Submitted: ApJ, May 2008

Sharpening the Tip of the Red Giant Branch

Barry F. Madore, Violet Mager & Wendy L. Freedman

The Observatories
Carnegie Institution of Washington
813 Santa Barbara St., Pasadena, CA 91101

barry@ociw.edu, vmager@ociw.edu, wendy@ociw.edu

ABSTRACT

We introduce a modified detection method for measuring the luminosity of the tip of the red giant branch (TRGB) by introducing the composite magnitude $T \equiv I - \beta[(V-I)_{\circ} - 1.50]$, where β is the slope of the tip magnitude as a function of color (or metallicity). The method is specifically designed to account for known systematics due to metallicity. In doing so, this simple transformation does away with arbitrary color selections in measuring the tip, and thereby significantly boosts the population of resolved stars that go into defining the TRGB distance. Moreover this method coincidentally reduces the impact of reddening on the true modulus as well as its final uncertainty.

Subject headings: Galaxies: Distances and Redshifts, Galaxies: Stellar Content, Stars: Population II

1. Introduction

The luminosity of the tip of the red giant branch (TRGB) has been profitably used by many groups to determine high-precision distances to nearby galaxies (for example, see the compilation of Ferarrese et al. 2001). And, by all measures this method continues to grow in popularity (e.g., Rizzi et al. 2007 and references therein). Its success is driven, in all probability, by its low cost in observing time, its conceptual simplicity, and its wide range of application (see Madore & Freedman 1998 for a review).

In our earliest paper on quantifying the procedures for measuring the magnitude of the TRGB tip (Lee, Freedman & Madore 1993) we first recommended the use of a digital

(Sobel) filter operating on a binned histogram representation of the RGB luminosity function. Being aware of the slight dependence of the RGB tip on metallicity (as manifest by a monotonic decline of the TRGB I-band magnitude with increasing color) we also outlined (a regretably convoluted) means of correcting the observed tip magnitude back to a fiducial metallicity/color-corrected tip magnitude based largely on theory. With the clarity of hindsight (drawing on better data, with continuing theoretical support) we now correct that shortcoming and suggest a means of measuring a TRGB distance modulus that is explicitly corrected for metallicity at the filtering level.

In correcting for metallicity the new methodology introduced below is simple, but its additional utility and power comes from the fact that it allows the use of a much enhanced sample of tip stars (independent of their metallicity) to bolster the statistical certainty of the distance modulus determination. The method outlined below is, at the same time, both systematically and statistically more powerful and robust than previously-used methods.

2. The T_{RGB} Magnitude

We begin by introducing the T magnitude for the red giant branch (RGB). T_{RGB} is the color-corrected I-band magnitude specifically designed to be independent of and insensitive to metallicity (see Figure 1). By construction

$$T_{RGB} \equiv I_\circ - \beta[(V - I)_\circ - \gamma]$$

where the color slope β is chosen to track the known, and now calibrated, (metallicity-induced) run of tip magnitude as a function of intrinsic color, and γ is the fiducial color (i.e., metallicity) to which the absolute magnitude of the TRGB distance scale is referred.

The value of β can be estimated from theory (see Mager et al. 2008 for a recent example) and/or derived from observations. For a restricted range of intrinsic colors, 1.5 < (V-I)$_\circ$ < 3.0 we provisionally adopt a slope of $\beta = 0.20 \pm 0.05$, which is consistent with both the value of 0.22 $\pm$ 0.02 given by Rizzi et al. (2008) for a sample 5 galaxies (NGC 0300, NGC 0598, NGC 1313, NGC 4258 and NGC 5128), and the value of 0.15 based on theory (Mager et al.) The fiducial metallicity, [Fe/H] = -1.7 dex, corresponds to the bright, low-metallicity end of the RGB which is marked by stars at the tip having a color of γ = (V-I)$_\circ$ = 1.5 mag. The range over which the calibration is valid extends from here to (V-I)$_\circ$ = 3.0 mag, which, for Pop II stars corresponds to a high metallicity of -0.5 dex. At the fiducial color of (V-I)$_o$ = 1.5 mag our absolute magnitude zero point is set to be M$_{TRGB}$ = -4.05 mag. Thus

$$\mu_{TRGB} \equiv T_{RGB} - M_{TRGB} = T_{RGB} + 4.05$$

$$\mu_{TRGB} = I(RGB)_{\circ} - 0.20[(V-I)_{\circ} - 1.50] + 4.05$$

and finally, for the observers:

$$\mu_{TRGB} = I(RGB)_{\circ} - 0.20(V-I)_{\circ} + 4.35$$

We have noted on previous occasions that, on astrophysical grounds, it only makes sense for a halo population to contain at least its most metal-poor stars, given that the low-mass component persists and they (or rather their originally coeval, high-mass counterparts) are responsible for the enriched material that may or may not produce later (higher metallicity) generations. That is, it would be very hard to have a pure population of high-metallicity Population II stars in the halo of a galaxy without its low-metallicity (progenitor) component. Thus one will always have a low-metallicity sample of stars; and they will be the first stars detected by the edge filter, because they are also the brightest TRGB population in any mixture (in the I band).

The real-world limitation comes in having enough stars to fill the luminosity function at the tip so as not to (downwardly) bias the detection magnitude. Simulations addressing this very point (Madore & Freedman 1995) suggested that about 100 stars in the magnitude bin just below the tip are mimimally required to give a credible detection at the ± 0.1 mag level. As we note below, that may be optimistic.

If one includes only stars of a given color/metallicity when measuring the tip one then selectively reduces the sample of stars available for the measurement. On the other hand, by using the metallicity-corrected sample as advocated here, one can take full advantage of all stars at the tip, not just those in a narrowly-defined window of color and metallicity space. If one were to simply open the color range admitted to the edge-detector without correcting for the color slope one would certainly increase the sample, but this would clearly be at the expense of blurring the TRGB discontinuity because of the progressive bias/contamination by fainter redder (higher-metallicity tip) stars in the transition region. By seeking a $1/\sqrt{N}$ decrease in the random error, one would not only be blunting the tool, but also implicitly inviting an increased systematic error. The method suggested here corrects for the systematics of metallicity and allows all available tip stars to contribute to the tip detection.

3. The New Filtering Methodology

In this section we discuss details of the application of this suggested method for TRGB detection and distance determination.

We first need to apply a reddening correction to the observed colors and apply the appropriate extinction correction to the apparent magnitudes. The halos of most galaxies are reasonably assumed to be dust and extinction free, but any given line of sight may have a Galactic foreground component. This is usually dealt with by adopting a predicted value either from the Burstein & Heiles (1982) or the Schlegel et al. (1998) Galactic extinction maps.[1]

We next adjust all of the I-band magnitudes using their color difference with respect to the fiducial color, mentioned above, $(V - I)_\circ = 1.50$ mag. The fiducial color corresponds to the most metal-poor and brightest population of TRGB stars expected in our galaxies. By differentially boosting the I-band magnitudes of the stars according to their individual colors by -0.20[(V-I)-1.50] mag, we are in effect forcing all tip stars, regardless of their metallicities, to take on the same TRBG magnitude.

We can now run the Sobel filter straight down in magnitude space and be confident that it will directly encounter the full TRGB at right angles. This guarantees complete and

[1]It would, of course, be desirable not to have to depend on foreground dust modelling to determine reddening corrections, especially if there were some unanticipated dust component in the host galaxy. Indeed, RGB data alone could be used to determine a total line-of-sight reddening. This would require adequate samples of stars below the TRGB to be observed to reasonably good levels of photometric precision. The method proposed here would equate the total line-of-sight reddening with the difference in color between the blue envelope of the observed population of RGB stars compared to the intrinsic color locus of the most metal-poor calibrating RGB population. Again, this first population must logically be in the stellar population mix and it will exclusively populate the blue edge. More metal-rich, intrinsically redder, stars will only fall in one direction, away (to the red) from this envelope and will not blur or bias any reddening determination if the blue envelope is well defined. Few data sets currently go deep enough below the TRGB, at the levels of photometric precision required, for this method to be widely applied to existing observations. That, of course, could change if the reddening correction were deemed to be critical in any given specific case. An actual implementation of a self-consistent reddening solution would need to be iterative with the distance determination itself. The method would boil down to fitting the RGB in both magnitude and color space by iteratively determining the apparent magnitude of the tip, and fitting the blue envelope until both the true modulus and total reddening are self-consistently solved for. Methods equivalent to the tip detection and its measurement would have to be deployed to detect and quantitatively measure the blue edge of the RGB. We defer that implemenation to a future paper, but note, for the interested reader, that Rizzi et al. (2007) give a detailed discussion of other methods for independently determining the reddening which include the red clump, the lower portion of the RGB and, when available in composite systems, the blue main sequence.

unbiased participation of all tip stars in the solution; it sharpens the filter output response; and it eliminates the need for subsequent corrections for mean color or mean metallicity of the population as a whole.

In practice the data can be binned to a resolution of 0.01 mag without compromising the precision of the method. This is because such a bin size exceeds the photometric precision for all but the very brightest stars in most studies. Typical photometric errors on stars in the data set to be discussed here are at the TRGB are ±0.03 mag. The basic filter used supports a traditional [-1, 0, +1] kernel; however, it can be adjusted to smooth over any number of bins, either to beat down photometric noise or to decrease the shot noise in sparse samples. That is, a 3-bin smoothing would effectively employ a [-1, -1, -1, 0, 0, 0, +1, +1, +1] kernel, smoothing the data and the output over 0.03 mag intervals. Other kernel weighting schemes are clearly possible. But the resolution of the tip will never be any better than the width of the kernel smoothing adopted.

It is useful to estimate the statistical uncertainty in the filter response to Poisson noise in the binned star counts. Since the filter response is basically N_3 - N_1, then the error on that difference is simply $\pm\sqrt{{N_3}^2 + {N_1}^2}$. The ratio of those two quantities, (N_3 - N_1)/$\sqrt{{N_3}^2 + {N_1}^2}$ is a form of χ^2, giving not just the output of the filter, but rather the statistically quantified significance of the output. We plot this ratio in all of the subsequent figures, and use it as a count of the number of "sigmas of significance" to be associated with any given response. Furthermore, measuring the semi-width of the response one unit down from the peak will be taken as the one-sigma uncertainty on that detection.

We now illustrate the efficacy of this method by examining the various options as applied to actual data for the halo of NGC 4258.

4. Experimenting on Real Data

In the following we make a first pass at illustrating some of the above claims about the reduced statistical and systematic uncertainties expected to be associated with the metallicity-corrected edge detector, as compared to its predecessor(s) which employed standard filtering at fixed metallicity or color.

For this demonstration we use the ACS data obtained by us for a TRGB distance determination to the maser galaxy NGC 4258 (Mager et al. 2008). Over 30,000 stars were in the analysis. These data provide a nice test case given that the sample size of RGB stars is large, the photometric errors (at the tip) are small (see below) and this particular population shows a significant, but not atypical, one-magnitude spread in (V-I) color.

4.1. Decreased Sample Size and Bias

In this section we quantify the effects of focussing the tip detection algorithm more and more narrowly on the fiducial metallicity (color) of [Fe/H] = -1.7 dex as represented by stars with colors of (V-I) = 1.5 mag. The filtering algorithm is fixed for all of these runs on the NGC 4258 dataset and only stars with formal errors of less than 0.30 mag are considered. Figures 2 through 5 each show the color-magnitude diagram in the left panel, and the edge-detection χ^2 filter output aligned to the right. As stated above the half-width of the filter response one unit down from its maximum will define our one-sigma figure of merit for the fitting.

We begin the process with no color selection imposed.

In this particular run there were 10,200 stars in the one-mag (fainter) interval leading up to the TRGB. The tip was measured at I = 25.39 ± 0.11 mag at a 2.6-sigma significance level (Figure 2). Individual stars at this same magnitude level have typical photometric errors of $\pm(0.02 - 0.03)$ mag. In other words, they are contributing about a third of the quoted variance (σ^2) for the tip magnitude. The projected (metallicity-spread induced) magnitude extent of the TRGB over the color range of 1.5 < (V-I) < 3.0 is about 0.25 mag, which would have an equivalent sigma of ±0.08. "Tip tilt" is clearly the dominant source of uncertainty imposed on this measurement of the TRBG magnitude: blurring (and biasing, see below) the value that would otherwise be obtained at the fiducial [Fe/H] = -1.7 value alone.

We now investigate the effect of decreased sample size on the precision and accuracy of the tip detection algorithm. In essence this is the same test that Madore & Freedman (1995) ran on controlled/simulated data; this test is run on real-world photometry with all of the subtleties and complexities that may or may not be in any given simulation. In any case, the NGC 4258 data were progressively restricted in sample size, holding all other parameters of the tip detection fixed (i.e., maximum error on the photometry of 0.3 mag, and three-bin smoothing of 0.03 mag on the Sobel filter).

Two examples are shown in Figures 4 and 5. The first illustrates a high significance (1.9 sigma) detection of the tip for a much reduced sample size, having only about 500 stars in the upper magnitude bin. The second figure illustrates the effect of reducing the sample further and the resulting bias in the most significant tip "detection" Only about 200 stars were in the magnitude bin below the (real) tip and the bias is significant.

The summary results for these, and 17 other test samples, are shown in Figure 6. The individual tip detections are shown as circled points. The dashed horizontal line shows the asymptotic value of the tip detection for the full sample of 7,500 stars in the one-magnitude bin below the TRGB. For sample sizes in excess of 400-500 stars the solutions agree to within ±0.1 mag (peak-to-peak) which brackets the commonly quoted statistical uncertainty in this method. However, for smaller sample sizes the detected tip magnitude begins to systematically deviate from the fiducial value. The bias should come as no surprise: as stars drop from the sample there is nowhere for the tip detector to go but to fainter magnitudes where there are stars and possibly structure in the RGB luminosity function to trigger on. Of course the significance of these false tips drops too. Indeed, at sample sizes below about 400 stars in the upper magnitude bin below the tip the significance of any "detection" starts to drop below the 1.5-sigma level.

In light of these experiments on real data we now revise and extend our statement concerning the minimum sample sizes needed for TRGB detection. For samples having more than 400 stars in the magnitude bin just below the TRGB the significance of the tip detection should be expected to be at the 1.5-sigma level or better, with a statistical uncertainty in the measured tip magnitude being less than ±0.1 mag. At this level of sample size there are enough stars to fill the luminsity function up to and including a definition of the discontinuity.

For sample sizes below this critical value the filling function pulls away from the discontuity and there is a steep and systematic roll-off of the (false) tip magnitude toward fainter apparent magnitudes, with the bias exceeding 0.5 mag at 200-300 stars. Not surprisingly these false detections will generally be at low (∼one-sigma) significance levels and should be treated with all due suspicion. Alternatively, at low count rates one should seriously consider using maximum-likelihood estimation techniques for measuring the tip, as discussed by Makarov et al. (2006); they demonstrate reliable detections when as few as 50 stars are in the one-magnitude bin below the TRGB.

One final cautionary note: The number of stars below the tip in these experiments was the number below the known level of the tip. In practice that value is not known a priori, and so in the regime where bias sets in that number will be artificially high and the a posteriori estimate of the bias will be artificially low.

4.2. Another Form of Bias

What we have not discussed yet, and what is not possible to address with the NGC 4258 data, is an opposite bias (toward incorrectly brighter tip magnitudes). This type of bias may be induced by young RGB and/or bright AGB stars, especially those resulting from a superimposed population of disk stars. Clearly using the T_{RGB} magnitude allows one to optimize the population of stars going into the tip definition. In practice this means that for a fixed field of view one can go to fields where the star density is lower than might have been required for a standard (color-restricted) tip detection. Moving further out into the halo will reduce any potential contamination due to disk stars (in those galaxies that have disks), and this may be of importance in planning future TRGB observations of composite population systems where the halo is weak and "disk" contamination is potentially large. Being able to (radially) distance one's self from the young and/or intermediate-aged populations will fast become important in these cases. False signals from AGB populations in barely resolved galaxies could result in erroneously small distance moduli, as may well be the case of the reported TRBG distance to the Antennae galaxies (Saviane et al. 2004), the data for which were extracted from a Population I rich "disk" field (Schweitzer et al., in preparation)

4.3. A Small Bonus

We close out this section with a note about the sensitivity of the T_{RGB} magnitude to reddening. It is a (happy) coincidence that the slope of the relation (magnitude versus color) that we are using to correct TRGB magnitude for metallicity is in the same general direction as the interstellar extinction/reddening trajectory. So when we correct the I-band magnitude by subtracting 0.2 (V-I) we are also implicitly subtracting 0.2E(V-I) from I. This effectively amounts to reducing the impact of reddening on the I-band magnitude from A= 1.4 E(V-I) to A_T = 1.26 E(V-I), with the same 10% downward scaling applying to propagating the uncertainty in the extinction. A small gain, but still in the right direction.

5. Discussion and Conclusions

As relatively insensitive to metallicity as the I-band tip of the red giant branch is, there is still observed to be a slight dependence of the TRGB magnitude on the metallicity as tracked by the intrinsic color. We have the means to account for it, since that dependence has been found to be extremely stable from galaxy to galaxy and is well calibrated (and indeed well understood). We have incorporated that new information into a very simply modified TRGB edge detector that makes distance determinations using the TRGB explicitly independent of the mean metallicity and/or metallicity distribution of the stars in a given halo population. In the process this new method allows all tip stars to add to the statistical significance of the detection of the core helium ignition discontinuity. As an added bonus the modified method is also slightly less sensitive (by 10%) to reddening corrections (and/or uncertainties in those same reddening corrections) as compared to the standard (fixed-metallicity) option.

The method does, of course, now require that both the V and I magnitudes be obtained for the color-correction to be applied. Previous applications measuring the tip discontiuity from only an I-band luminosity function could not be corrected for metallicity effects, were always met with justifiable skepticism, and are not to be recommended.

We thank the referee, Brent Tully, for his careful reading of the paper and his useful suggestions and comments.

REFERENCES

Burstein, D., & Heiles, C. 1982, AJ, 87, 1165

Lee, M.G., Freedman, W.L., & Madore, B.F. 1993, ApJ, 417, 553

Madore, B.F., & Freedman, W.L. 1998, "Stellar Astrophysics for the Local Group" VII Canary Islands Winter School, eds. A. Aparicio, A. Herroro & F. Sanchez, Cambridge University Press, Cambridge, p. 234

Madore, B.F., & Freedman, W.L. 1995, ApJ, 109, 1645

Mager, V., Madore, B.F., & Freedman, W.L. 2008, ApJ, in press

Rizzi, L., Tully, R.B., Makarov, D., Makarova, L., Dolphin, A.E., Sakai, S., & Shaya, E.J. 2007, ApJ, 661, 816

Makarov, D., Makarova, L., Rizzi, L., Tully, R.B., Dolphin, A.E., Sakai, S., & Shaya, E.J. 2006, ApJ, 132, 2729

Saviane, I., Hibbard, J.E., & Rich, R.M. 2004, AJ, 127, 660

Schlegel, D.J., Finkbinder, D.P., & Davis, M. 1998, ApJ, 500, 525

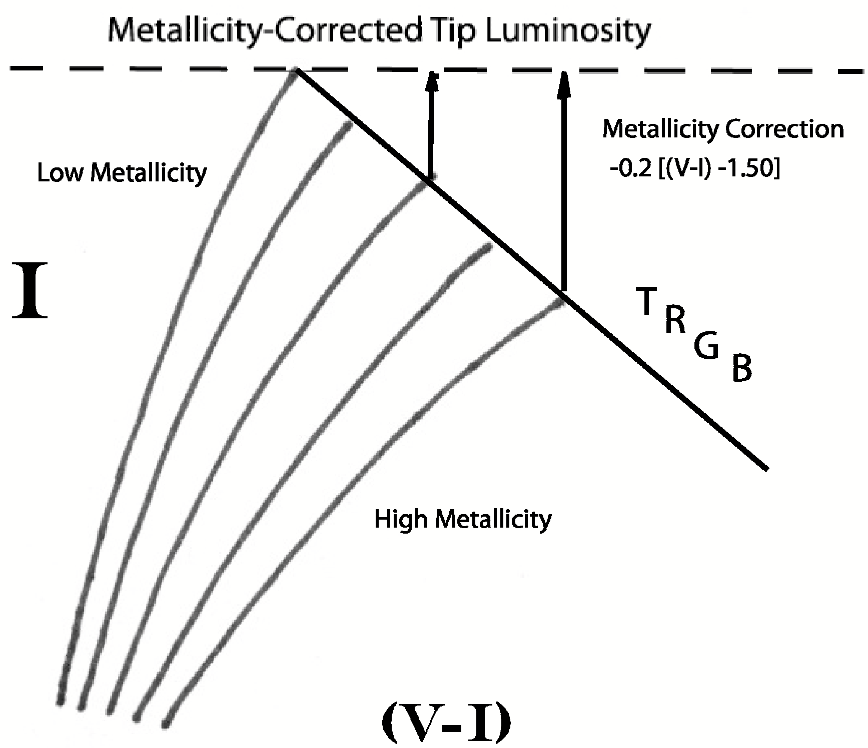

Fig. 1.— Schematic representation of a variety of red giant branches in the I-(V-I) Color-Magnitude Diagram. The terminal points of the RGB are delineated by the downward sloping line marked TRGB. Low-metallicity (high-luminosity) giant branches are shown to the left (blue); high-metallicity giant branches arc up to the right (red). A color-dependent metallicity correction as applied to the I-band lumniosity are shown by vertical arrows. These corrections flatten the TRGB in magnitude space and scale them all to the lowest-metallicity track.

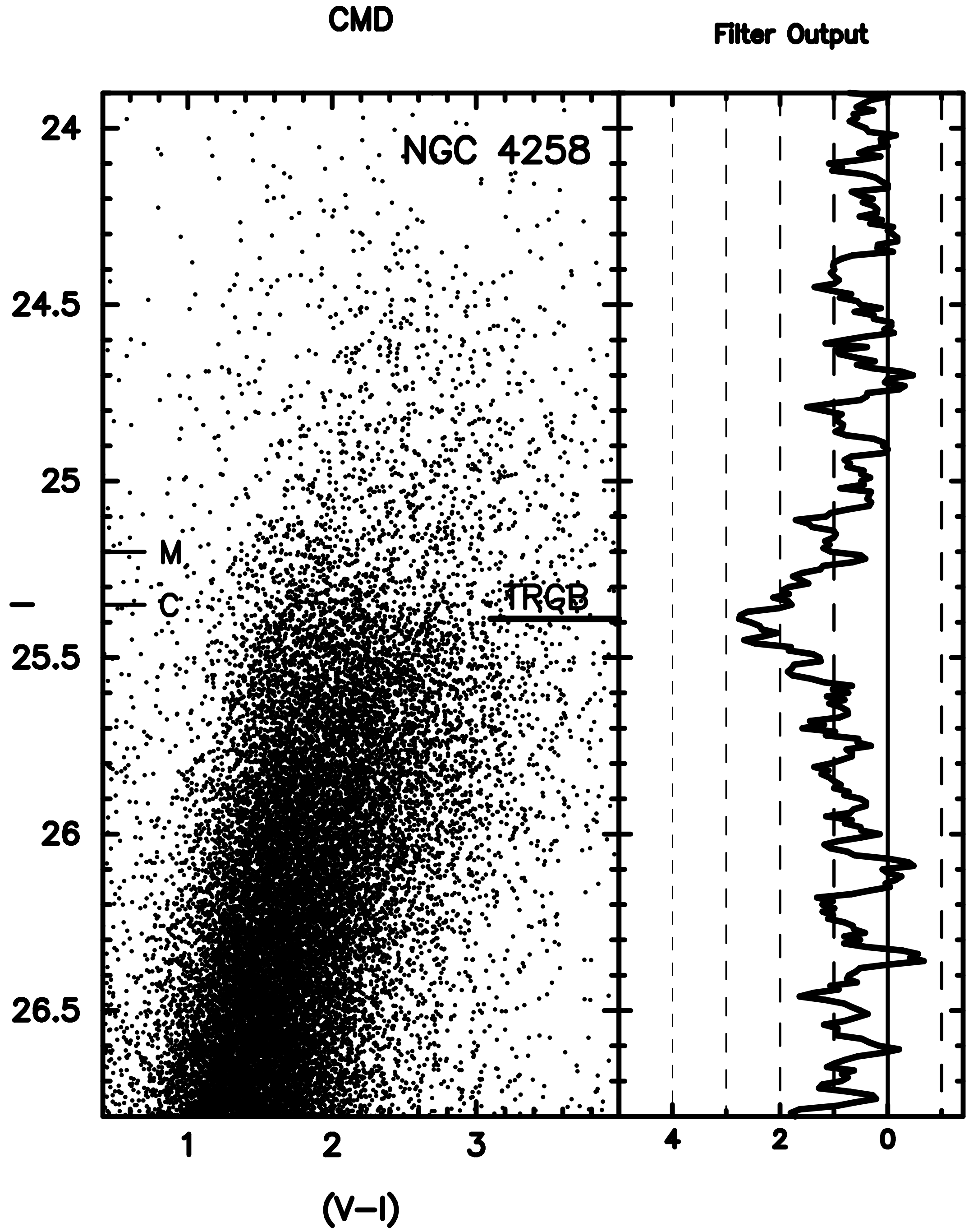

Fig. 2.— I vs (V-I) Color-Magnitude Diagram (CMD) for NGC 4258 to the left, and the normalized Sobel filter response shown to the right. The tip is measured to be at I = 25.39 ± 0.11 mag at the 2.6-sigma significance level. M and C mark the expected magnitude of the tip based on the maser and Cepheid distances, respectively.

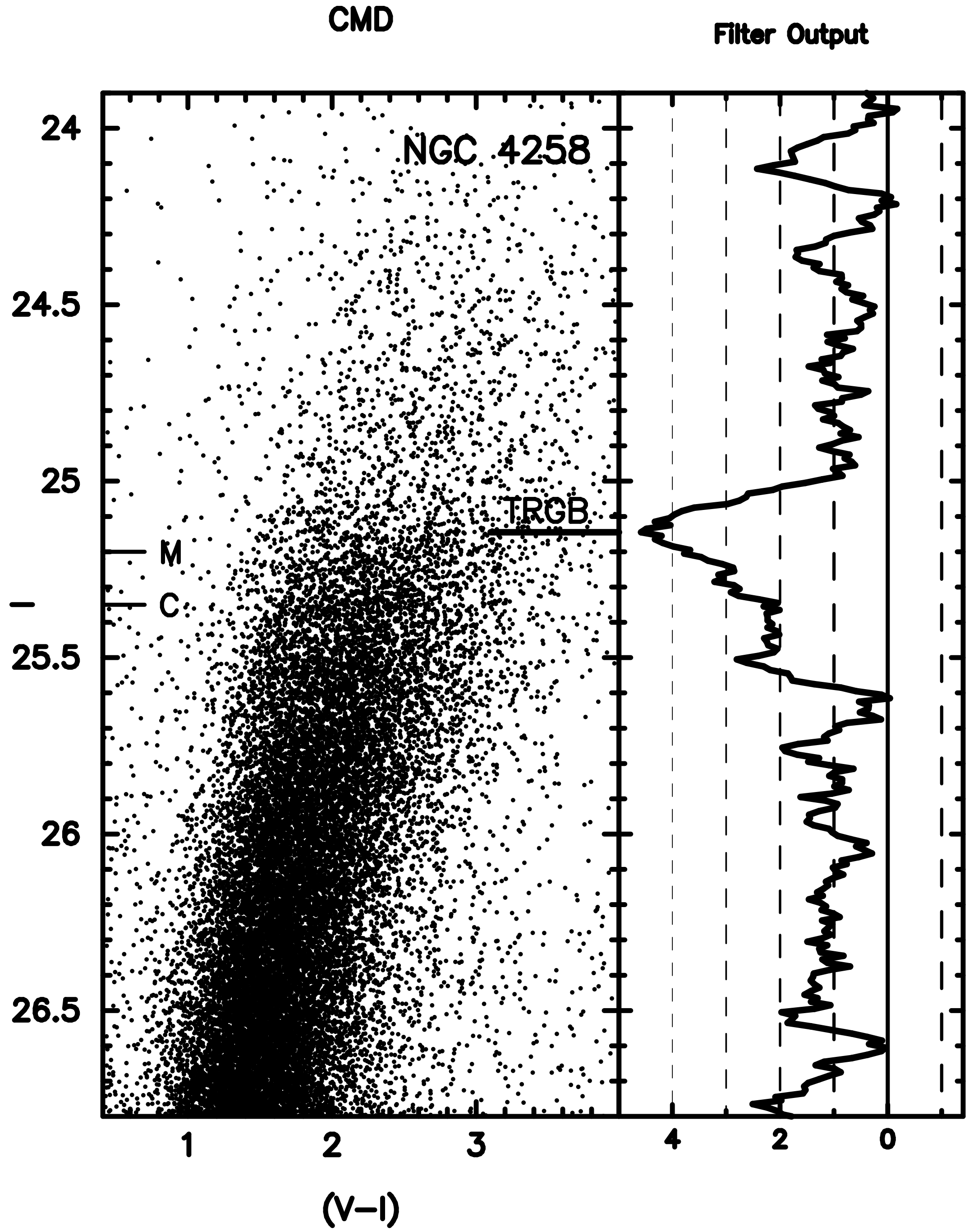

Fig. 3.— T_{RGB} vs (V-I) Metallicity-Corrected Color-Magnitude Diagram for NGC 4258 (see text) to the left, and the normalized Sobel filter response shown to the right. The response of highest (relative) significance is marked at I = 25.21 mag with a reported significance of 4.8 sigma.

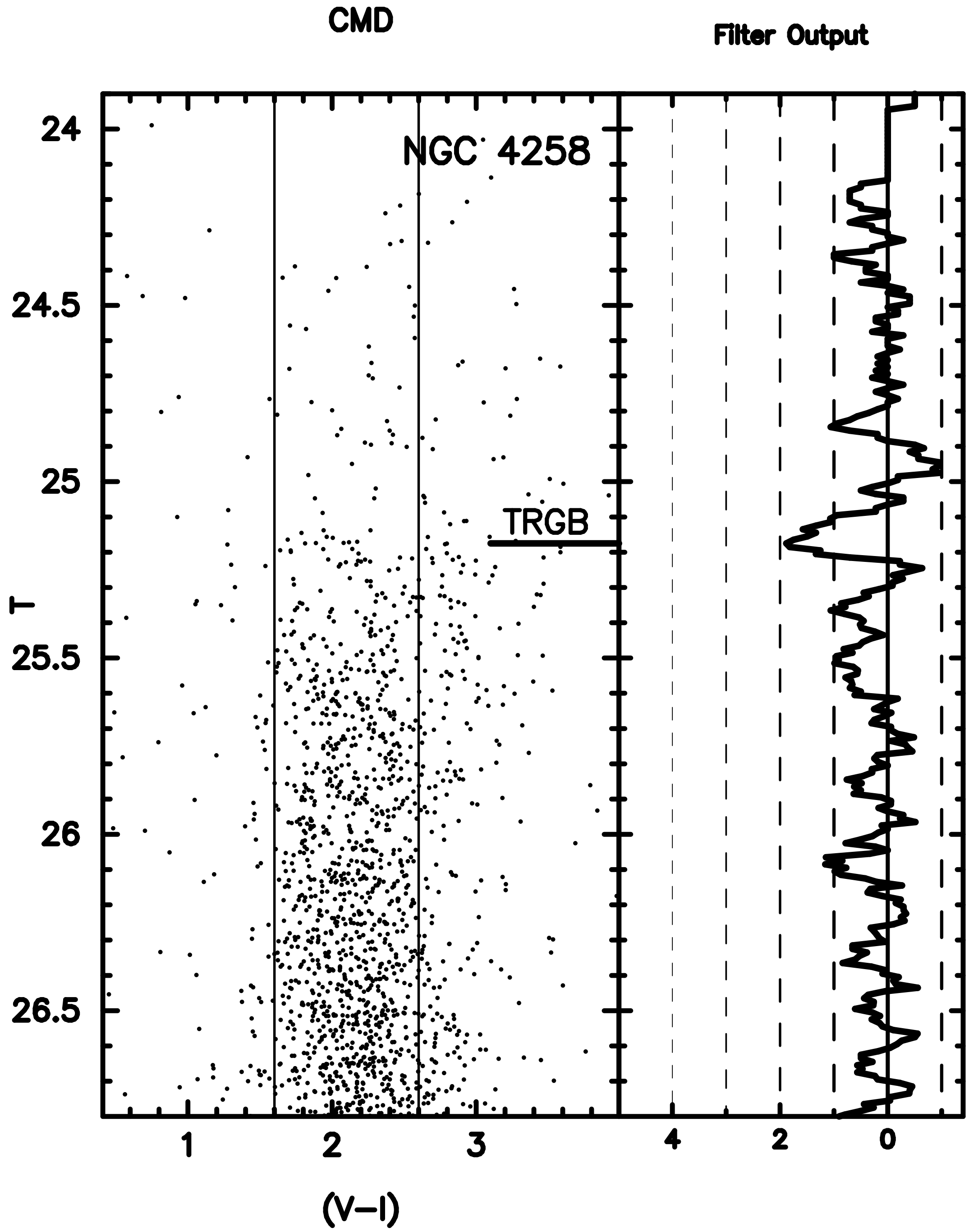

Fig. 4.— Same as Figure 3 except for a reduced sample of NGC 4258 stars. Only one star in 14 was used (amounting to 540 stars in the one-magnitude bin below the tip) for this tip-detection test. The response of highest (relative) significance is marked at I = 25.18 mag with a reported significance of 1.9 sigma.

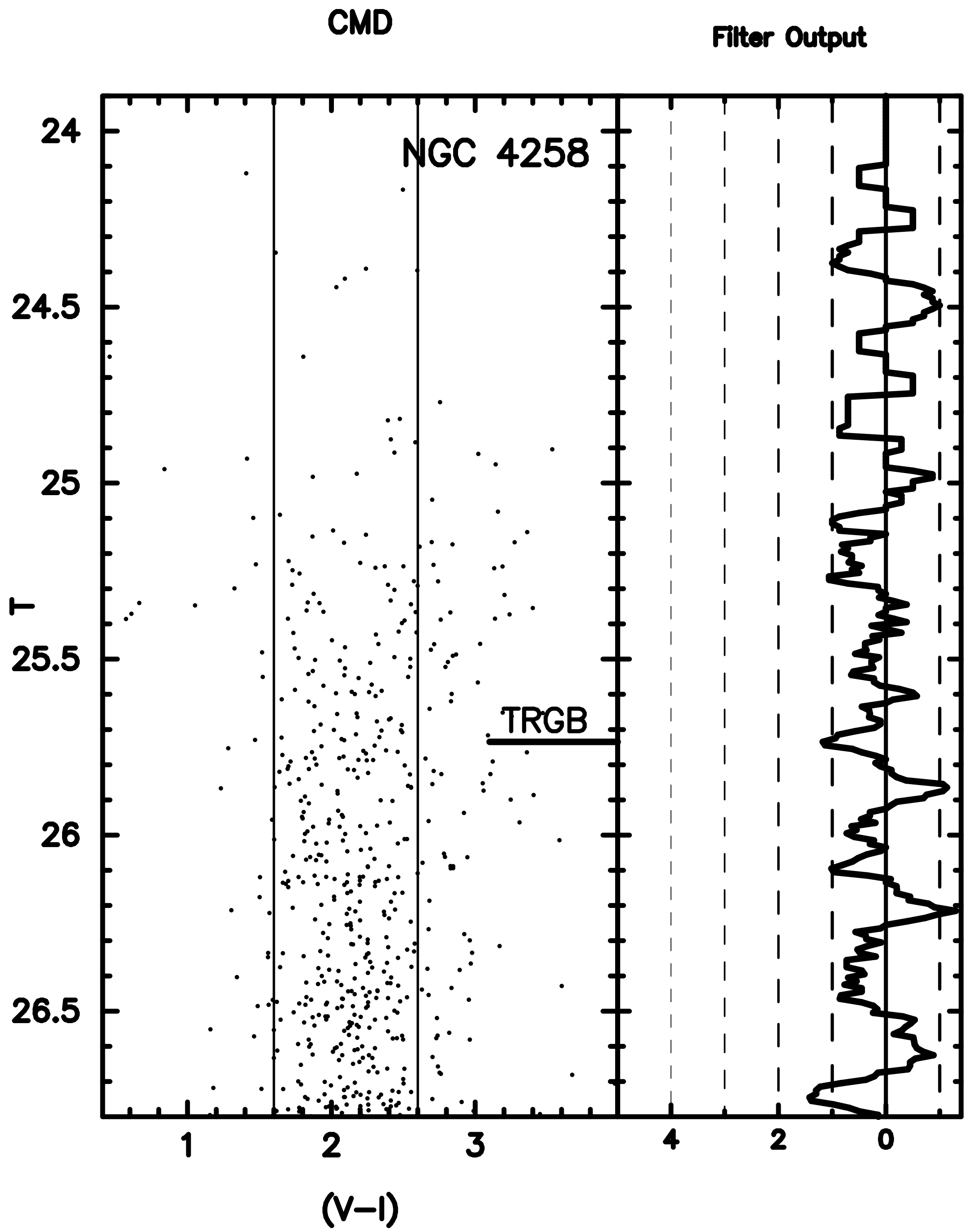

Fig. 5.— Same as Figure 3 except for a much reduced sample of NGC 4258 stars. Only one star in 40 (amounting to only 192 stars in the one-magnitude bin below the tip) was used for this tip-detection test. The response of highest (relative) significance is marked at I = 25.74 mag but its absolute significance is low (1.2 sigma) and a number of other false positives of similar significance are seen throughout the plot.

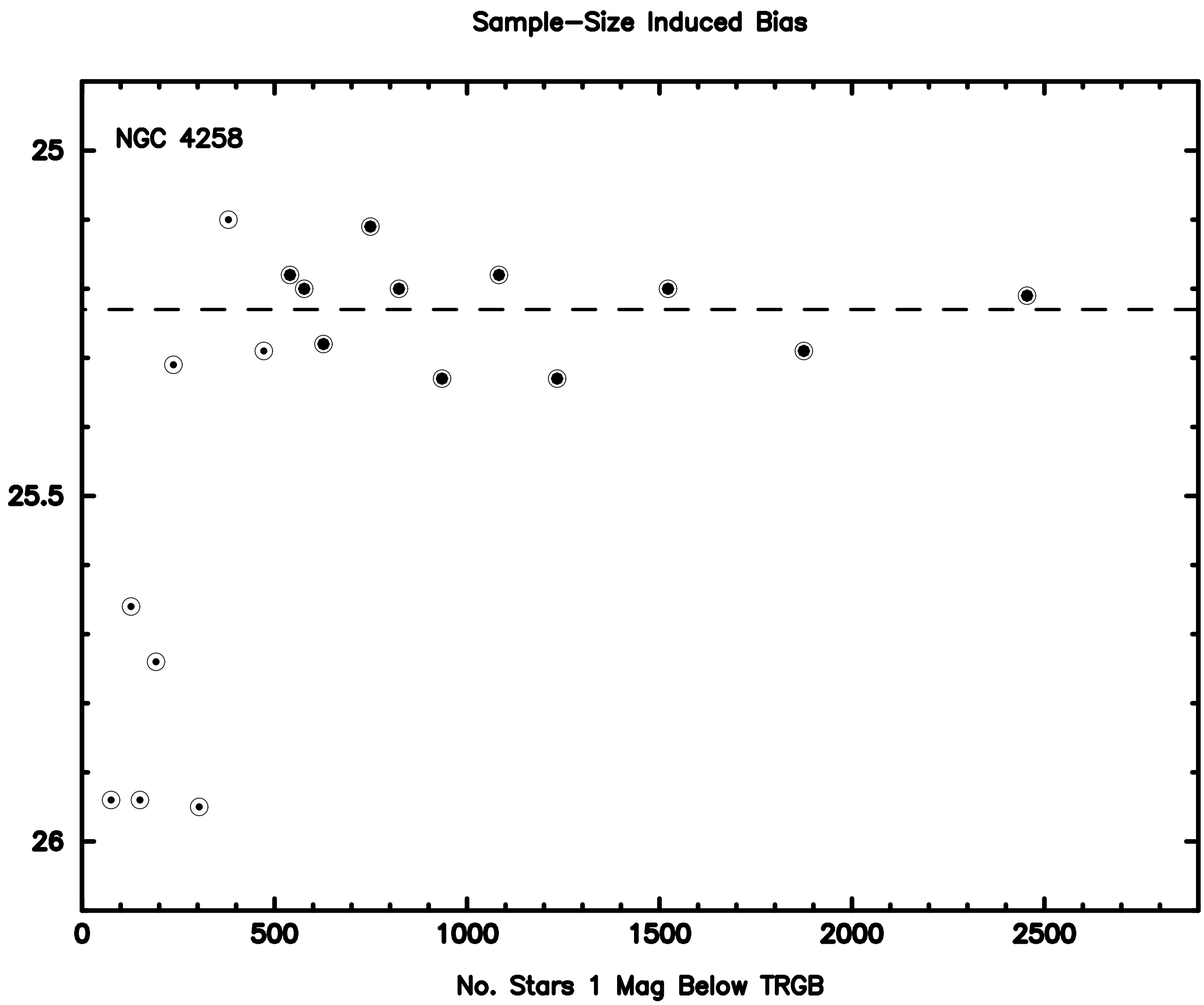

Fig. 6.— The run of derived TRGB magnitude with the number of stars in the magnitude bin below the tip. The dashed line indicates the true value of the tip magnitude as defined by the largest sample size available to us (around 7,000 stars in the one-magnitude bin below the tip). With Poisson noise rising up to about 0.1 mag at the 400-500 star level the tip detections are relatively unbiased for larger samples. Below that level there is a precipitous fall-off in the apparent magnitude of the tip resulting in a bias exceeding 0.5 mag for sample sizes less than 200-300 stars in the upper magnitude bin. Large filled circles have detections that are at, or above, the 1.5-sigma (∼85%) confidence level.