding new contribution tax rate equals $\tau^N = 1.96$ percent which corresponds closely to the value of 2.02 percent derived by Feldstein and Samwick (1997) using a more complex multi-period model.[19] Prefunding reduces unfunded liabilities by only 13.7 percent while privatization reduces them by only 13.2 percent, or by one-eighth. Notice that (i) the reduction in unfunded liabilities is small for both policies at this contribution rate and (ii) the

[19] Feldstein and Samwick (p. 10) also derived 1.96 percent using a two-period model and they were the first to note the ability of the two-period model to generate fairly accurate tax rates.

reduction is about the same for each policy choice. The reason for the small change in unfunded liabilities in the case of privatization is that the tacit put option associated with the minimum benefit guarantee is very much "in the money" at the high strike price equal to $\$1\left[\frac{\chi}{\psi}(1+\bar{e})\right] = \$(1+\bar{e})$ for $\psi = \chi = 1$. This option places a very expensive liability on future generations (see Figure 2). The reason why prefunding Social Security does not do much better is because the tacit call option, given by current workers to future workers—associated with prefunding but not privatization—is not very valuable; i.e., it is very much "out of the money." Figure 2 shows that, at such a high strike price, the value of the put option for a 30-year period is about $6.36 for each dollar invested in equities whereas the value of the corresponding call option is only $0.03.

We can perform a rough check on these calculations for the case of prefunding Social Security's defined benefit for these choices of policy parameters. Since benefits are fixed at the same level as under Social Security ($\chi = 1$), the 13.7 percent reduction in unfunded liabilities predicted above should, after divided by $(1+r)/(1+\bar{g})$ where r and g are 30-year rates, be directly proportional to the mandatory contribution level in the new system divided by the previous pay-as-you-go tax rate.[20] Note that a 2 percent payroll tax is about 11 percent of the 18.75 percent payroll tax (in the out years) that is being replaced. This is very close to $13.7\ /\ [(1+r)/(1+\bar{g})] \approx 10.5$ percent. This comparison suggests that the simple two-period model, with each period representing 30 years, appears to generate very plausible predictions.

Consider what happens as we increase the contribution level in the new system in order to increase the expected benefit level (ψ *times* the current expected Social Security benefit). Table 2 reports values for ψ equal to 2, 3, 5, 8 and 15 along with a guaranteed benefit level equal to the current Social Security level ($\chi = 1$) and $\bar{e}$ equal to 9 percent. The value of $\psi = 8$ is particularly

[20] The factor, $(1+r)/(1+\bar{g})$, reflects the stochastic dominance of bonds over pay-as-you-go financing.

relevant because, for $\bar{e} = 0.09$ and $r = 0.02$, one dollar invested in equities today is expected to be worth almost 7½ times the value of a dollar invested in the risk-free asset. The simulation results are as follows. For $\psi = 2$, the contribution rate equals 3.9 percent and prefunding reduces liabilities by 27 percent while privatization reduces liabilities by 23 percent. For $\psi = 3$, the contribution rate equals 5.9 percent and prefunding reduces liabilities by 41 percent while privatization reduces liabilities by 30 percent. For $\psi = 5$, the contribution rate equals 9.8 percent and prefunding reduces liabilities by 68 percent while privatization reduces liabilities by 35 percent. For $\psi = \left[(1+\bar{e})/(1+r)\right]$ ≈ 7.32 (not shown in Table 2) where each period equals 30 years, the contribution rate equals 14.4 percent and prefunding reduces liabilities by exactly 100 percent while privatization reduces liabilities by only 36 percent. For $\psi = 8$, the contribution rate equals 15.7 percent and prefunding reduces liabilities by 109 percent while privatization reduces liabilities by only 36 percent. For $\psi = 15$, the contribution rate equals 37.4 percent and prefunding reduces liabilities by 204 percent while privatization reduces liabilities by only 37 percent.

Notice that as the value of ψ increases, prefunding Social Security's current defined benefit substantially reduces unfunded liabilities. The higher contribution level reduces the likelihood that future generations will face a positive contingent tax to finance the guaranteed defined benefit. This result can be best understood with reference to Figure 2. Consider the implicit strike price equal to $\$1\left[\frac{\chi}{\psi}(1+\bar{e})\right]$ $= 1 + r$ for $\psi = \left[(1+\bar{e})/(1+r)\right] \approx 7.32$ and $\chi = 1.0$, where each period equals 30 years. In this case, the tacit put and call options underlying prefunding exactly cancel since any expected return above r represents pure compensation for risk. Prefunding the fixed benefit, exactly eliminates all of Social Security's unfunded liabilities because the risk-adjusted value of obligations being passed forward in time (the put option) exactly equals the risk-adjusted value of promises being passed forward in time (the call option). The decline in unfunded liabilities continues to dramatically increase at higher values of ψ, exceeding 100 percent for values of ψ above 7.32, reflecting the crossing of the put and call option curves. In sum, oversaving relative to the guarantee does an excellent job in reducing unfunded liabilities in the case of prefunding.

In sharp contrast, privatization has little impact on unfunded liabilities at higher values of ψ. Moreover, the marginal impact that increasing ψ has on unfunded liabilities drops as the value of ψ increases. These surprising results can also be best understood with reference to Figure 2. Recall that in the case of $\psi = 1.0$, the strike price of the tacit put option equals $\$1\left[\frac{\chi}{\psi}(1+\bar{e})\right] = 1 + \bar{e}$ and privatization reduces unfunded liabilities by only 13 percent. This small reduction was traced to the fact that the tacit put option was worth a considerable amount at such a high tacit strike price. As shown in Figure 2, increasing ψ from 1 to 7.32 substantially lowers the tacit strike price which, in turn, lowers the insured value of each dollar placed in stocks. In fact, the value of the put option decreases from almost \$6.36 to \$0.64 (Figure 2), or a factor of ten. However, the sheer amount of dollars being insured increases by a factor of 7.32, as reflected by the increase in the contribution rate from 2 percent to 14.4 percent. It follows that a considerable amount of risk is still being passed forward in time. The situation gets even worse at larger values of ψ due to the decrease in slope of the put curve shown in Figure 2. Unfunded liabilities hardly budge when the value of ψ is increased to 15 with a 37.4 percent contribution rate: the reduction in the price of the put option associated with each dollar is almost exactly offset by the sheer number of additional dollars being insured.

Table 3 reports the numbers plotted in Figure 2 (which, in order to fit on a page, is not drawn to scale), including the ratios of the option values at a given value of ψ relative to their value evaluated at $\psi = 1$. Notice that at $\psi = 15$, the value of the implied put option is only about 1/21 the value of the option evaluated at $\psi = 1$. However, this large decline is still not enough to reduce *total* unfunded liabilities very much after privatization because the amount of assets being insured is 15 times larger.

The problem with attempting to use higher contribution levels to reduce risk is that the assets associated with higher contribution levels are perfectly correlated with the assets accumulated at lower contribution levels. High contribution levels offer no hedging value. The method of oversaving for retirement should not, therefore, be confused with the general wisdom of investing in a broad portfolio that spans the space of state-contingent returns. They are fundamentally different approaches to limiting risk. (Indeed, the calculations above, by focusing on the S&P500, already assume that agents are required to invest in a fairly broad portfolio of assets.) Oversaving simply involves betting more dollars. Of course, each dollar does not have to perform as well to satisfy the minimum benefit. But, on net, oversaving has little impact on unfunded liabilities.

The situations for both prefunding and privatization gets worse as the guarantee level increases. Policymakers might be tempted (or compelled) to offer guarantees in excess of the current Social Security level, especially in the case of privatization where expected benefits are much larger than the current Social Security benefit. Table 2 shows that at $\psi = 8$, increasing the guaranteed benefit level to 150 percent of the current Social Security benefit ($\chi = 1.5$) means that prefunding now reduces liabilities by only 59 percent (instead of 109 percent). By contrast, privatization now reduces unfunded liabilities by only 3 percent (instead of 36 percent). Table 2 also shows that considerable gains can be achieved by guaranteeing a benefit equal to only 75 percent of the current Social Security benefit ($\chi = 0.75$). For example, for $\psi = 3$, prefunding now reduces unfunded liabilities by 66 percent (compared to 50 percent for $\chi = 1.0$) and privatization reduces unfunded liabilities by 50 percent (compared to 30 percent for $\chi = 1.0$). Whether policymakers can credibly commit to this low of a benefit level, however, is unclear since the average monthly OASI benefit per recipient (workers and spouses) in December, 1994, was only $644.30.

Finally, for the purpose of sensitivity analysis, Table 2 reports changes in unfunded liabilities when using a smaller value for $\bar{e}$, set equal to 7 percent. In this case, unfunded liabilities are reduced ceteris paribus but at the cost of a higher contribution level. For example, with $\psi = 5$ and $\chi = 1.0$, prefunding Social Security's defined benefit reduces unfunded liabilities by 119 percent (compared to 68 with $\bar{e} = 0.09$) but the contribution rate increases to 17.1 percent (compared to 9.8 percent). The impact on privatization, however, is again hardly noticeable at this high of a value for ψ. Privatization reduces unfunded liabilities by 36 percent (compared to 35 percent with $\bar{e} = 0.09$). This small reduction is due to the almost doubling in total size of assets being insured associated with the higher contribution rate. The value of $\bar{e}$, however, becomes a little more important at lower values for ψ. With $\psi = 1$, privatization now reduces unfunded liabilities by 21 percent (compared to 13 percent with $\bar{e} = 0.09$).

In sum, for high values of ψ, the substantially higher expected benefits achieved under privatization relative to prefunding Social Security's existing defined benefit comes at a zero-sum cost to future generations in the form of much higher insurance obligations.

VI. CONCLUSIONS

This paper argues that prefunding Social Security's existing fixed defined benefit has the advantage over privatization in that, for identical guarantee levels, prefunding the fixed benefit is able to reduce unfunded liabilities by much more than privatization. This is because that while both policies insure against downside risk, prefunding the existing defined benefit passes the upside potential to future workers whereas privatization does not. The differences in the reduction in unfunded liabilities is quite sharp at especially high contribution rates. It is unlikely that modifying the analysis in this paper in various directions will change this key result.

The comparative advantage of prefunding the existing defined benefit in reducing unfunded liabilities, however, critically assumes that the policymakers can credibly "lock in" the associated trust fund, making it unavailable to finance new spending (including increasing future benefits or other spending) or tax cuts. It also assumes that the government earns market rates on its investments and does not use political criteria in selecting stocks and bonds for its portfolio. Both of these assumptions are, at a minimum, very debatable (see, e.g., Bosworth, 1996; White, 1996). Considerable caution should, therefore, be exercised in interpreting these mathematical results; alternative assumptions about the political economy could change the results.

References

1994-1995 Advisory Council on Social Security, The. *Findings, Recommendations, and Statements*, 1996.

Aaron, Henry. "Privatizing Social Security: A Bad Idea Whose Time Will Never Come." *The Brookings Review*, (Summer, 1997): 17-23.

Black, F. and M.J. Scholes. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy*, 1973, 81(3): 637-54.

Bohn, Henning. "Risk Sharing in a Stochastic Overlapping Generations Economy." Mimeo, UC Santa Barbara, January, 1998.

Bosworth, Barry. "Fund Accumulation: How Much? How Managed?" in *Social Security: What Role for the Future*. Peter Diamond, David Lindeman and Howard Young, eds. Distributed by the Brookings Institution for the National Academy of Social Insurance, 1996.

——— and Gary Burtless. "Budget Crunch: Population Aging in Rich Countries." *The Brookings Review*, (Summer, 1997): 10-15.

Brooks, Robin. "Baby Boom, Population Aging and Asset Prices," Yale University mimeo, 1996.

Campbell, John and Jonathan Shiller. "Valuation Ratios and the Long-Run Stock Market Outlook." *Journal of Portfolio Management*, 1998.

Diamond, Peter. "A Framework for Social Security Analysis." *Journal of Public Economics*; 8(3), Dec. 1977, pages 275-98.

———. "Macroeconomic Aspects of Social Security Reform." *Brookings Papers on Economic Activity*, 1998.

——— and Salvador Valdés-Prieto. "Social Security Reforms." in *The Chilean Economy: Policy Lessons and Challenges*, Bosworth, Dornbusch and Labán, eds., The Brookings Institution: Washington, DC, 1994.

Feldstein, Martin. "Transition to a Fully Funded Pension System: Five Economic Issues." NBER Working Paper 6149, 1997.

——— and Andrew Samwick. "The Economics of Prefunding Social Security and Medicare Benefits." in B. Bernanke and J. Rotemberg (eds.) NBER Macroeconomics Annual 1997. Cambridge: MIT Press, 115-148.

Gordon, Roger and John Wilson. "Measuring the Efficiency Cost of Taxing Risky Capital Income," *American Economic Review*; 79(3), June 1989, pages 427-39.

Ibbotson, Roger and Gary Brinson. *Global Investing: The Professional's Guide to the World Capital Markets*. NY: McGraw-Hill, Inc., 1993.

Ingersoll, Jonathan. "Option Pricing Theory." The New Palgrave Dictionary of Money & Finance: 199-212, 1989.

Jagannathan, Ravi and Narayana Kocherlakota. "Why Should Older People Invest Less in Stocks Than Younger People?" *Quarterly Review*, Federal Reserve Bank of Minneapolis, Summer, 1996.

Kocherlakota, Narayana. "The Equity Premium: It's Still a Puzzle." *Journal of Economic Literature*, 44 (March, 1996): 42-71.

Kotlikoff, Laurence. "Privatizing Social Security: How It Works and Why It Matters," in James M. Poterba (ed.), Tax Policy and the Economy, 10, Cambridge, Mass: MIT Press, 1996.

Kotlikoff, Laurence., Kent Smetters and Jan Walliser. "Privatizing US Social Security—A Simulation Study," forthcoming in: Klaus Schmidt Hebbel (ed.), Pension Systems: From Crisis to Reform, Washington, DC: The World Bank, 1997.

Merton, Robert. "Theory of Rational Option Pricing." *Bell Journal of Economics*; 4(1), Spring, 1973, pages 141-83.

———. "On the Role of Social Security as a Means for Efficient Risk Sharing in an Economy Where Human Capital is Not Tradable." Bodie, Zvi; Shoven, John, eds. *Financial Aspects of the United States Pension System*. NBER Project Report Series, Chicago and London: University of Chicago Press, 1983, pp. 325-358.

Poterba, James. "The Rate of Return to Corporate Capital and Factor Shares: New Estimates Using Revised National Income Accounts and Capital Stock Data," NBER Working Paper #6263, 1997.

Shiller, Robert J. "Price-Earnings Ratios as Forecasters of Returns: The Stock Market Outlook in 1996." Mimeo, Department of Economics, Yale University, 1996.

Stokey, Nancy and Robert Lucas. *Recursive Methods in Economic Dynamics*. Cambridge, MA: Harvard University Press, 1989.

Vial Ruiz-Tagle, Joaquín and Francisca Castro. "The Chilean Pension System." Meeting of Senior Budget Officials, 18th Annual Meeting, Paris, Château de la Muette, June 10-11, 1997.

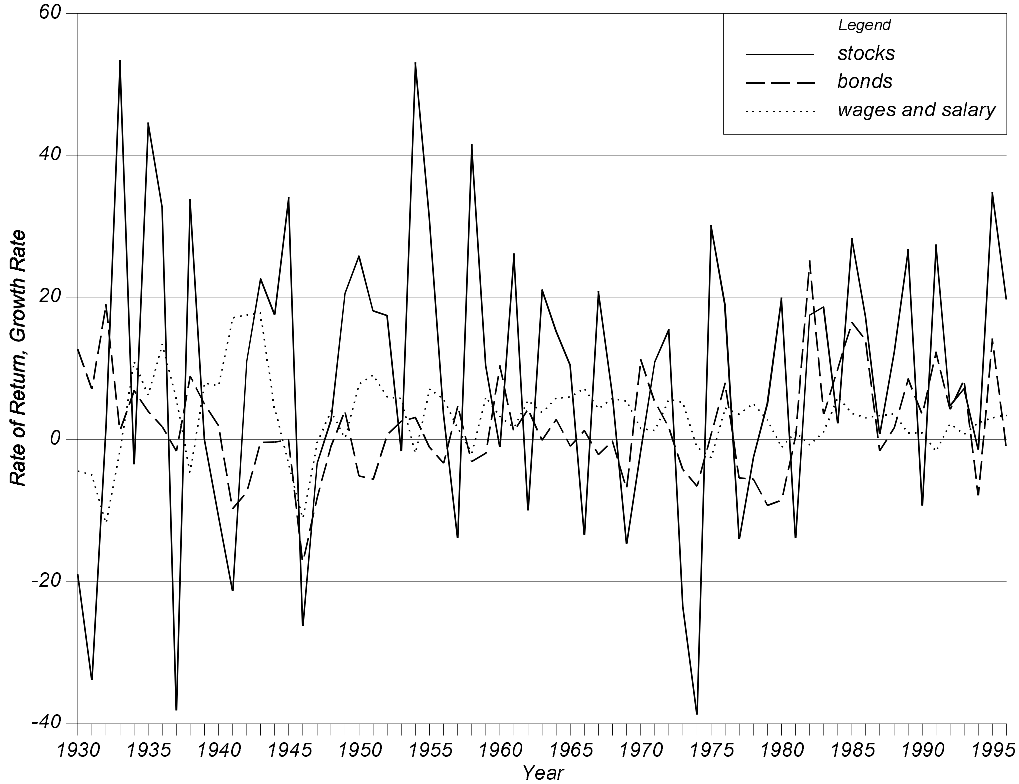
White, Lawrence. 1996. "Investing the Assets of the Social Security Trust Funds in Equity Securities: An Analysis." *Investment Company Institute Perspective*, 2(4): 1-20.

World Bank. *Averting the Old Age Crisis: Policies to Protect the Old And Promote Growth*. NY: Oxford University Press, 1994.

Zeldes, Stephen. "Consumption and Liquidity Constraints: An Empirical Investigation." *Journal of Political Economy*; 97(2), April 1989, pages 305-46.

FIGURE 1

Annual Real Yields to Stocks and Annual Growth in Wages and Salaries, 1930 to 1996[1]



Notes:

[1] **Data sources.** Stock and bond real yields: *Stocks, Bonds, Bills and Inflation 1997 Yearbook*, Ibbotson Associates: Chicago, Illinois. The rate of return to stocks is represented by the Standard and Poors 500. The returns include both capital gains and dividend yields. The bonds series corresponds to intermediate 5-year securities issued by the US Treasury. The bond yields assume that the coupon payments are re-invested at the prevailing bond prices. Wages and Salary base: U.S. Department of Commerce, Bureau of Economic Analysis. Wages and salary data converted into real values using a chain-weighted price index.

TABLE 1

The Impact of Privatization on Unfunded Liabilities Under Benchmark Policy: Expected Retirement Income Equal to Social Security Coverage, Guaranteed Benefit Equal to Social Security Coverage
($\bar{e}$ = 0.09 ; r = 0.02)

	TYPE OF GUARANTEE	
PRIVATIZATION METHOD	Fixed Benefit	Minimum Benefit
"Recognition Bond"	No Change	1 % Increase
"Substitution"	13.7 % Decrease	13.2 % Decrease

TABLE 2

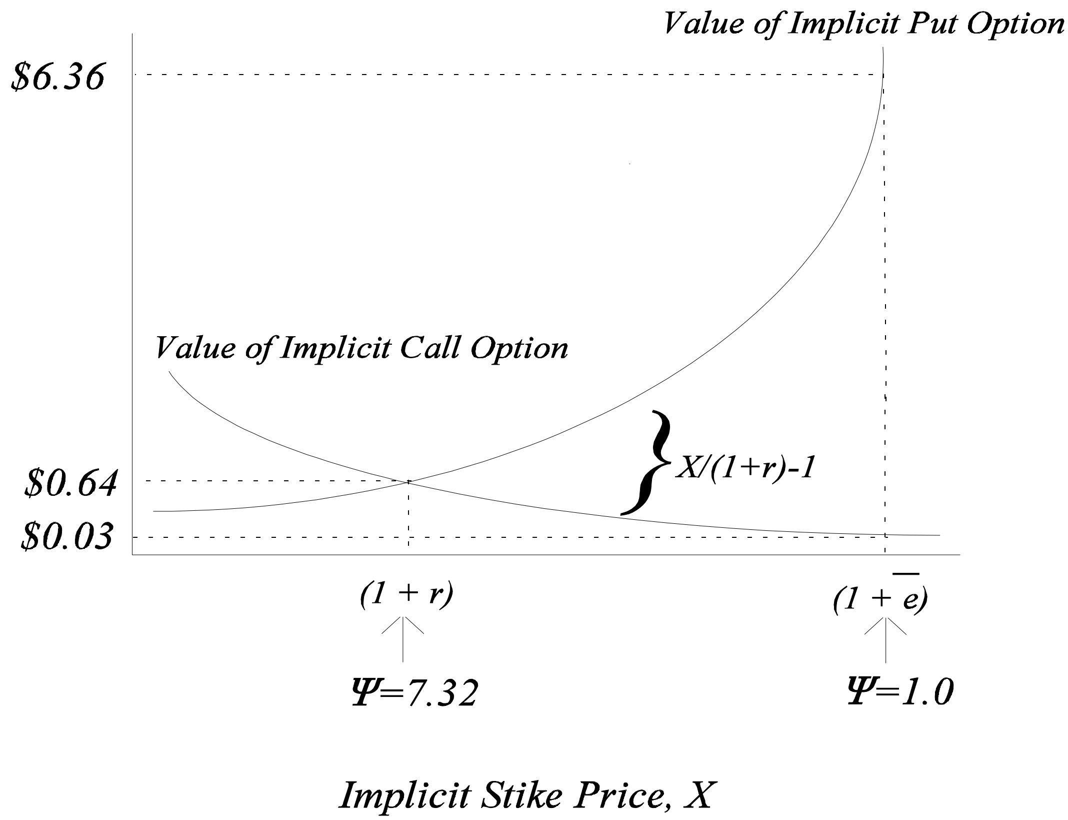
New Contribution Tax Rate for Private Account, τ^N,
Percent Reduction in Unfunded Liabilities With a Minimum Benefit, $\%\Delta_M$, and
Percent Reduction in Unfunded Liabilities With a Fixed Benefit, $\%\Delta_F$
as a Function of
Expected Retirement Income Equal to ψ times Social Security Coverage,
Minimum Guaranteed Benefit Equal to χ times Social Security Coverage, and
Expected Rate of Return on Equities, $\bar{e}$,
For Privatization Via "Substitution"

Exogenous Parameters			New Payroll Tax Rate	Minimum Benefit	Fixed Benefit
ψ	χ	$\bar{e}$	τ^N	$\%\Delta_M$	$\%\Delta_F$
1	0.75	0.07	0.0342	44.3	48.8
1	0.75	0.09	0.0196	37.7	38.7
1	1.00	0.07	0.0342	21.1	23.8
1	1.00	0.09	0.0196	13.2	13.7
2	0.75	0.07	0.0684	51.0	72.6
2	0.75	0.09	0.0392	45.9	52.3
2	1.00	0.07	0.0684	31.6	47.6
2	1.00	0.09	0.0392	23.3	27.3
2	1.25	0.07	0.0684	10.7	22.6
2	1.25	0.09	0.0392	-0.4	2.3
2	1.50	0.07	0.0684	-11.5	-2.4
2	1.50	0.09	0.0392	-24.5	-22.7
3	0.75	0.07	0.1026	52.3	96.4
3	0.75	0.09	0.0589	49.9	66.0
3	1.00	0.07	0.1026	35.4	71.4
3	1.00	0.09	0.0589	29.6	41.0
3	1.50	0.07	0.1026	-2.5	21.4
3	1.50	0.09	0.0589	-15.1	-9.0
3	1.75	0.07	0.1026	-23.3	-3.6
3	1.75	0.09	0.0589	-38.6	-34.0
5	1.00	0.07	0.1710	36.4	119.0
5	1.00	0.09	0.0981	35.1	68.3
5	1.50	0.07	0.1710	3.9	69.0
5	1.50	0.09	0.0981	-3.4	18.3
5	2.00	0.07	0.1710	-31.9	19.0
5	2.00	0.09	0.0981	-46.3	-31.7
8	1.00	0.07	0.2736	36.9	190.4
8	1.00	0.09	0.1570	36.4	109.2
8	1.50	0.07	0.2736	4.5	140.4
8	1.50	0.09	0.1570	3.3	59.2
8	2.00	0.07	0.2736	-27.3	90.4
8	2.00	0.09	0.1570	-33.5	9.2
15	1.00	0.07	0.5130	46.5	356.9
15	1.00	0.09	0.2943	37.3	204.8
15	2.00	0.07	0.5130	-26.7	256.9
15	2.00	0.09	0.2943	-27.2	104.8
15	3.00	0.07	0.5130	-90.9	156.9

Notes: Risk-free rate, *r*, equals 2%. Expected growth rate of tax base equals 1.1%. Social Security tax rate, τ, equals 0.1875. Length of each period equals 30 years. The second moment for the stock price is discussed in the paper.

FIGURE 2

Value of Implicit Put and Call Options Underlying Performance Guarantees for $\chi = 1$



Note: not drawn to scale.

TABLE 3

Implicit Strike Price, *X*,
Value of Implicit Put Option, P^P, and
Value of Implicit Call Option, P^C,
for an Expected Benefit Level Equal to ψ *times* the Social Security Benefit
(Guaranteed Benefit Level Equal to Current Social Security Benefit: χ = 1)

Expected Benefit	Implicit Strike Price	Implicit Put Option Price	Implicit Call Option Price	Memo: Put Value as Fraction of Value at ψ = 1	Memo: Call Value as Fraction of Value at ψ = 1
ψ	X	P^P	P^C	$P^P(\psi)/P^P(1)$	$P^C(\psi)/P^C(1)$
1	13.27	6.36	0.03	1 / 1.0	1.0
2	6.63	2.81	0.15	1 / 2.3	4.4
3	4.42	1.72	0.28	1 / 3.7	8.2
4	3.32	1.22	0.39	1 / 5.2	11.6
5	2.65	0.95	0.49	1 / 6.7	14.4
6	2.21	0.78	0.56	1 / 8.1	16.6
7	1.90	0.67	0.62	1 / 9.5	18.4
7.32	1.81	0.64	0.64	1 / 10.0	18.9
8	1.66	0.58	0.67	1 / 10.9	19.8
9	1.47	0.52	0.70	1 / 12.3	20.9
10	1.33	0.47	0.73	1 / 13.6	21.8
11	1.21	0.42	0.76	1 / 15.0	22.5
12	1.11	0.39	0.78	1 / 16.4	23.1
13	1.02	0.36	0.79	1 / 17.8	23.6
14	0.95	0.33	0.81	1 / 19.3	23.9
15	0.88	0.31	0.82	1 / 20.8	24.3

Notes: Risk-free rate, r, equals 2 percent. Length of each period equals 30 years. Expected return to equities equals 9 percent. The second moment for the stock price is discussed in the paper.