Inflation scenario via the Standard Model Higgs boson and LHC*

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Abstract

We consider a quantum corrected inflation scenario driven by a generic GUT or Standard Model type particle model whose scalar field playing the role of an inflaton has a strong non-minimal coupling to gravity. We show that currently widely accepted bounds on the Higgs mass falsify the suggestion of the paper arXiv:0710.3755 (where the role of radiative corrections was underestimated) that the Standard Model Higgs boson can serve as the inflaton. However, if the Higgs mass could be raised to ~ 230 GeV, then the Standard Model could generate an inflationary scenario with the spectral index of the primordial perturbation spectrum $n_s \simeq 0.935$ (barely matching present observational data) and the very low tensor-to-scalar perturbation ratio $r \simeq 0.0006$.

1 Introduction

This is a challenging task to understand the nature of a fundamental particle physics model that underlies the inflationary scenario of the early Universe [1, 2, 3, 4, 5, 6] leading to generation of scalar [7, 8, 9, 10, 11] and tensor [12] perturbations, the former leading to the formation of observable structure of the Universe. Early attempts to model inflation in terms of a self-interaction Higgs-like scalar field φ minimally coupled to gravity faced the necessity to assume an extremely small coupling constant $\lambda \sim 10^{-13}$ of its quartic self-interaction $\lambda\varphi^4/4$ – a natural candidate for the inflaton potential motivated by particle phenomenology [6]. Initially this situation was considered very unfavorable from the viewpoint of post-inflationary reheating, whereas now this simple model is actually ruled out by the present observational data, see e.g. [13].

It was also observed long ago that the problem of small λ can be circumvented by adding to the Einstein term in the action the non-minimal coupling term $\xi\varphi^2 R/2$ with a very large coupling constant ξ, because in this case the CMB anisotropy $\Delta T/T \sim 10^{-5}$ is given by the ratio $\sqrt{\lambda}/\xi$ [14, 15, 16, 17]. Therefore, such smallness of $\Delta T/T$ can be obtained even for λ close to unity (but still small enough to justify perturbative expansion in λ) if ξ is taken very large.

*To the memory of John Archibald Wheeler.

This model was considered from the viewpoint of quantum cosmology with the tunneling cosmological wave function in [18, 19, 20], where it was shown that quantum effects of matter fields are crucial both for the formation of the initial conditions for inflation [18] and its dynamics [19]. Since inflation may well be related to the GUT scale of particle physics, in [18, 19] the matter content of the model was taken to be of a generic GUT type with the inflaton belonging to the scalar multiplet of the GUT theory.

Recently it was advocated that in fact this non-minimally coupled inflaton can be the Higgs boson of the Standard Model (SM), and no new particles besides already present in the electroweak theory are required to produce inflation with cosmological perturbations in accordance with the CMB data [21]. This conclusion was achieved within a tree-level approximation of this theory, because its radiative corrections were claimed to be strongly suppressed by a large value of the non-minimal coupling constant ξ.

The purpose of this paper is to show that this conclusion of [21] is erroneous – radiative corrections are actually enhanced by a large ξ. They strongly affect the inflationary dynamics of the Universe in a controllable way, and therefore can be probed by current and future CMB observations and LHC experiments testing SM. In particular, we will show that with a widely accepted upper bound on the Higgs mass, $m_H \simeq 180$ GeV [23], the model of [21] is falsified, but with $m_H \geq 230$ GeV the SM Higgs can drive inflation scenario with a low spectral index $n_s \geq 0.935$ and a very low tensor-to-scalar perturbation ratio $r \simeq 0.0006$.

We consider the cosmological model with the classical Lagrangian density

$$\begin{aligned} \boldsymbol{L}(g_{\mu\nu}, \varphi, \chi, A_\mu, \psi) = & \left(\frac{m_P^2}{16\pi} + \frac{1}{2}\xi\varphi^2\right) R - \frac{1}{2}(\nabla\varphi)^2 - \frac{\lambda}{4}(\varphi^2 - \nu^2)^2 \\ & - \frac{1}{2}\sum_\chi (\nabla\chi)^2 - \frac{1}{4}\sum_A F_{\mu\nu}^2(A) - \sum_\psi \bar{\psi}\hat{\nabla}\psi \\ & + \boldsymbol{L}_{\rm int}(\varphi, \chi, A_\mu, \psi) \end{aligned} \qquad (1)$$

containing the graviton-inflaton sector with a big non-minimal coupling constant[1] $\xi \gg 1$, and a generic GUT or SM sector of Higgs χ, vector gauge A_μ and spinor fields ψ coupled to the inflaton φ via the interaction term

$$\boldsymbol{L}_{\rm int} = -\sum_\chi \frac{\lambda_\chi}{2}\chi^2\varphi^2 - \sum_A \frac{1}{2}g_A^2 A_\mu^2\varphi^2 - \sum_\psi f_\psi \varphi\bar{\psi}\psi + \text{derivative coupling}, \qquad (2)$$

whose structure is dictated by the local gauge invariance. In (1) the inflaton φ can be regarded generically as a component of one of the scalar multiplets, which has a non-vanishing expectation value in the cosmological quantum state. After inflation it settles in the minimum of its potential at the symmetry breaking scale $\varphi = \nu$. This scale is small enough, $\nu^2 \ll M_P^2/\xi$, so that at present the gravitational interaction is mediated by the effective Planck mass squared $M_{\rm eff}^2(\nu) = M_P^2 + \xi\nu^2 \simeq M_P^2$, $M_P \equiv m_P/\sqrt{8\pi} = 2.4 \times 10^{18}$ GeV, and large enough to generate the mass of the φ-particle, $m_\varphi^2 = 2\lambda\nu^2$, which renders its interaction short-ranged and not violating stringent bounds from Solar system tests of gravity.

[1] For our choice of signs, the case of the conformally invariant massless scalar field corresponds to the coupling $\xi = -1/6$.

The paper is organized as follows. In Sec. 2 we present the quantum effective action for the model (1- 2), previously obtained in [19], and show that the contribution from its radiative corrections was underestimated in [21]. In Sec. 3 we investigate how the inflation dynamics of the model is modified by these radiative corrections. In Sec. 4 power spectra of primordial scalar and tensor perturbations are presented and compared with the recent WMAP bounds. It follows that the present observational data put strong constraints on the parameters of the model. In Sec. 5 we show that the Standard Model within the accepted range of the Higgs mass does not satisfy these constraints, but it can fit the CMB data for $m_H \geq 230$ GeV. We finish in Sec. 6 with a general conclusion that the fate of the model may be resolved by LHC tests of electroweak theory, and if nevertheless proved to be viable, this model predicts a very small ratio of tensor to scalar primordial perturbation $0.0006 < r < 0.001$.

2 Effective action

The quantum effective action for the model (1) was calculated in [19]. For a large and slowly varying mean scalar field φ and a large ξ, this calculation is facilitated by the number of properties. Firstly, the non-minimal coupling efficiently implies the replacement of the original Planck mass parameter by $M^2_{\rm eff}(\varphi) = M_P^2 + \xi\varphi^2 \gg M_P^2$. This means that the contribution of the graviton and inflaton quantum loops is essentially suppressed by powers of $1/M^2_{\rm eff} \sim 1/\xi\varphi^2$ [24, 19], and the main contribution comes from quantum loops of matter sector of the model (1) – scalar fields χ, vector bosons A_μ and spinor fields ψ.

Secondly, due to the Higgs mechanism on the background of φ, all these fields acquire large masses, $m(\varphi) \sim \varphi$, following from the non-derivative part of the interaction Lagrangian (2):

$$m^2_\chi = \lambda_\chi\, \varphi^2,\ \ m^2_A = g^2_A\, \varphi^2,\ \ m^2_\psi = f^2_\psi\, \varphi^2. \tag{3}$$

The scale of these masses are much larger than the characteristic scale of the spacetime curvature $R \sim \lambda(\varphi^2 - \nu^2)^2/12M^2_{\rm eff} \sim \lambda\varphi^2/\xi \ll \varphi^2$, and therefore the quantum effective action can be found as a local $1/m^2$-expansion in powers of the curvature, its gradients and the gradients of the background scalar field $\nabla\varphi$. In the approximation linear in $R(g_{\mu\nu})$ and $(\nabla\varphi)^2$, the answer reads [19]

$$S[g_{\mu\nu},\varphi] = \int d^4x\, g^{1/2} \left(-V(\varphi) + U(\varphi)\, R(g_{\mu\nu}) - \frac{1}{2}\, G(\varphi)\, (\nabla\varphi)^2 \right), \tag{4}$$

where the coefficient functions $V(\varphi)$, $U(\varphi)$ and $G(\varphi)$ together with their classical parts contain one-loop radiative corrections of the form

$$V(\varphi) = \frac{\lambda}{4}(\varphi^2 - \nu^2)^2 + \frac{\lambda\varphi^4}{128\pi^2} \left(\boldsymbol{A} \ln\frac{\varphi^2}{\mu^2} + \boldsymbol{B} \right), \tag{5}$$

$$U(\varphi) = \frac{1}{2}(M_P^2 + \xi\varphi^2) + \frac{\varphi^2}{384\pi^2} \left(\boldsymbol{C} \ln\frac{\varphi^2}{\mu^2} + \boldsymbol{D} \right), \tag{6}$$

$$G(\varphi) = 1 + \frac{1}{192\pi^2} \left(\boldsymbol{F} \ln\frac{\varphi^2}{\mu^2} + \boldsymbol{E} \right). \tag{7}$$

Here $\boldsymbol{A}$ is the following combination of Higgs, vector gauge boson and Yukawa coupling constants of the GUT-inflaton interaction Lagrangian (2)

$$\boldsymbol{A} = \frac{2}{\lambda}\left(\sum_{\chi}\lambda_{\chi}^{2} + 3\sum_{A} g_{A}^{4} - 4\sum_{\psi} f_{\psi}^{4}\right). \tag{8}$$

$\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$, $\boldsymbol{E}$ and $\boldsymbol{F}$ are other five combinations of the powers of these constants and their logarithms [22, 9, 19], whose specific form will be inessential in what follows[2], and μ^2 is a normalization point. On the contrary, the constant (8) is very important because, as we will see, its effect is enhanced due to multiplication by $\xi \gg 1$. This constant determines the anomalous scaling behavior of the theory, or its local conformal anomaly. In fact it arises as a coefficient of φ^4 in the sum of quartic powers of particle masses

$$\frac{1}{64\pi^2}\,\mathrm{tr}\sum_{\mathrm{particles}}(\pm 1)\, m^4(\varphi) = \frac{\lambda\varphi^4}{128\pi^2}\boldsymbol{A}, \tag{9}$$

where summation takes into account boson/fermion statistics and the trace is taken over spin-tensor indices.[3] For $\xi \gg 1$ this quantity gives a dominant contribution to the spacetime integrand of the one-loop ζ-function of the theory (subdominant contributions are suppressed by powers of the curvature to mass squared ratio $\sim 1/\xi$) and also determines the coefficient of the logarithmic Coleman-Weinberg potential in (5)

$$\mathrm{tr}\sum_{\mathrm{particles}}(\pm 1)\,\frac{m^4(\varphi)}{64\pi^2}\,\ln\frac{m^2(\varphi)}{\mu^2} = \frac{\lambda\boldsymbol{A}}{128\pi^2}\,\varphi^4\ln\frac{\varphi^2}{\mu^2} + \ldots\,. \tag{10}$$

Note that in the leading order of $\xi \gg 1$ the coefficient $\boldsymbol{A}$ does not have a contribution from the graviton-inflaton sector, which as was mentioned above is suppressed by powers of $1/\xi\varphi^2$. In particular, the expression (8) does not contain in parentheses a typical contribution λ^2 of the Higgs field itself.

Quantum corrections are obviously small for $\boldsymbol{A}/32\pi^2 \ll 1$, but the authors of [19] claimed an additional mechanism of their strong suppression by a large ξ, based on calculations in the Einstein frame of the classical model (1). This frame can be obtained by the conformal transformation, $g_{\mu\nu} \to \hat{g}_{\mu\nu}$, $\hat{g}_{\mu\nu} = \Omega^2(\varphi) g_{\mu\nu}$, $\Omega^2(\varphi) = 1 + \xi\varphi^2/M_P^2$, and a relevant reparametrization of the inflaton field, $\varphi \to \hat{\varphi}$, rendering $\hat{\varphi}$ a canonical normalization of its kinetic term. Under this transformation all the particle masses $m(\varphi)$ in the Einstein frame get rescaled as $\hat{m}(\varphi) = m(\varphi)/\Omega(\varphi) \sim 1/\sqrt{\xi}$ and for $\Omega \gg 1$ become very small and actually independent of φ. This makes according to [21] their Coleman-Weinberg potential small and very flat, so that quantum corrections are not significant for the inflationary dynamics.

However, one should bear in mind that the contribution of this potential to the effective action enters with the factor $\hat{g}^{1/2} = \Omega^4(\varphi)\, g^{1/2}$ which cancels the

[2] In slightly different notations, these combinations were obtained in [19] including the non-logarithmic part of (7). Logarithmic parts of the coefficients $\boldsymbol{A}$, $\boldsymbol{C}$ and $\boldsymbol{F}$ induced by one-loop radiative corrections from vector fields were earlier presented in [9] (see also [22]).

[3] With the masses (3) this yields the expression (8) in which the coefficients 1, 3 and 4 imply respectively one, three and four degrees of freedom of a real scalar field, massive vector field and charged (Dirac) spinor field.

dominant effect of the decrease in mass,

$$\hat{g}^{1/2}\,\hat{m}^4(\varphi)\ln\frac{\hat{m}^2(\varphi)}{\mu^2}=g^{1/2}\,m^4(\varphi)\ln\frac{\hat{m}^2(\varphi)}{\Omega^2(\varphi)\mu^2}\simeq \mathrm{const}\,g^{1/2}\,m^4(\varphi). \tag{11}$$

Therefore, for large φ quantum corrections calculated in the Einstein frame differ from those of the Jordan frame (10) by replacing the log factors with some constants. This is not unusual that the covariant renormalization in different conformally related frames leads to different results, because the theory has a conformal anomaly, and this anomaly yields a weak logarithmic frame dependence. This means that the mechanism of suppression for radiative corrections advocated in [21] should be much weaker – instead of suppression by a power of the conformal factor $\Omega^2\sim\xi\varphi^2/M_P^2$ the radiative corrections in the Einstein frame are suppressed only by its logarithm.

Below we will see that the dominant contribution of quantum corrections to the inflaton rolling force originates from differentiating namely the log factor in the effective potential (5). Therefore the disappearance of these factors in the Einstein frame raises the question of which frame is appropriate for calculating the quantum effects. The original Jordan frame is the right one, because it determines the physical distances in terms of the original metric $g_{\mu\nu}$. In particular, physical (atomic) clocks measure the proper time of just this frame. Covariant renormalization which introduces the logarithmic factors and the normalization scale μ should be performed in terms of this physical metric. This justifies the original Jordan frame and the expressions (5) and (6) containing the correct logarithmic terms.

3 Inflation

We apply the effective action (4)-(7) with $\xi\gg 1$ to study inflationary dynamics in the range of the inflaton field much beyond its current value at the minimum of the classical potential, $\varphi^2\gg M_P^2/\xi\gg\nu^2$. Thus we assume smallness of the following two parameters

$$\frac{M_P^2}{\xi\,\varphi^2}\ll 1,\quad \frac{\boldsymbol{A}}{32\pi^2}\ll 1. \tag{12}$$

It is also natural to assume that other combinations of coupling constants are of the same order of magnitude as $\boldsymbol{A}$, so that the second of bounds above also holds for all of them $(1/32\pi^2)(\boldsymbol{B},\boldsymbol{C},\boldsymbol{D},\boldsymbol{E},\boldsymbol{F})\ll 1$.

In fact (12) guarantees the regime of the slow roll approximation for the system (4). In the leading order corresponding to the omission of $\dot\varphi^2$ terms the equations of motion read [19, 25]

$$\ddot\varphi+3H\dot\varphi-F=0, \tag{13}$$

$$H^2=\frac{V}{6U}-\frac{U'F}{3U}. \tag{14}$$

Here $H=\dot a/a$ is the Hubble parameter, $U'\equiv dU/d\varphi$, and the rolling force $F=F(\varphi)$ in this approximation depends only on φ and reads[4]

$$F(\varphi)=\frac{2VU'-V'U}{GU+3U'^2}. \tag{15}$$

[4]Strictly speaking, in the leading order of the slow roll approximation $\ddot\varphi$ in (13) should

If the inequalities (12) are satisfied, it reduces to:

$$F = -\frac{\lambda M_P^2}{6\,\xi^2}\,\varphi\left(1 + \frac{\boldsymbol{A}}{64\pi^2}\frac{\xi\varphi^2}{M_P^2}\right) \equiv -\frac{\lambda M_P^2}{6\,\xi^2}\,\varphi\left(1 + \frac{\varphi^2}{\varphi_I^2}\right), \tag{16}$$

where

$$\varphi_I^2 = \frac{64\pi^2 M_P^2}{\xi \boldsymbol{A}}. \tag{17}$$

This expression for the rolling force was suggested in Eq.(6.7) of [19] along with the scale of inflation field φ_I which was derived from the principles of quantum cosmology — the probability distribution maximum at φ_I of the quantum corrected tunneling state of the Universe [18]. Note that while the second term in the right-hand side of (14) is small as compared with the first one, the second term in the denominator in the right-hand side of (15), just the opposite, dominates the first one. Thus, during inflation the effective Brans-Dicke parameter $\omega_{BD} \equiv U/2U'^2$ is very small in this model.

For large φ the strongest (cubic in φ) *classical* term of the rolling force (15) identically cancels out and F gets dominated by the second term of (16). The latter is cubic in φ, too, but it is essentially quantum, and it originates from the cross term $-V'U$ in the numerator of (15) dominated by the product of $\xi\varphi^2/2$ in U and the derivative of the logarithmic factor in V'. It is important that the resulting ratio in the parenthesis of (16),

$$\frac{\varphi^2}{\varphi_I^2} = \frac{\boldsymbol{A}/64\pi^2}{M_P^2/\xi\,\varphi^2}\ , \tag{18}$$

is in fact the ratio of two smallness parameters (12). This ratio is a priori not small even despite naively small quantum corrections – the result of multiplication of a small $\boldsymbol{A}/32\pi^2$ by a very large ξ.

The meaning of restrictions (12) becomes transparent in the Einstein frame of fields $\hat{g}_{\mu\nu}$, $\hat{\varphi}$ related to the Jordan frame of (4) by the equations

$$\hat{g}_{\mu\nu} = \frac{2U(\varphi)}{M_P^2} g_{\mu\nu}, \tag{19}$$

$$\left(\frac{d\hat{\varphi}}{d\varphi}\right)^2 = \frac{M_P^2}{2}\frac{GU + 3U'^2}{U^2}. \tag{20}$$

The action (4) in the Jordan frame, $\hat{S}[\hat{g}_{\mu\nu}, \hat{\varphi}] = S[g_{\mu\nu}, \varphi]$, has a minimal coupling, $\hat{U} = M_P^2/2$, canonically normalized inflaton field, $\hat{G} = 1$, and the new inflaton potential

$$\hat{V}(\hat{\varphi}) = \left(\frac{M_P^2}{2}\right)^2 \frac{V(\varphi)}{U^2(\varphi)}\Bigg|_{\varphi=\varphi(\hat{\varphi})}. \tag{21}$$

be discarded on equal footing with $\dot{\varphi}^2$, and it was retained only to emphasize a second order derivative structure of the equation of motion. In particular, the second term of (14), which originates from the $\dot{a}\dot{U} = aH\dot{\varphi}U'$ term of the Friedmann equation in this model, results from expressing $\dot{\varphi}$ via Eq.(13) as $\dot{\varphi} = F/3H$.

This potential is very flat[5] and monotonically growing at least below the exponentially big value of the inflaton, $\varphi < \varphi_* \simeq \mu^2 \exp(192\pi^2\xi/\boldsymbol{C})$. This provides us with a very big domain of the slow roll inflation. The corresponding slow roll parameters for moderate (not exponentially large) values of φ read[6]

$$\hat{\varepsilon} \equiv \frac{M_P^2}{2}\left(\frac{1}{\hat{V}}\frac{d\hat{V}}{d\hat{\varphi}}\right)^2 = \frac{4M_P^4}{3\xi^2\varphi^4}\left(1+\frac{\varphi^2}{\varphi_I^2}\right)^2 = \frac{4}{3}\left(\frac{M_P^2}{\xi\,\varphi^2}+\frac{\boldsymbol{A}}{64\pi^2}\right)^2, \quad (22)$$

$$\hat{\eta} \equiv \frac{M_P^2}{\hat{V}}\frac{d^2\hat{V}}{d\hat{\varphi}^2} = -\frac{4M_P^2}{3\xi\varphi^2}\;. \quad (23)$$

For φ below φ_I and for larger φ the smallness of these parameters reduces respectively to the first and the second of restrictions (12). Thus, these restrictions are nothing but the conditions of sub-Planckian slow roll inflation.

As it follows from (13) and (15) the e-folding number of the inflation stage beginning with φ and ending at $\varphi_{\rm end}$ equals

$$N = \int_{\varphi}^{\varphi_{\rm end}} d\varphi'\,\frac{3H^2(\varphi')}{F(\varphi')} = \frac{48\pi^2}{\boldsymbol{A}}\ln\frac{1+\varphi^2/\varphi_I^2}{1+\varphi_{\rm end}^2/\varphi_I^2}\;. \quad (24)$$

This equation was used in [19] (Eq. (6.8)) with the initial inflaton field $\varphi = \varphi_I$, the quantum scale of inflation derived from the tunneling state of the Universe, and $\varphi_{\rm end} \simeq 0$. When the both fields are small, $\varphi^2, \varphi_{\rm end}^2 \ll \varphi_I^2$, all the dependence on $\boldsymbol{A}$ cancels out and N reduces to Eq.(11) of [21]

$$N \simeq \frac{48\pi^2}{\boldsymbol{A}}\frac{\varphi^2-\varphi_{\rm end}^2}{\varphi_I^2} = \frac{3}{4}\,\frac{\xi}{M_P^2}\,(\varphi^2-\varphi_{\rm end}^2)\;. \quad (25)$$

However, only $\varphi_{\rm end}^2$ a priori satisfies this bound, because it follows from $\hat{\varepsilon} \simeq 1$ it that $\varphi_{\rm end}^2 \simeq M_P^2/\xi$, and $\varphi_{\rm end}^2/\varphi_I^2 \simeq \boldsymbol{A}/64\pi^2 \ll 1$ according to (12). Therefore,

$$N \simeq \frac{48\pi^2}{\boldsymbol{A}}\ln\left(1+\frac{\varphi^2}{\varphi_I^2}\right), \quad (26)$$

and

$$\frac{\varphi^2}{\varphi_I^2} = e^x - 1, \quad (27)$$

where we introduced a new parameter

$$x \equiv \frac{N\boldsymbol{A}}{48\pi^2}. \quad (28)$$

This parameter relates the initial value of the inflaton to the quantum cosmological scale φ_I^2. In quantum cosmology the normalizability of the cosmological

[5] Flatness of this potential explains the cancellation of strongest classical terms in the rolling force (15) mentioned above. Note that the numerator of (15) is proportional to the gradient of the Einstein frame potential (21) which in the classical approximation tends to a constant for $\varphi \to \infty$.

[6] Exponentially big values of φ near φ_* and beyond can also generate inflation which in particular becomes interminable at the negative slope of $\hat{V}(\hat{\varphi})$ for $\varphi > \varphi_*$. But in this domain the one-loop approximation for radiative corrections (5) - (7) breaks down definitely, not to say about unobservable range of the relevant e-folding numbers. Therefore we disregard this domain in what follows.

quantum state requires $\boldsymbol{A}$ to be positive [18, 19], so that $\varphi_I^2 > 0$, and $\varphi = \varphi_I$ corresponds to $x = \ln 2$. Here we adopt an alternative approach and deduce the value of x from angular properties of CMB. In particular, we relax the requirement of positivity for $\boldsymbol{A}$ and admit also its negative values when the parameter x is negative and $\varphi^2 < |\varphi_I^2|$ ($x \to -\infty$ for $\varphi^2 \to |\varphi_I^2|$).

In the limit of large φ ($\varphi^2 \gg \varphi_I^2 > 0$), $N \propto \ln(|\varphi|/\varphi_I)$. Such dependence arises purely due to one-loop quantum corrections to the potential V. Thus, in this case one can indeed try to relate evolution of the Universe during inflation to the renormalization group flow, in contrast to usual inflation in the Einstein gravity where it is not possible [26].

4 Primordial perturbation spectra and observational bounds

Initial conditions for perturbations are chosen deep in the WKB-regime before the first Hubble radius crossing during inflation, where the perturbations themselves, as well as their energy density, are conformally invariant approximately. On the other hand, at the post-inflationary epoch the Einstein frame nearly coincides with the Jordan one because the Higgs-inflaton settles at the minimum of the classical potential $\varphi = \nu$ and $\nu^2 \ll M_P^2/\xi$. Therefore, observable cosmological perturbations can be directly derived in the Einstein frame by standard formulae based on the slow roll parameters (22)-(23) and the Einstein frame potential (21). In particular, the amplitude of perturbations reads $\zeta^2(k) \equiv k^3\zeta_{\mathbf{k}}^2 = \hat{V}/24\pi^2 M_P^4\hat{\varepsilon}$, where the right-hand side is taken at the moment $t = t(k)$ of the first Hubble radius crossing $k = aH$ that relates the comoving perturbation wavelength k^{-1} to the e-folding number N from the end of inflation.

This expression can be also re-written in the form following from the general δN formalism [9, 27, 28] (see also [29, 30] for its recent developments):

$$\zeta^2 = \left(\frac{dN}{d\varphi}\right)^2 (\delta\varphi)^2 \ , \quad \delta\varphi = \frac{H}{2\pi}\frac{1}{\sqrt{1+\frac{3U'^2}{U}}} \ . \tag{29}$$

Here $\delta\varphi \equiv (k^3\delta\varphi_{\mathbf{k}}^2)^{1/2}$ is the rms fluctuation of a free minimally coupled scalar field in the de Sitter background with the curvature H – the multiplier $(1 + 3U'^2/U)^{-1/2}$ is due to a non-standard kinetic term in the Einstein frame arising from the non-minimal coupling $U(\varphi)$ in the physical (Jordan) frame, and we put $G = 1$. [7]

With (5) and (22) (or (24) as well) in the range (12), this gives the relation

$$\zeta^2 = \frac{\lambda}{96\pi^2\xi^2\hat{\varepsilon}} = \frac{N^2}{72\pi^2}\,\frac{\lambda}{\xi^2}\left(\frac{e^x - 1}{x\,e^x}\right)^2 . \tag{30}$$

In view of the WMAP+BAO+SN normalization $\zeta^2 \simeq 2.5 \times 10^{-9}$ at the pivot point $k_0 = 0.002$ Mpc^{-1} [32] which we choose to correspond to $N \simeq 60$, this

[7]This result is valid in both the Jordan and Einstein frames. The difference between the number of e-folds in both frames is an effect of a higher order in the slow-roll and loop expansions. In this connection, see also a more detailed discussion of this topic in the recent paper [31] which appeared when the present paper was prepared for submission.

yields the following estimate on the ratio of coupling constants

$$\frac{\lambda}{\xi^2} \simeq 0.5 \times 10^{-9} \left(\frac{x\, e^x}{e^x - 1} \right)^2 . \tag{31}$$

Thus, the estimate $\lambda/\xi^2 \sim 10^{-10}$ known from [14, 15, 16, 17, 18, 19, 21] is modified here by the factor $(x\, e^x/(e^x-1))^2$ of entirely quantum origin. This factor approaches unity for $x \ll 1$ but grows $\propto x^2$ for $x \gg 1$. In quantum cosmology of the tunneling state [18, 19] with the initial value $\varphi = \varphi_I$, it is equal to $(2 \ln 2)^2 \simeq 1.92$. As we will see now, observational constraints lead to a much wider admissible range of x corresponding to the observed window of scales.

The spectral index n_s of the power spectrum of primordial scalar (adiabatic) perturbations

$$\begin{aligned} n_s &\equiv 1 + \frac{d \ln \zeta^2(k)}{d \ln k} \approx 1 - \frac{d \ln \zeta^2(N)}{dN} \\ &= 1 - \frac{2}{e^x - 1}\, \frac{\boldsymbol{A}}{48\pi^2} = 1 - \frac{2}{N}\, \frac{x}{e^x - 1} \ . \end{aligned} \tag{32}$$

Note that $n_s = 1 - 2/N$ for $x \ll 1$, as in the $m^2\varphi^2$ or $R + R^2/6M^2$ inflationary models (the latter model is the simplified variant of the one introduced in [1]).

The power spectrum of primordial tensor perturbations (gravitational waves) is

$$h_g^2(k) \equiv \sum_{\text{polarizations}} k^3 < h_{\mu\nu}(\mathbf{k}) h^{\mu\nu}(\mathbf{k}) >= \frac{16 G_{\rm eff} H^2}{\pi} \approx \frac{V}{6\pi^2 U^2} \approx \frac{\lambda}{6\pi^2 \xi^2} \ , \tag{33}$$

where $G_{\rm eff}$ is the effective large-scale Newton gravitational constant in the physical frame. As a result, the tensor-to-scalar ratio r is given by the slow roll parameter $\hat{\varepsilon}$ [33]:

$$r \equiv \frac{h_g^2}{\zeta^2} = 16\hat{\varepsilon} = \frac{12}{N^2} \left(\frac{x e^x}{e^x - 1} \right)^2 . \tag{34}$$

Due to the N^{-2} dependence, it is much smaller than in $m^2\varphi^2/2$ and $\lambda\varphi^4/4$ models of inflation, but it exactly coincides with the value of r in the $f(R) = M_P^2(R + R^2/6M^2)/2$ model [1, 34] for $x \ll 1$. The fact that, in the limit $\varphi \ll |\varphi_I|$ when radiative corrections are small, the model (4) - (7) produces the same predictions for n_s and r as this $f(R)$ model is a consequence of the latter model being equivalent to the former one with $\lambda = \xi = M_P^2/3M^2$ (so that the effective coupling constant λ/ξ^2 remains small) and $G(\varphi) = \nu = \boldsymbol{A} = \boldsymbol{B} = \boldsymbol{C} = \boldsymbol{D} = \boldsymbol{E} = \boldsymbol{F} = 0$. Note also that the consistency condition $r = 8n_t$ is satisfied in the model (4- 7), too, in spite of its non-Einsteinian (in fact, scalar-tensor gravity) nature. To obtain it, the quantum one-loop contribution to the potential V should be taken into account.

Let us now compare the spectral index n_s (33) to the present observational data. Using the WMAP+BAO+SN constraint from [32] at the 2σ confidence level, we get

$$0.934 < n_s(k_0) < 0.988 \ . \tag{35}$$

Note that these bounds were obtained assuming $r = 0$ (otherwise, they have to be shifted up by about 0.01). However, since in our model r appears to be much smaller than the upper 95% confidence level upper bound $r < 0.2$ following from the same data, the estimate (35) is just suitable for our purpose. For $N(k_0) = 60$, it leads to $0.36 < x/(e^x - 1) < 1.98$ or $-1.57 < x < 1.79$. So, $\boldsymbol{A} = 48\pi^2 x/N$ and r belong to the following ranges:

$$-12.4 < \boldsymbol{A} < 14.1\ , \tag{36}$$

$$0.0006 < r < 0.015\ . \tag{37}$$

Therefore, there is no "tensor desert" [35] in this model.

The corresponding spectral scalar index running

$$\alpha = -\frac{dn_s}{dN} = -\frac{2}{N^2}\,\frac{x^2 e^x}{(e^x - 1)^2} = -\frac{e^x(n_s - 1)^2}{2} \tag{38}$$

is negative but negligible, and lies in the range $-5.6 < \alpha \times 10^4 < -4.3$ (the most negative value is reached for $\boldsymbol{A} = 0$). This is significantly lower than the present observational upper bound on $|\alpha|$ (though the negative sign of α is slightly favoured by data).

Thus, cosmic data leaves a rather wide window for possible values of the quantity $\boldsymbol{A}$ and the tensor-to-scalar ratio r with the latter one varying by more than an order of magnitude. This window includes also the case of inflation scale generated by the tunneling state in quantum cosmology, $\varphi = \varphi_I$, corresponding to $x \simeq 0.693$, $\boldsymbol{A} = 5.47$, $n_s = 0.977$ and $r = 0.0064$. This window can be compared to constraints coming from the concrete particle model – the Standard Model with the Higgs field playing the role of the inflaton.

5 Standard Model bounds

Since the particle masses induced by the Higgs effect scale as $m(\varphi) \sim \varphi$, we can reduce the calculation of $\boldsymbol{A}$, given by (9), to the present moment when the Higgs field is in the vacuum state with $\varphi = \nu$. Then the masses $m(\nu)$ comprise a well known set of the presently observable Z boson, $W^{\pm}$ boson and top quark masses, $m_Z = 91$ GeV, $m_W = 80$ GeV and $m_t = 171$ GeV, accompanied by much lighter masses of other quarks and leptons giving a negligible contribution. Taking into account three polarizations for massive vector bosons and four polarizations times three colors for quarks, we finally have

$$\boldsymbol{A} = \frac{2}{\lambda\nu^4}\mathrm{tr}\sum_{\mathrm{particles}}(\pm 1)\, m^4(\nu) \simeq \frac{6}{\lambda\nu^4}\left(m_Z^4 + 2m_W^4 - 4m_t^4\right). \tag{39}$$

The scale of the present symmetry breaking is known from the measurement of the Fermi constant, $\nu = 247$ GeV, while λ can be expressed in terms of the Higgs mass, $m_H^2 = 2\lambda\nu^2$, which is currently believed to be in the range 115 GeV$\leq m_H \leq 180$ GeV [23]. Therefore, the total anomalous scaling constant

$$\boldsymbol{A} \simeq \frac{12}{m_H^2\nu^2}\left(m_Z^4 + 2m_W^4 - 4m_t^4\right) \tag{40}$$

turns out to belong to the following range

$$-48 < \boldsymbol{A} < -20. \tag{41}$$

Unfortunately, it does not overlap with the range (36) suggested by the CMB data. If the theory and future LHC experiments on Standard Model could push the value of the Higgs mass up to ~ 230 GeV, then SM generated inflation could yield an observationally viable scenario with parameters at the lower limit of the ranges in (35) and (37), i.e. with a low n_s and a very low tensor-to-scalar ratio $r \simeq 0.0006$. [8]

6 Conclusions

Thus, in principle the Standard Model Higgs can be the source of inflation. However, the mechanism of this phenomenon is very different from the suggestion of [21], because it is dominated by the quantum rather than by the tree-level part of the effective action. In particular, the relation between the e-folding number and the initial value of the inflaton (27) is determined by the parameter of the quantum anomalous scaling $\boldsymbol{A}$. Therefore CMB data necessarily probe not only the graviton-inflaton sector of SM or GUT type theory (1), but also all its heavy massive particles coupled to the inflaton. The deviation of the CMB spectral index

$$n_s(N) = 1 - \frac{2a}{e^{Na} - 1} \tag{42}$$

from unity is determined by the quantum conformal anomaly $a \equiv \boldsymbol{A}/48\pi^2$.

Currently, however, this model of SM Higgs driven inflation seems falsified. In particular, it strongly contradicts predictions of quantum cosmology with the tunneling state, whose normalizability at $\varphi \to \infty$ requires positive $\boldsymbol{A}$ [18, 19, 24], though this conclusion might be revised in view of the recent model of cosmological initial conditions in the form of the microcanonical density matrix [36, 37].

More important is that the cosmological range (36) of $\boldsymbol{A}$ does not overlap with the SM range (41) within the widely accepted rather strong upper bound on the Higgs mass $m_H \leq 180$ GeV. This bound follows from the arguments of [23] justifying, in particular, the electroweak perturbation theory with a small $\lambda \leq 0.26$. Precision tests of this theory [38] give at 95% confidence level a much weaker bound of 285 GeV which already provides a big overlap with the cosmic range (36), $-12.4 < \boldsymbol{A} < -8.0$. Within this overlap

$$0.934 < n_s < 0.95, \tag{43}$$

$$0.0006 < r < 0.001. \tag{44}$$

Thus, it is up to the anticipated Higgs particle discovery at LHC, which will or will not finally falsify the SM driven inflation. It is important that in the latter case a possible tensor-to-scalar ratio (44) is very small. This opens a big reserve in future experiments for possible smallness of not yet observed tensor perturbations.

Also, it should be noted that this inflationary model is a representative of a broad class of models with a red tilted spectrum and a small value of r

[8] In this range of m_H the inflaton self-coupling becomes large, $\lambda \sim \mathcal{O}(1)$, so that the running of coupling constants can slightly shift the above numerical bounds – the authors are grateful to F. Bezrukov for this observation. The RG improved analysis of radiative corrections in this range goes beyond the scope of this paper and is currently under study.

which belong neither to "small-field", nor to "large-field" models, and are, in some respects, intermediate between them. Namely, in the Einstein frame these models have a practically constant potential in the inflationary range, like small-field models, which however extends over a large, or even semi-infinite, range of inflaton field values, like in large-field models. The $R + R^2/6M^2$ model, as well as the induced gravity model considered recently in [39] fall into this class, too.

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