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Comparative Accuracy of Diffuse Radiative Properties Computed Using Selected Multiple Scattering Approximations

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ABSTRACT

Computational results have been obtained for the spherical albedo, global transmission, and global absorption of plane-parallel layers composed of cloud droplets. These computations, obtained using the doubling method for the entire range of single scattering albedos ($0 \leqslant \omega_0 \leqslant 1$) and for optical depths between 0.1 and 100, are compared with corresponding results obtained using selected multiple scattering approximations. Both the relative and absolute accuracies of asymptotic theory for thick layers, three diffuse two-stream approximations, and two integrated two-stream approximations are presented as a function of optical thickness and single scattering albedo for a scattering phase function representative of cloud droplets at visible wavelengths. The spherical albedo and global absorption computed using asymptotic theory are found to be accurate to better than 5% for all values of the single scattering albedo, provided the optical thickness exceeds about 2. The diffuse two-stream approximations have relative accuracies that are much worse than 5% for the spherical albedo over most of the parameter space, yet are accurate to within 5% in the global absorption when the absorption is significant. The integrated delta-Eddington scheme appears to be the most suitable model over the entire range of variables, generally producing relative errors of less than 5% in both the spherical albedo and global absorption.

1. Introduction

The role of clouds in determining the earth's radiation budget has led to increased interest in the parameterization of the radiative properties of cloud layers in numerical atmospheric models. Recent work has been concerned with relating cloud microphysics to optical properties (Slingo 1989) that can then be used in radiative transfer schemes within models. Most models now use some form of approximation to compute cloud radiative properties, such as the plane albedo from a given set of optical properties (optical thickness, single scattering albedo, etc.). Whereas in the past these optical properties were generally fixed, there is now increasing use of interactive schemes in which cloud optical properties are generated internally by the model (Charlock and Ramanathan 1985; Harshvardhan et al. 1989).

As cloud fields evolve during a model integration, the optical properties of the generated clouds and models of gaseous absorption are used in a radiative-transfer scheme to provide the shortwave and longwave radiative-energy field through the atmosphere. These computations need to be carried out at each model grid point at least every time the model cloud fields are updated. In models that resolve the diurnal cycle, this could be every three hours of simulated time, or even hourly. The computational burden is such that rapid, yet accurate, techniques are essential. In the shortwave, a common procedure is the computation of cloud-layer properties by a two-stream method and the adding of radiative fluxes through the atmosphere in an energy-conserving scheme (Lacis and Hansen 1974; Coakley et al. 1983; Charlock and Ramanathan 1985; Geleyn and Hollingsworth 1979; Harshvardhan et al. 1987), although the two-stream equations can also be solved directly for multiple layers using matrix solvers (Wiscombe 1977; Toon et al. 1989). The flux adding method is essentially a severely truncated form of the adding–doubling method (Hansen and Travis 1974), using upward and downward fluxes instead of intensities.

In order to compute radiative fluxes through several atmospheric layers by the flux adding method, the radiative properties of cloud layers for two different sources are required (Harshvardhan et al. 1987; Kiehl et al. 1987). When collimated solar radiation is incident on an isolated cloud layer at some zenith angle with respect to the vertical direction, the fluxes emergent

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from the layer in the upward and downward directions are determined by the plane albedo and total transmission of the layer, respectively. If the incident source is diffuse, the emergent flux may be obtained by an angular integration over the incident intensity field. In two-stream methods, the angular distribution of the incident intensity field is not resolved, and a common practice is to assume an isotropic diffuse source. For example, in a multilayer cloud system, the diffuse solar flux transmitted through the upper layer is the incident source for the lower layer. Also, in the case of a cloud layer overlying a reflecting ground surface, multiple reflections between the cloud and ground are considered by assuming an isotropic diffuse source at the bottom boundary of the cloud layer. These diffuse radiative properties have also been used in the past to provide estimates of global effects of aerosol layers (Chýlek and Coakley 1974). A comprehensive study of the accuracy of various multiple scattering approximations for the plane albedo, total transmission, and fractional absorption of isolated cloud layers corresponding to incident collimated radiation was presented by King and Harshvardhan (1986a,b). The present study complements the earlier one in assessing the accuracy of various approximations for calculating the radiative properties of cloud and aerosol layers for an incident isotropic diffuse source.

The presentation follows the organization of King and Harshvardhan (1986a, hereafter referred to as KH). Section 2 discusses multiple scattering calculations used to obtain the diffuse radiative properties of cloud layers of varying optical thicknesses and single scattering albedos. These computational results, obtained with the doubling method, will be considered the benchmark solutions with which various multiple scattering approximations will be compared. Section 3 introduces the asymptotic theory approximation and the general class of two-stream approximations that we will consider. Section 4 presents the results of the comparison between the approximate and exact results in terms of absolute and relative differences. A discussion of the results follows in section 5. Section 6 is a summary including recommendations for using these approximations.

2. Multiple scattering computations

To assess the accuracy of various multiple scattering approximations, radiative transfer computations were performed using the doubling method described by Hansen and Travis (1974), together with the invariant embedding initialization described by King (1983). These computations were performed for a cloud drop size distribution typical of fair weather cumulus (FWC) clouds (Hansen 1971), and were performed at a wavelength $\lambda = 0.754$ μm assuming a refractive index of liquid water $m = 1.332$. A detailed description of the cloud model, together with an illustration of the single scattering phase function, can be found in KH. The azimuth-independent terms of the reflection and transmission functions were used to obtain the plane albedo $r(\tau_t, \mu_0)$ and total transmission $t(\tau_t, \mu_0)$ as a function of τ_t, the total optical thickness of the layer, and μ_0, the cosine of the solar zenith angle. In terms of these functions the spherical albedo, global transmission, and global absorption of the layer are given by

$$\bar{r}(\tau_t) = 2\int_0^1 r(\tau_t, \mu_0)\mu_0 d\mu_0, \qquad (1)$$

$$\bar{t}(\tau_t) = 2\int_0^1 t(\tau_t, \mu_0)\mu_0 d\mu_0, \qquad (2)$$

$$\bar{a}(\tau_t) = 1 - \bar{r}(\tau_t) - \bar{t}(\tau_t). \qquad (3)$$

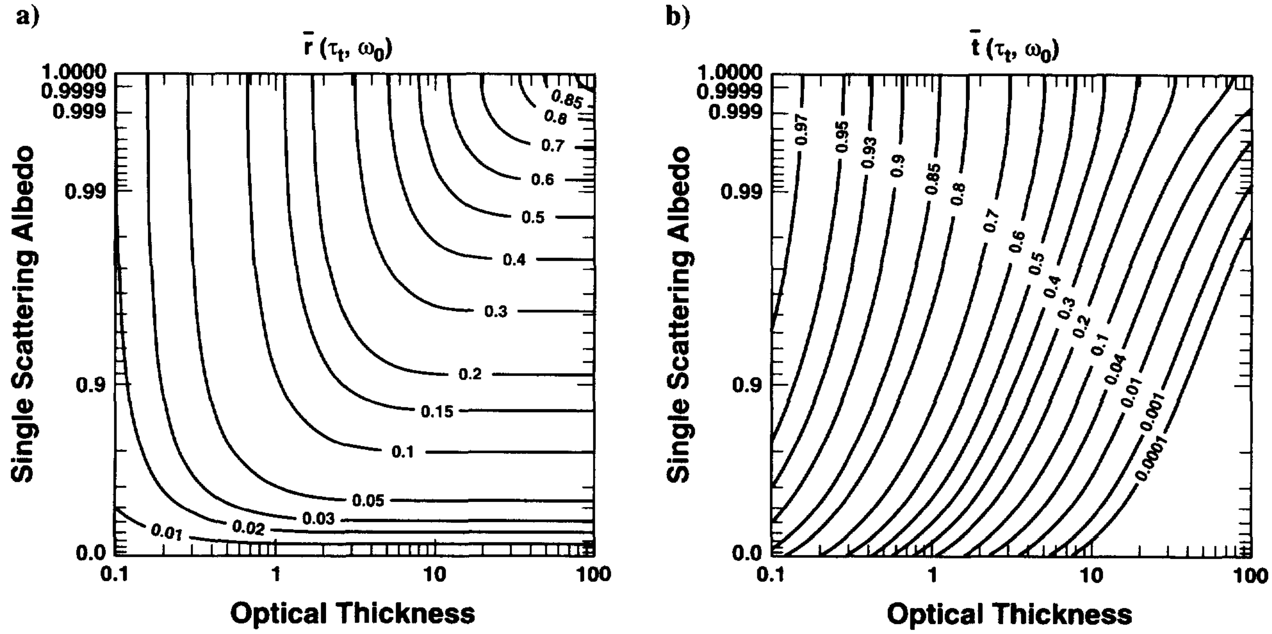
In order to cover a wide range of applications, these computations were performed for values of the single scattering albedo ranging from pure absorption ($\omega_0 = 0$) to conservative scattering ($\omega_0 = 1$). The single scattering phase function was left unchanged such that all computations apply to a phase function having an asymmetry factor $g = 0.843$.

Figure 1 illustrates numerical computations of the spherical albedo $[\bar{r}(\tau_t)$, global transmission $[\bar{t}(\tau_t)]$, and global absorption $[\bar{a}(\tau_t)]$ as a function of ω_0 and τ_t. The doubling computations used to generate these results were obtained at 12 optical depths 0.0625, 0.125, . . . , 128 interleaved with another set of 11 optical depths 0.0884, 0.1768, . . . , 90.51. Each set of doubling computations was itself made at each of 31 values of the single scattering albedo. The single scattering albedo scale is linear in the similarity parameter s, defined by

$$s = \left(\frac{1-\omega_0}{1-\omega_0 g}\right)^{1/2}. \qquad (4)$$

This makes it possible to expand the scale in the vicinity of conservative scattering ($\omega_0 = 1$) and still to span the full range $0 \leq \omega_0 \leq 1$. The angular computations, including the integration in (1) and (2), were performed at 80 Gaussian quadrature points. As in KH, the computed results were first interpolated to generate a 300 $\times$ 300 matrix prior to plotting. The interpolated arrays represent the exact results to which the radiative transfer approximations are compared in section 4.

It is perhaps pertinent to point out certain features of the radiative properties illustrated in Fig. 1. For conservative or very weakly absorbing layers, the spherical albedo increases rapidly with increasing optical thickness for small values of τ_t and then much more slowly as τ_t becomes large. This is the well-known nonlinear behavior that leads to problems in estimating area-averaged albedos for a nonhomogeneous cloud layer (Harshvardhan and Randall 1985). For moderate to strong absorption, the saturation of both the spherical



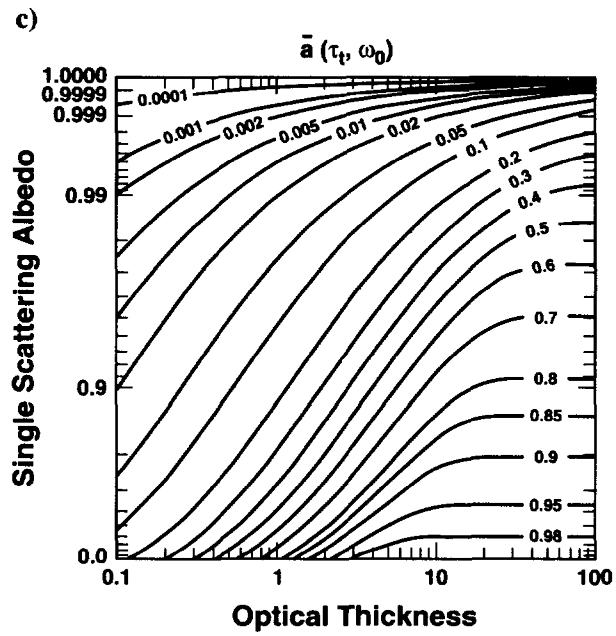


FIG. 1. Doubling computations of the (a) spherical albedo $\bar{r}(\tau_t, \omega_0)$, (b) global transmission, $\bar{t}(\tau_t, \omega_0)$, and (c) global absorption $\bar{a}(\tau_t, \omega_0)$ as a function of optical thickness and single-scattering albedo for the FWC phase function. The single scattering albedo scale is linear in the similarity parameter, defined by Eq. (4).

albedo and global absorption at optical thicknesses of about 10 or even less is the most striking feature of Fig. 1. In the near-infrared, this implies that cloud absorption is primarily a function of the single scattering albedo and not the optical thickness once the cloud layer is several hundred meters thick (Twomey 1976). The importance of determining the spectral dependence of ω_0 for cloud layers and the development of accurate parameterizations for inclusion in radiative transfer models follows from this observation (King et al. 1990; Fouquart et al. 1991).

3. Radiative-transfer approximations

Three classes of approximations will be considered here for comparison with the multiple scattering results presented above. In all cases, analytic or easily integrable functions relate the radiative properties to the optical properties. The three approximations we will consider are asymptotic theory for thick layers, diffuse two-stream approximations, and integrated two-stream approximations. Although there are several variations of two-stream approximations, only a few common and representative models will be considered.

a. Asymptotic theory

Asymptotic theory is a rigorous solution to the equation of transfer in optically thick layers, and as such, makes no assumption about the angular distribution of scattered radiation within the medium. Expressions for the plane albedo and total transmission of an optically thick layer under collimated illumination conditions can be found in KH and will not be repeated here. From these expressions, it can be shown that the asymptotic theory approximations for the spherical albedo $[\hat{\bar{r}}(\tau_t)]$, global transmission $[\hat{\bar{t}}(\tau_t)]$, and global absorption $[\hat{\bar{a}}(\tau_t)]$ are given by

$$\hat{\bar{r}}(\tau_t) = \bar{r}_\infty - \frac{mn^2 l e^{-2k\tau_t}}{1 - l^2 e^{-2k\tau_t}}, \qquad (5)$$

$$\hat{\bar{t}}(\tau_t) = \frac{mn^2 e^{-k\tau_t}}{1 - l^2 e^{-2k\tau_t}}, \qquad (6)$$

$$\hat{\bar{a}}(\tau_t) = 1 - \hat{\bar{r}}(\tau_t) - \hat{\bar{t}}(\tau_t), \qquad (7)$$

for nonconservative scattering ($\omega_0 < 1$). In these expressions, $\bar{r}_\infty$ is the spherical albedo of a semi-infinite atmosphere and m, n, l, and k are constants (coefficients) that depend primarily on the similarity parameter given by (4). All of the functions and constants that appear in these expressions can be computed by equating asymptotic formulas and doubling results at three values of the optical thickness for which asymptotic theory is valid (viz., τ_t = 8, 16, and 32), as first pointed out by van de Hulst (1968). Similarity relations for calculating $\bar{r}_\infty$ (denoted A^* by van de Hulst 1968), m, n, l, and k as a function of s for the full range $0 \leqslant s \leqslant 1$ can be found in Table 1 of King et al. (1990). Once these coefficients have been computed, expressions for all of the radiative properties are analytic functions that can be computed rapidly within a radiative transfer code.

For the special case of conservative scattering ($\omega_0 = 1$), Eqs. (5) and (6) reduce to

$$\hat{\bar{r}}(\tau_t) = 1 - \frac{4}{3(1-g)(\tau_t + 2q_0)}, \qquad (8)$$

$$\hat{\bar{t}}(\tau_t) = \frac{4}{3(1-g)(\tau_t + 2q_0)}, \qquad (9)$$

where q_0 is the extrapolation length. The reduced extrapolation length $q' = (1-g)q_0$ is known to range between 0.709 and 0.715 for all possible phase functions (van de Hulst 1980), and has the value $q' = 0.715$ for the phase function used here. Again, one is left with simple analytic functions describing the variation of $\hat{\bar{r}}(\tau_t)$ and $\hat{\bar{t}}(\tau_t)$ as a function of τ_t for a given asymmetry factor g. The set of equations (5)–(9) forms the approximations for the diffuse radiative properties of a medium based on asymptotic theory.

b. Diffuse two-stream approximations

In the absence of any direct collimated beam, the two-stream equations of radiative transfer result in a set of differential equations for the upward and downward diffuse fluxes $F^{\pm}(\tau)$ (Coakley and Chýlek 1975; Meador and Weaver 1980)

$$\frac{dF^-(\tau)}{d\tau} = \gamma_1 F^-(\tau) - \gamma_2 F^+(\tau), \qquad (10)$$

$$\frac{dF^+(\tau)}{d\tau} = \gamma_2 F^-(\tau) - \gamma_1 F^+(\tau), \qquad (11)$$

where $F^-(\tau)$ represents the upward flux and $F^+(\tau)$ the downward flux at optical depth τ. The equations can easily be solved subject to the boundary conditions

$$F^+(0) = F_0, \qquad (12)$$

$$F^-(\tau_t) = 0, \qquad (13)$$

for a diffuse isotropic source incident at the top boundary of the layer (or cloud) and for which no illumination is incident from below. The spherical albedo is thus obtained from the expression

$$\hat{\bar{r}}(\tau_t) = F^-(0)/F_0, \qquad (14)$$

and the global transmission from

$$\hat{\bar{t}}(\tau_t) = F^+(\tau_t)/F_0. \qquad (15)$$

For nonconservative scattering ($\omega_0 < 1$), the solution may be obtained in the form (Coakley and Chýlek 1975; Meador and Weaver 1980)

$$\hat{\bar{r}}(\tau_t) = \frac{\gamma_2(1 - e^{-2k\tau_t})}{k + \gamma_1 + (k - \gamma_1)e^{-2k\tau_t}} \qquad (16)$$

$$\hat{\bar{t}}(\tau_t) = \frac{2ke^{-k\tau_t}}{k + \gamma_1 + (k - \gamma_1)e^{-2k\tau_t}}, \qquad (17)$$

and for conservative scattering ($\omega_0 = 1$)

$$\hat{\bar{r}}(\tau_t) = \frac{\gamma_1 \tau_t}{1 + \gamma_1 \tau_t} \qquad (18)$$

$$\hat{\bar{t}}(\tau_t) = 1 - \hat{\bar{r}}(\tau_t). \qquad (19)$$

In (16)–(19), the coefficients γ_1 and γ_2 depend on the particular two-stream approximation, with the diffusion exponent k defined as

$$k = (\gamma_1^2 - \gamma_2^2)^{1/2}. \qquad (20)$$

Table 1 lists three diffuse two-stream models used for this study and the corresponding values of γ_1 and γ_2. The discrete ordinates model is identified as the quadrature scheme by Meador and Weaver (1980) and Toon et al. (1989). The hemispheric-mean model defined by Toon et al. (1989) is similar to the Coakley–Chýlek model II referred to by KH and first introduced by Chýlek and Coakley (1974). The two-stream model used by Sagan and Pollack (1967) has coefficients similar to those of both of the aforementioned models. Instead of the asymmetry parameter g, some two-stream models use the average backscatter fraction $\bar{\beta}$, which is defined in KH and readily computed from the backscatter fraction $\beta(\mu_0)$, introduced by Coakley and Chýlek (1975) and Zdunkowski et al. (1980) to compute the radiative properties of layers for collimated incident sources. The Eddington model has, of course, been used widely (Shettle and Weinman 1970). The set of equations (16)–(20) is used to compute the diffuse radiative properties for the two-stream approximations. It should be noted that these expressions have fairly simple analytic forms that favor rapid computation.

c. *Integrated two-stream approximations*

Extensive discussion of two-stream approximations for a collimated source can be found in KH as well as in earlier work, in particular the comprehensive treatment by Meador and Weaver (1980). Expressions for the approximate plane albedo $[\hat{r}(\tau_t, \mu_0)]$, total transmission $[\hat{t}(\tau_t, \mu_0)]$, and fractional absorption $[\hat{a}(\tau_t, \mu_0)]$ are the set of equations (21)–(29) in KH. These expressions include the transformations that are required in the case of delta scaling (Joseph et al. 1976). To obtain comparable expressions for the diffuse radiative properties, $\hat{r}(\tau_t, \mu_0)$ and $\hat{t}(\tau_t, \mu_0)$ must be integrated according to Eqs. (1) and (2). These expressions, however, are quite complicated, and thus integration in a closed form is not generally practical.

An analytic expression for the spherical albedo in the Eddington and delta-Eddington approximations has been obtained by Wiscombe and Warren (1980) and involves exponential integrals that are not conducive to rapid computation within a model. For this study, $\hat{r}(\tau_t, \mu_0)$ and $\hat{t}(\tau_t, \mu_0)$ obtained by the delta-Eddington approximation were numerically integrated to provide $\hat{\bar{r}}(\tau_t)$ and $\hat{\bar{t}}(\tau_t)$. King and Harshvardhan found that the delta-Eddington approximation for collimated illumination conditions is quite accurate over a wide range of τ_t and μ_0, especially when ω_0 is near unity. A model that performs well for optically thin layers over the limited range of ω_0 studied by KH is the plane albedo scheme of Coakley and Chýlek (1975), designated Coakley–Chýlek model I by KH. Two-stream methods for collimated sources require a third coefficient, γ_3, which appears with the source term and is thus not included in Eqs. (10) and (11). The expressions for γ_3 used by the two integrated models presented here are given in Table 1.

The integrations in Eqs. (1) and (2) required to obtain the diffuse properties are performed using 80-point Gaussian quadrature, and the results should be considered identical to an analytic solution for all practical purposes. The general form of the quadrature summation is

$$\bar{r}(\tau_t) = 2 \sum_{i=1}^{N} r(\tau_t, \mu_i)\mu_i w_i, \tag{21}$$

where μ_i are the Gaussian quadrature points on the half space and w_i are the corresponding Gaussian weights. This detailed integration, however, is of no practical value because the computational burden is onerous when applied to a global climate model. We have, therefore, also included results for the delta-Eddington and Coakley–Chýlek (I) models integrated using two-point and four-point quadrature, respectively. The diffuse radiative properties can then be obtained with a computational effort comparable to that required to compute properties for collimated radiation.

4. Results

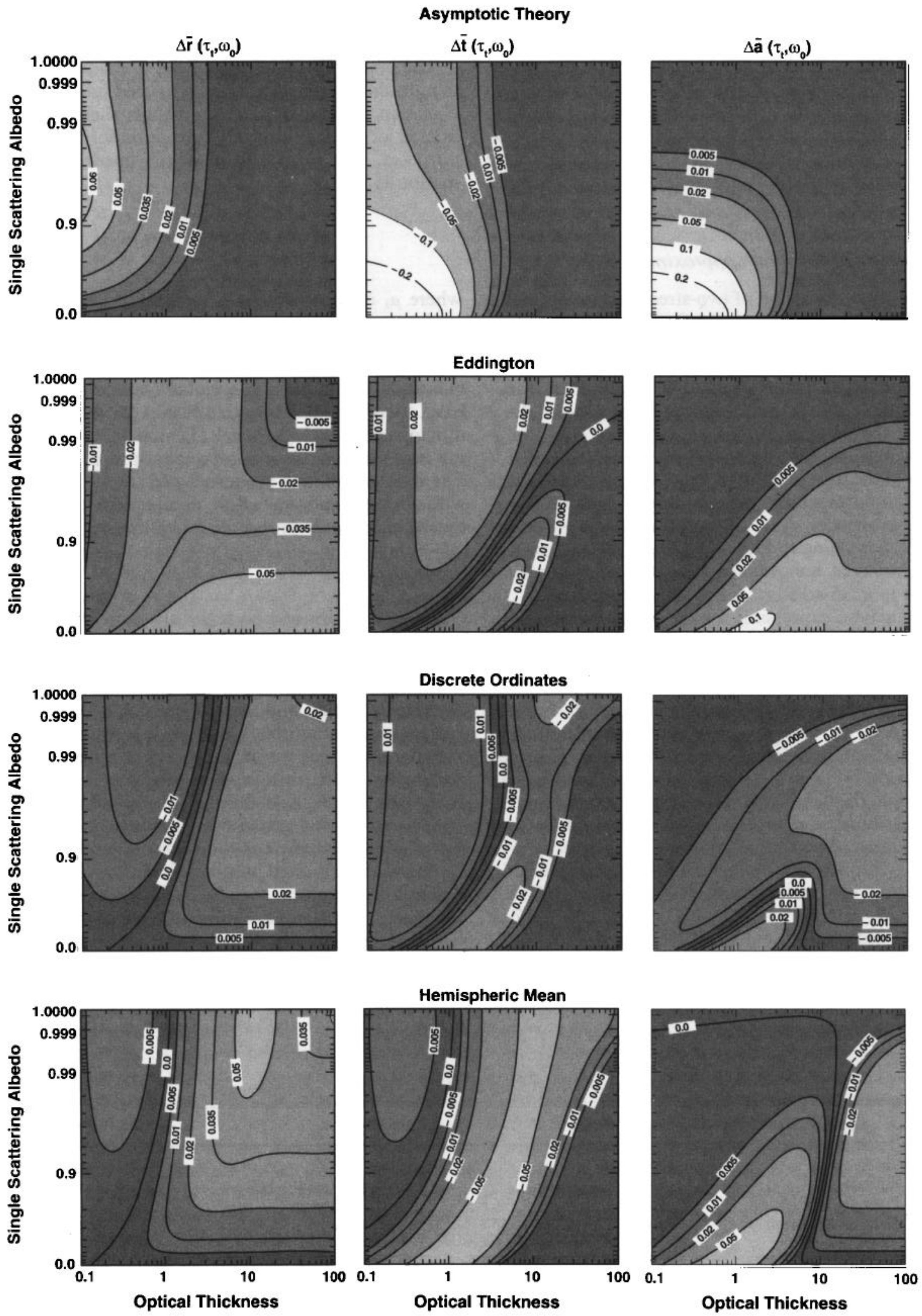
We have examined both the absolute and relative accuracies of the spherical albedo, global transmission, and global absorption as a function of τ_t and ω_0 for the asymptotic approximation, as well as the Eddington, discrete ordinates, and hemispheric-mean diffuse two-stream approximations. Other diffuse two-stream approximations that we have examined generally yield somewhat poorer results when compared to our doubling benchmark calculations. In addition, we have considered the integrated delta-Eddington and Coakley–Chýlek (I) approximations computed using both 80 points and a limited number of Gaussian quadrature points for integration over the solar zenith angle.

Figure 2 illustrates a 4 × 3 plot composite of results for the absolute difference in the spherical albedo, global transmission, and global absorption for four of these models, where the first row applies to asymptotic theory and succeeding rows to the Eddington, discrete

TABLE 1. Summary of γ_i coefficients in selected two-stream approximations.

Method	γ_1	γ_2	γ_3
Diffuse			
Eddington	$1/4[7 - \omega_0(4 + 3g)]$	$-1/4[1 - \omega_0(4 - 3g)]$	—
Discrete ordinates	$\sqrt{3}/2[2 - \omega_0(1 + g)]$	$\sqrt{3}/2[\omega_0(1 - g)]$	—
Hemispheric mean	$2 - \omega_0(1 + g)$	$\omega_0(1 - g)$	—
Integrated			
Delta-Eddington	$1/4[7 - \omega_0'(4 + 3g')]$	$-1/4[1 - \omega_0'(4 - 3g')]$	$1/4(2 - 3g'\mu_0)$
Coakley–Chýlek (I)	$\{1 - \omega_0[1 - \beta(\mu_0)]\}/\mu_0$	$\omega_0\beta(\mu_0)/\mu_0$	$\beta(\mu_0)$

$\omega_0' = (1 - g^2)\omega_0/(1 - \omega_0 g^2)$
$g' = g/(1 + g)$


Asymptotic Theory
Eddington
Discrete Ordinates
Hemispheric Mean
$\Delta\bar{r}\ (\tau_t,\omega_0)$
$\Delta\bar{t}\ (\tau_t,\omega_0)$
$\Delta\bar{a}\ (\tau_t,\omega_0)$
Single Scattering Albedo
1.0000
0.999
0.99
0.9
0.0
Optical Thickness
0.1
1
10
100

ordinates, and hemispheric-mean approximations. Individual plots in the first column of Fig. 1 represent absolute errors in the spherical albedo, defined as

$$\Delta\bar{r}(\tau_t, \omega_0) = \hat{\bar{r}}(\tau_t, \omega_0) - \bar{r}(\tau_t, \omega_0), \qquad (22)$$

with succeeding columns representing corresponding errors in global transmission [$\Delta\bar{t}(\tau_t, \omega_0)$] and global absorption [$\Delta\bar{a}(\tau_t, \omega_0)$]. With these definitions, positive (negative) errors indicate that the radiative transfer approximation overestimates (underestimates) the exact solution, taken as the computational results presented in Fig. 1. The relative errors in the spherical albedo, global transmission, and global absorption are presented in Fig. 3, and are given in percent. It is necessary to consider the performance of a particular model in both a relative and an absolute sense in order to delineate a range of acceptability.

Individual contour plots in Figs. 2 and 3 have been shaded to draw attention to those regions of greatest accuracy. For example, asymptotic theory is seen to be accurate to within 5% in reflection and absorption for $\tau_t \gtrsim 2$ and for all values of ω_0. In transmission, relative errors exceed 5% for $\omega_0 < 0.90$ and $2 \leqslant \tau_t < 8$, but the absolute errors are so small (<0.03) that the approximation could probably still be used without serious adverse results. It is evident from Figs. 2 and 3 that the asymptotic approximation provides accurate results for all three diffuse radiative properties over the entire range of ω_0 as long as $\tau_t \gtrsim 2$.

The three diffuse two-stream models considered here are seen to yield unacceptable errors in one or more of the radiative properties over regions that would normally be encountered in modeling applications. Although the range of acceptability will depend on the particular application, one can consider a 5% error in the spherical albedo as a standard for comparison. The spherical albedo is usually the parameter of choice in estimating the sensitivity of any radiative perturbation. When the value itself is small, however, an absolute error criterion is more useful. For optically thin layers, the absolute errors in spherical albedo are generally less than 0.01 for the discrete ordinates and hemispheric-mean approximations. Errors in global transmission are similar for all three models, while the Eddington and hemispheric-mean models are successful in estimating the global absorption of a layer when $\omega_0 \gtrsim 0.99$ and $\tau_t \leqslant 10$ with errors of less than 1%. If the range of acceptability is relaxed to 5%, then the Eddington and hemispheric-mean models can be used for absorption when ω_0 is as low as 0.95 except for optically thick layers. This covers the range of single scattering albedo encountered in water clouds throughout the visible and near-infrared spectrum (King et al. 1990).

The two integrated two-stream methods studied in this investigation provide more accurate results for all three diffuse radiative properties as shown in Figs. 4 and 5. The delta-Eddington model was shown by KH to be highly successful in estimating the plane albedo for conservative scattering. There was a marked degradation of performance when nonconservative cases were considered. The present study shows that this model, when integrated over an isotropic diffuse incident source, provides excellent results for the spherical albedo and global transmission over most of the range of τ_t and ω_0. Errors in excess of 10% in global absorption are present for moderate optical depths ($0.5 \leqslant \tau_t \leqslant 5$) when ω_0 exceeds about 0.95. It may be seen from Fig. 4, however, that the absolute errors in global absorption are less than 0.02 throughout this region. In addition, the large relative error in global transmission for optically thick absorbing layers is irrelevant since the global transmission is itself close to zero, as is the absolute error. The Coakley–Chýlek (I) model provides results of comparable accuracy for optically thin layers. This is not surprising since KH showed that it was the most accurate of the two-stream models for this case. The delta-Eddington model, however, when integrated over all incident angles, is nearly as accurate as the Coakley–Chýlek (I) model for optically thin layers. Moreover, the accuracy of the integrated delta-Eddington model does not degrade as rapidly at higher optical depths.

As mentioned previously, these two models would only be of academic interest if a rigorous numerical integration were required for every computation of the diffuse radiative properties. We have, therefore, also presented results obtained using a limited number of quadrature points in the integration over solar zenith angle [cf. Eq. (21)]. As can be seen from the second panel of Figs. 4 and 5, a two-point integration of the delta-Eddington models yields accuracies that are comparable to the accuracy obtained using an 80-point integration. For the Coakley–Chýlek (I) model, however, it is necessary to use a four-point integration to obtain results that are of comparable accuracy.

5. Discussion

Although the results presented here are not exhaustive in the sense that all possible approximations have not been tested, we feel they are representative of what one might expect for any class of model. All the schemes are computationally efficient, and it is not necessary to perform a rigorous integration for the models based on incident collimated sources. The approximations presented here can be incorporated into

FIG. 2. Absolute accuracy of asymptotic theory, Eddington, discrete ordinates and hemispheric-mean approximations to the spherical albedo, global transmission and global absorption as a function of optical thickness and single scattering albedo. The FWC phase function is assumed throughout.

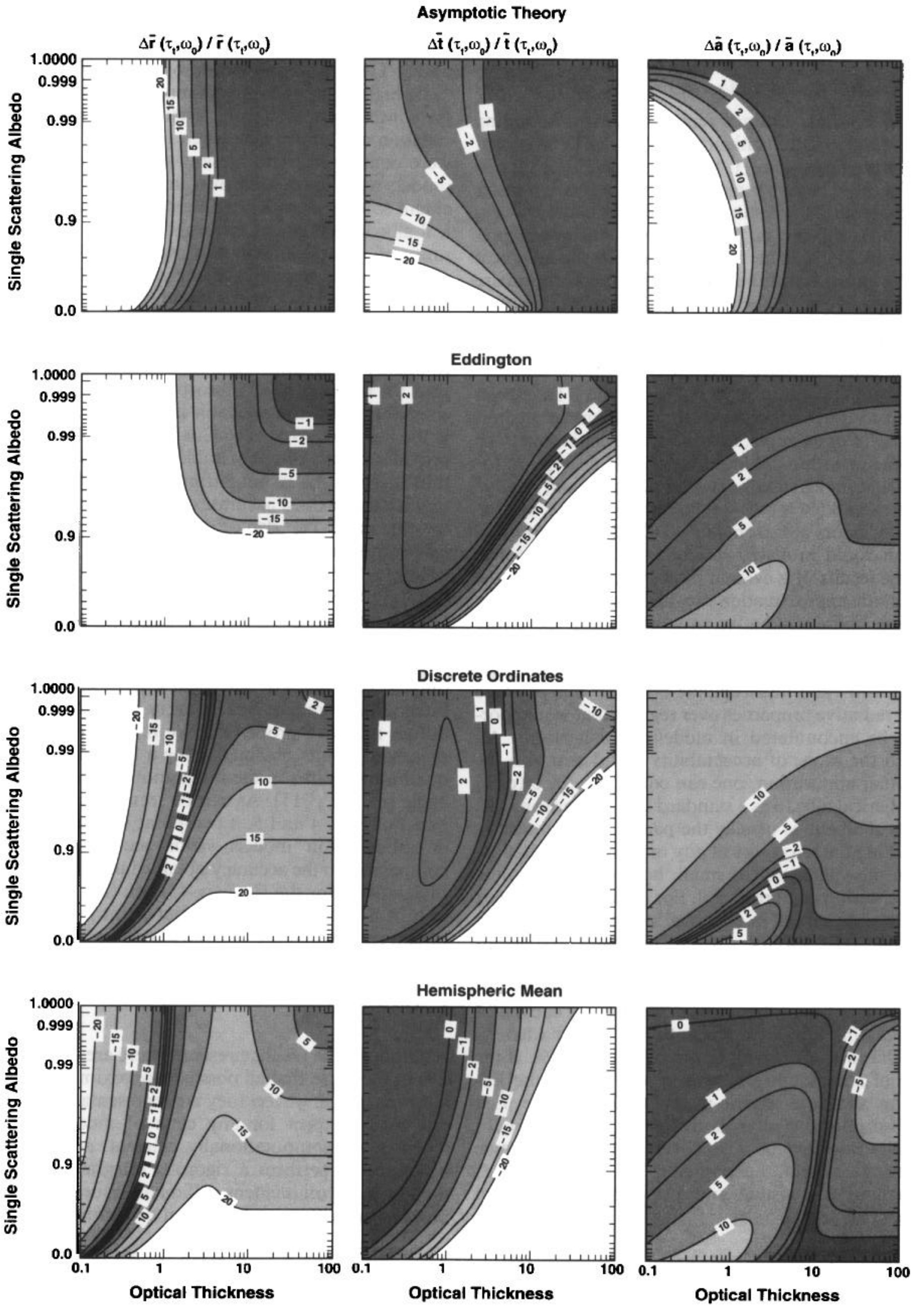


FIG. 3. As in Fig. 2 except for relative accuracies (in percent).

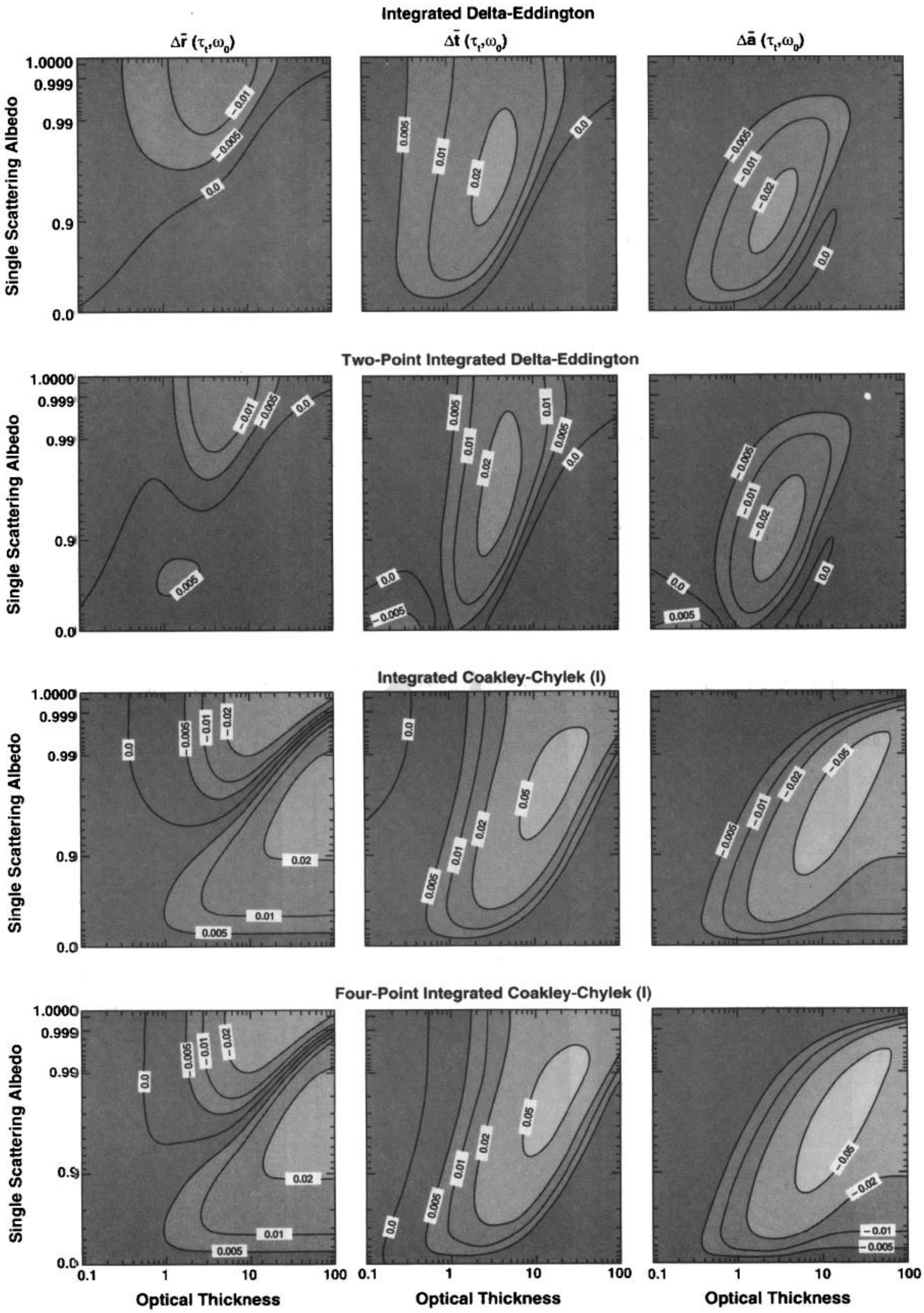


FIG. 4. As in Fig. 2 but for the integrated delta-Eddington, two-point–integrated delta-Eddington, integrated Coakley–Chýlek (I), and four-point integrated Coakley–Chýlek (I) approximations.

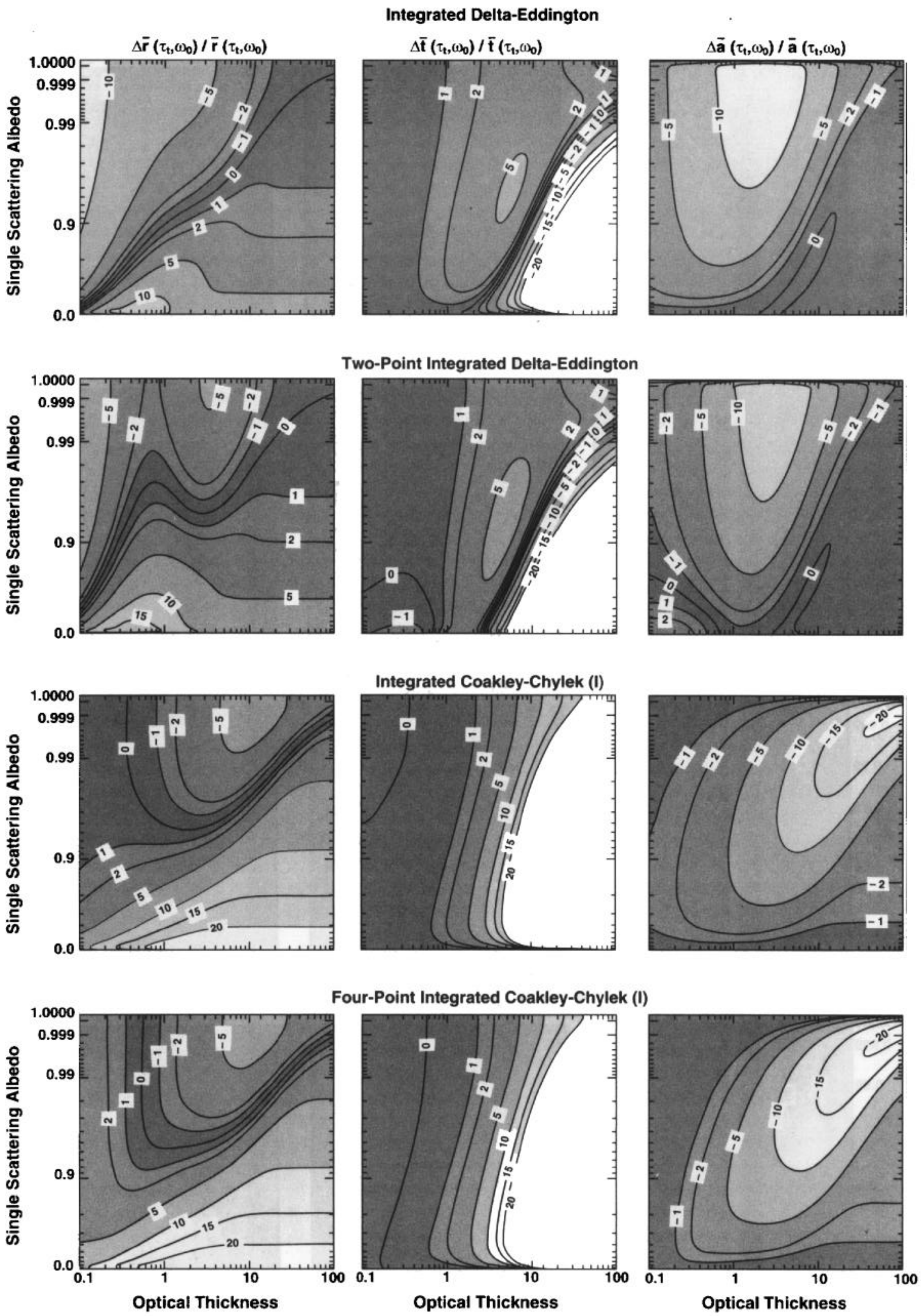


FIG. 5. As in Fig. 4 except for relative accuracies (in percent).

a multilayer radiative transfer module that may be added to the radiation code in a numerical model.

All computations presented here were obtained for a FWC drop size distribution having an asymmetry factor $g = 0.843$. Variations along the ω_0 axis can therefore be viewed as representing the effect of altering the gaseous absorption in the layer at a particular wavelength, or to some extent, variations in the wavelength if g does not vary too greatly. This would cover the solar near-infrared spectrum over which $(1 - \omega_0)$ varies by several orders of magnitude, while g generally lies between 0.80 and 0.90 (cf. King et al. 1990).

As found by KH for collimated radiative properties, the asymptotic approximation yields consistently excellent results for optically thick layers, regardless of single scattering albedo and solar zenith angle. Figures 2 and 3 show that the same is true for the diffuse radiative properties as long as $\tau_t \geqslant 2$. In a numerical model with internally generated cloud optical properties, this requirement will not always be met. Errors become unacceptably large when $\tau_t < 1$. For this reason, the asymptotic approximation should only be used when it is known a priori that $\tau_t \geqslant 2$ at all times. This is the one serious shortcoming of an otherwise simple and accurate model. The method also requires a precomputed table of coefficients m, n, k, l, and $\bar{r}_\infty$, or analytic forms that compute these quantities within the program. Analytic expressions for these coefficients in terms of the similarity parameter can be found in King et al. (1990), which further discusses a remote sensing application of asymptotic theory.

The three diffuse two-stream models presented here are the simplest to implement in a numerical atmospheric model and are the most computationally efficient, but their accuracy is limited to certain regions of the parameter space. They are also not uniformly accurate for all three radiative properties. This is especially true in the Eddington approximation, where the spherical albedo is frequently too inaccurate to be of any value in a numerical model. In addition, the Eddington model yields unphysical values of the spherical albedo and global absorption when absorption is very large (King and Harshvardhan 1986b). This situation arises occasionally in the water vapor bands of the near-infrared and frequently in the thermal infrared. The problem can be rectified in a computer code with the addition of a check for unphysical values that could then be forced to the condition of zero reflection. The discrete ordinates model does not suffer from this limitation and generally yields better results for the spherical albedo than does the Eddington approximation. The somewhat poorer results for global absorption are not too important since the absolute errors are small in this case. The hemispheric-mean model yields results very similar to the discrete ordinates model, except for global absorption. The smaller relative errors for weak absorption are an especially attractive feature of the hemispheric-mean model, which otherwise suffers from the fact that it tends to overestimate the spherical albedo by more than 5% for the very important case of nearly conservative optically thick layers.

The integrated delta-Eddington model yields excellent results for all three radiative properties over the entire range of optical properties that are encountered in the radiation code of a numerical atmospheric model. In fact, errors in the diffuse radiative properties are generally smaller than the errors found by KH for collimated radiative properties, with no unphysical results anywhere in the parameter space. There has obviously been some cancellation of errors in the angular integration. As mentioned earlier, the one error-prone region is moderate optical thickness and weak absorption. This was also true for the errors in fractional absorption for a collimated source. Since the direct beam is usually handled by a delta-Eddington or similar approximation, the coefficients and functions used for this model are usually already present in a numerical model. There is, however, an extra computational overhead in the angular integration, in that planar properties need to be computed at several angles and then numerically integrated. As seen in Figs. 4 and 5, however, these computations need be carried out at only two points to yield results comparable to a detailed numerical integration.

The integrated Coakley–Chýlek (I) model is of limited value, except perhaps for optically thin, weakly absorbing layers. There is also an added computational burden since at least four angular computations are required for the phase function used here. For collimated radiative properties and for optically thin layers, KH found that this model was superior to the delta-Eddington model. For diffuse radiative properties, on the other hand, we find that there is little advantage in using the Coakley–Chýlek (I) model, even for optically thin layers.

6. Summary and recommendations

In the present study the spherical albedo, global transmission, and global absorption computed by various radiative transfer approximations have been compared with doubling computations as a function of optical thickness and single scattering albedo. Since the entire range of ω_0 has been considered for optical depths from 0.1 to 100, the results presented here can be utilized to decide which approximate method is the most accurate for a particular application. The results presented here should be considered in parallel with the findings of KH regarding the plane albedo, total transmission, and fractional absorption for a collimated incident source.

In order to summarize the results of this study, it is useful to present composite figures extracted from the individual figures to highlight regions of highest accuracy. Following van de Hulst (1980), we show in

Fig. 6 the regions for which a particular model is accurate to within 1% and 5%. Only those models that are reasonably accurate in the particular radiative property have been included. These models include asymptotic theory, the two-point delta-Eddington method, and the four-point Coakley–Chýlek (I) method. Although the hemispheric-mean model yields acceptable results for the global absorption, it is not included here because results for the spherical albedo are generally poor.

At the 1% (5%) level, asymptotic theory can be used for all ω_0 as long as $\tau_t \geqslant 3.5$ (2). For smaller optical depths, there is a choice that can be made between the delta-Eddington and Coakley–Chýlek (I) models, but our recommendation is to use the delta-Eddington method. Many general circulation models are already using this method to compute collimated radiative properties, and the additional overhead incurred in the two-point integration should be minimal. If a scheme is needed to span the entire domain, the asymptotic method should not be used since its performance deteriorates very rapidly for $\tau_t \leqslant 3$. For this situation, typical of GCM applications, the integrated delta-Eddington scheme should yield acceptable results.

The overall errors for a multilayer cloud system over a reflecting surface will depend on the optical thickness and single scattering albedo of the individual layers. At present, it is felt that errors in parameterizing the band-averaged single scattering albedo of cloud layers in the near-infrared will dominate errors in approximating the radiative properties of individual layers (Fouquart et al. 1991). For example, the use of a single value of ω_0 to represent the entire solar near infrared can result in errors in the layer absorption of several

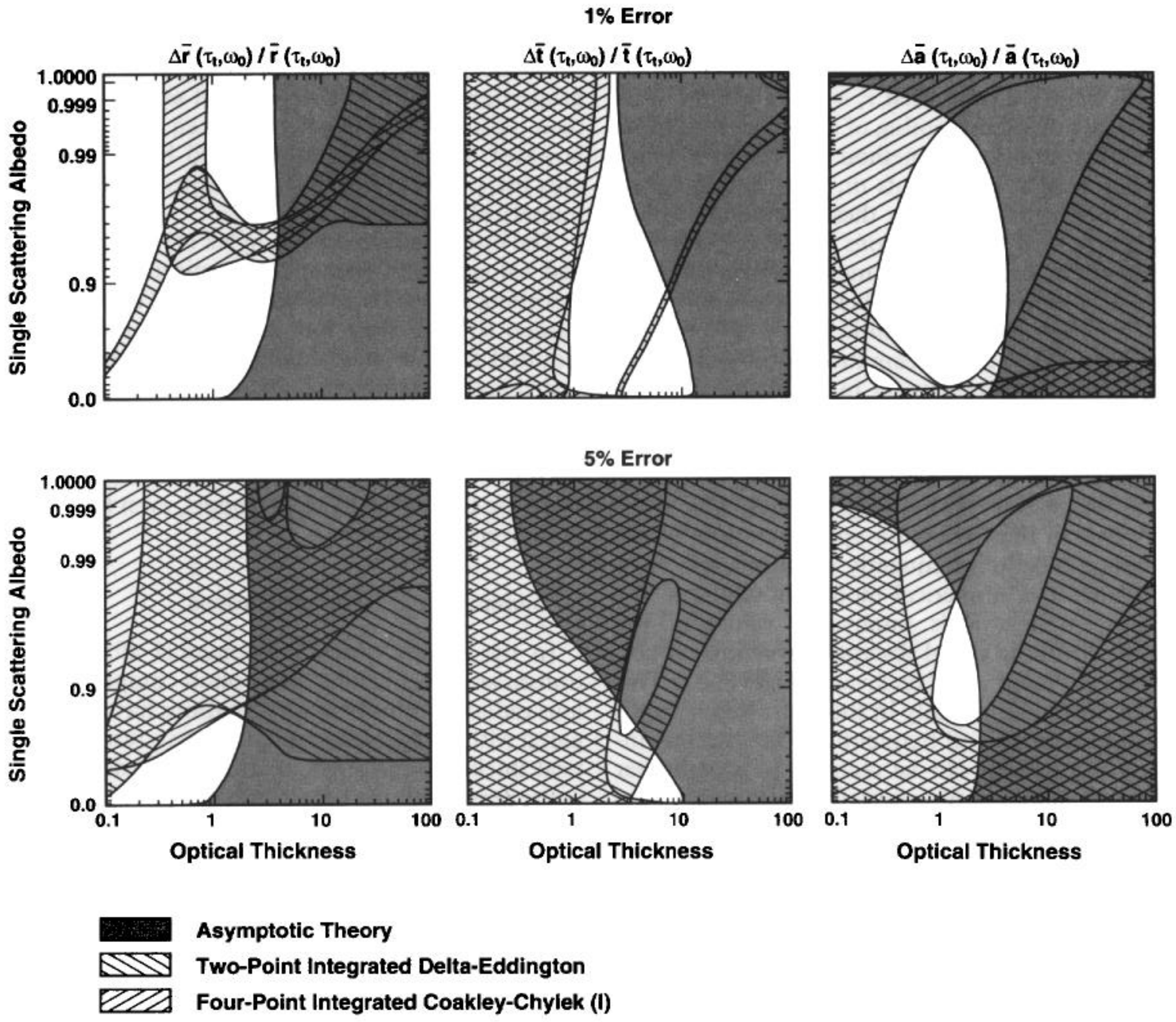


FIG. 6. The domain of validity of selected approximations to the diffuse radiative properties of cloud layers as a function of optical thickness (τ_t) and single scattering albedo (ω_0). The upper panels correspond to a relative accuracy of 1% and the lower panels to 5%. The single scattering albedo scale is linear in the similarity parameter. The FWC phase function is assumed throughout.

hundred percent (Slingo 1989). The sensitivity of all radiative properties to ω_0 can be appreciated by inspection of Fig. 1. Since any scheme has to limit the number of bands for computational efficiency, the selection of these bands and the average absorbing properties used could determine the overall accuracy. For a given set of τ_t and ω_0, however, the results presented in this study could act as a guide for choosing an appropriate model. Finally, it is pertinent to mention that these accuracies refer to an idealized plane parallel model. There is, of course, the additional problem of representing inhomogeneous cloud systems including geometric effects (Harshvardhan and Thomas 1984; Stephens 1988), a problem not considered in this study.

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