NOTES AND CORRESPONDENCE

On the Linear Approximation of Gravity Wave Saturation in the Mesosphere[1]

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21 July 1983 and 17 November 1983

ABSTRACT

Lindzen's model of gravity wave breaking is shown to be inconsistent with the process of convective adjustment and associated turbulent outbreak. The K-theory turbulent diffusion model used by Lindzen implies a spatially uniform turbulent field which is not in agreement with the fact that gravity wave saturation and the associated convection produce turbulence only in restricted zones. The Lindzen model may be corrected to some extent by taking the *turbulent* Prandtl number for a diffusion acting on the wave itself to be very large. The eddy diffusion coefficients computed by Lindzen then become a factor of 2 larger and *eddy* transports of heat and constituents by wave fields vanish to first order.

1. Introduction

After the seminal papers by Lindzen (1981) and Matsuno (1982) it has been generally agreed that the Rayleigh friction used in two-dimensional models of the zonal mean circulation of the middle atmosphere (Leovy, 1964; Schoeberl and Strobel, 1978; Holton and Wehrbein, 1981) has its physical origin in the turbulent breakdown of gravity waves in the mesosphere. These gravity waves are assumed to originate in the troposphere. Thus, breaking gravity waves apply a drag, whose origin characterizes tropospheric flows, directly to the mesospheric circulation.

It is not entirely clear that the mesosphere is the only locus of the breaking waves. In fact, VHF radars have observed the rapid attenuation of vertically propagating waves in the lower stratosphere (Gage *et al.*, 1981) as well as the mesosphere (Klostermeyer and Lin, 1978). Although anomalous deceleration of the stratospheric zonal wind is not required to obtain the observed stratospheric conditions in the numerical models mentioned above, some of the more recent models which use improved IR radiation algorithms do show excessively strong polar night jets (Wehrbein and Leovy, 1982; Mahlman and Umscheid, 1984). It may very well be, then, that momentum stress provided by breaking gravity waves is critical to the climatology of the stratosphere as well as the mesosphere.

Schoeberl and Strobel (1984) have shown that only the fast moving gravity waves with large intrinsic phase speeds ($|\bar{u} - c| > 15$ m s^{-1}) can penetrate to the mesosphere. It is also likely that the energy in these waves is considerably smaller than the energy in the slower phase speed topographically forced waves, most of which are trapped in "shear wave guides" near the surface (see Smith, 1979, for a comprehensive review of topographically forced disturbances). Thus, the faster moving waves will preferentially saturate at mesospheric altitudes where the density is low and the background lapse rate is closer to adiabatic. Slower moving waves will probably be absorbed in the stratosphere.

As previously mentioned, initial papers describing the effect of gravity wave breaking and momentum deposition on the mesosphere were presented by Lindzen (1981) and Matsuno (1982). While their basic approaches are rather similar, the fundamental assumptions of the two studies are quite different.

Matsuno (1982) assumed that a permanent turbulent field was present in the mesosphere so that gravity waves encountering this turbulence dissipate. The wave–mean flow interaction which results from the dissipation of the waves decelerates the mesosphere toward the horizontal phase speed of the gravity wave. Aside from the background zonal wind speed and the phase speed of the gravity wave, Matsuno's model requires the magnitude of the background turbulent field and the amplitude of the gravity waves launched from the troposphere.

Lindzen's (1981) formulation suggests that most of the mesospheric turbulent field owes its existence to the breaking gravity wave and is a response to the super adiabatic lapse rates generated by the large gravity wave amplitudes in the mesosphere. The convection thus generated limits the amplitude growth of the wave with altitude. Under these assumptions the turbulent field intensity is coupled to the wave amplitude at and above the breaking zone; therefore, Lindzen's model requires one less parameter than Matsuno's. Lindzen's formulation does not necessarily require knowledge of the amplitude of the gravity wave. Instead, since the wave is assumed to be breaking, the

[1] Contribution No. 12 of the Stratospheric General Circulation with Chemistry Project, NASA/GSFC.

amplitude is known but the breaking height is the (unknown) variable which would determine wave amplitude at the surface.

A third, somewhat more pragmatic, approach has been followed by Dunkerton (1982). He simply noted that the ultimate effect of wave breaking is to deposit momentum into the mean flow below critical lines. The changes in the mean flow occur in Dunkerton's model through an approximation to "sustained transience" which occurs as the wave approaches the critical line where the group velocity slows to zero and the effects of wave transience become more or less permanent. The mechanics of wave absorption are, of course, irrelevant as far as momentum deposition is concerned, but, as will be shown later, can be critical to the heat and constituent transport process.

In both Lindzen's and Matsuno's models, K-theory or eddy diffusion is used to model the effect of the Reynolds stresses associated with turbulent dissipation upon the waves. Schoeberl *et al.* (1983) showed that this type of formulation also implied significant downward eddy heat and tracer transport by the waves—as distinguished from turbulent transport of mean quantities by the turbulence due to the wave breaking. It has also been shown that the presence of large eddy diffusion produced by the breaking waves in the mesosphere will have significant impact on the global mean temperature (Apruzese *et al.*, 1984) and presumably on the constituent structure.

The purpose of this paper is to explore the underlying assumptions of wave breaking and the applicability of K-theory models to this process. Implicit in Lindzen's formulation is the assumption that eddy diffusion acts equally in both momentum and thermodynamic equations. In other words, the turbulent Prandtl number is taken to be unity. Assuming a sinusoidal structure for the gravity wave in the vertical, convection which develops in the breakdown of the gravity wave occurs only where $\partial\theta/\partial z$ minimizes, where θ is the potential temperature and z the log pressure height. On the other hand, the Reynolds stresses parameterized as eddy diffusion suppress both minimums and maximums in θ_{zz} which appear one quarter of a vertical wavelength above and below the θ_z minimum.

Clearly a fundamental problem exists in the K-theory formulation for saturating gravity waves. In this paper an improvement is described which makes the linear models of gravity wave saturation consistent with convective adjustment. We will also discuss the impact of breaking waves on the zonal mean structure of the mesosphere.

2. The gravity wave model

The equations of motion for a gravity wave propagating with phase speed c and zonal wavenumber m in a resting atmosphere are described by

$$imc\theta' + w'\bar{\theta}_z = DP^{-1}\theta'_{zz}, \quad (1a)$$

$$imu' + \rho^{-1}(\rho w')_z = 0, \quad (1b)$$

$$imcu' = -im\phi' - Du'_{zz}, \quad (1c)$$

$$\theta' = T' \exp(\kappa z/H), \quad (1d)$$

$$\phi'_z = RT'/H. \quad (1e)$$

The notation is standard: $z = H \ln(p_0/p)$, P is the turbulent Prandtl number, D the diffusion coefficient, $\kappa = R/c_p$; the dry air gas constant is R, c_p the specific heat of air at constant pressure and H a scale height (~7 km). Primes represent perturbation quantities. The buoyancy frequency N is related to θ by $N^2 = R\bar{\theta}_z H^{-1} \exp(-\kappa z/H)$; N becomes imaginary when $(\theta' + \bar{\theta})_z < 0$ which is a sufficient condition for instability as the Richardson number (Ri) is less than zero (Miles, 1961).

If D is small and $\bar{N}^2$ roughly constant, then Eqs. (1a)–(1d) may be solved for ϕ'. The solution is

$$\phi' = \phi_0 \exp\left[\left(i\gamma + \frac{1}{2}H\right)z\right] \exp[im(x + ct)], \quad (2)$$

where $\gamma = \pm(N^2/c^2 - \frac{1}{4}H^2)^{1/2}$ is the vertical wavenumber. In (1a) and (1c) it has been implicitly assumed that $\gamma \gg H^{-1}$ so the variation in density with height need not be considered in the diffusion terms. For positive, real values of γ, gravity waves are upward propagating and ϕ_0 is specified using the lower boundary condition.

a. The potential temperature equation

The combined gravity wave–mean flow system becomes convectively unstable when $\theta'_z \leq -\bar{\theta}_z$ so turbulence will appear first at the minimum of θ'_z, which we will refer to as the initial convection point. With the assumed structure (2), the eddy diffusion terms in (1a) are almost zero at this point and are largest approximately one quarter of a vertical wavelength above and below the turbulent region. Clearly, the form of the eddy diffusion in (1a) is not consistent with the idea of convective breakdown.

Presuming that turbulence is confined to a region where θ'_z is a minimum, then $\theta' \approx 0$ at the convection point which is also the point where w' has its largest negative value. In the vicinity of that point, the turbulence will mix the smaller values of θ' above the region with the larger values of θ' below. Even after such mixing θ' is still nearly zero since to first order, θ' has only been vertically averaged by the turbulence. Since $\overline{(w'\theta')} = 0$ in the absence of wave dissipation, *the presence of turbulence at the zones where θ' is nearly zero should not produce a significant correlation between w' and θ'.*

For a more intuitive view of this result, consider the displacement of air parcels by the gravity wave in the absence of a turbulent field. As a parcel bobs up and down, the temperature changes in response to adiabatic compression and rarefaction. The temperature excursions are largest when the parcel displacement is largest; the excursion is zero when the parcel displacement is zero. The vertical velocity is also largest when the displacement is zero. Therefore, the temperature and vertical velocity fields are uncorrelated [i.e., $(\overline{w'\theta'}) = 0$].

If turbulence is present everywhere with equal intensity (as the K-theory of turbulence implies), then the parcel mixes slightly with its surrounding. Thus, a parcel displaced upward returns with a slightly higher temperature and a parcel displaced downward returns with a slightly lower temperature. The warm parcels are moving downward and cool parcels are moving upward, on the average, so $(\overline{w'\theta'}) < 0$.

For gravity wave breaking, the turbulence is confined to the region where θ_z is minimum which is also the zero parcel displacement zone where $\theta' \approx 0$. The turbulent mixing creates no significant change in θ', so to first order $(\overline{w'\theta'}) = 0$. A significant correlation between w' and θ' may only appear when the turbulence encompasses a significant portion of the vertical structure of the wave.

b. A Lagrangian view of gravity wave breaking

Now we consider whether the breaking gravity wave has any effect on the mean field. Clearly from the previous argument we expect $(\overline{w'\theta'})$ to be much smaller than that derived from Lindzen's approach. However, is there any net heat flux due to the breaking process? The answer can be determined by examining Fig. 1a, which gives a Lagrangian view of the gravity wave packet in the process of breaking. The curves in this figure represent particle trajectories in a vertical plane. The "particle" here actually represents the center of mass of an air parcel. The wave amplitude is larger at middle level. The line A^1B^1 represents a stably stratified material line (i.e., θ at point A^1 is smaller than θ at B^1).

We now consider the wave breaking process in three stages. First, the gravity wave folds the line from A^1B^1 to A^2B^2 such that the conditions for convective instability are met. During this process a local downward heat transfer takes place as a warmer parcel B^1 is forced below the colder parcel A^1. Next, convection develops between A^2 and B^2 resulting in much reduced θ difference between A^2 and B^2. The particles A^2, B^2 are now different from the particles A^1, B^1, as the convective mixing implies an exchange of mass between the particles. During this second stage the local heat transfer is upward as the warmer air parcels rise and colder air parcels sink in the convection zone. In the final stage, the gravity wave unfolds line A^2B^2 back to its original orientation A^3B^3. If the gravity wave did not undergo convection, then the third stage would be an exact reversal of the first. Thus the net transfer of heat would be zero. However, in the breaking wave case, the unfolding in the third stage involves less transfer of heat than the downward heat transfer in the first stage because the particles A^2, B^2 have very little potential temperature difference. In fact, the transfer of heat in the convective stage must always be less than the transfer of heat in the first folding stage because convective processes can only neutralize (at least partially) the unstable fluid.

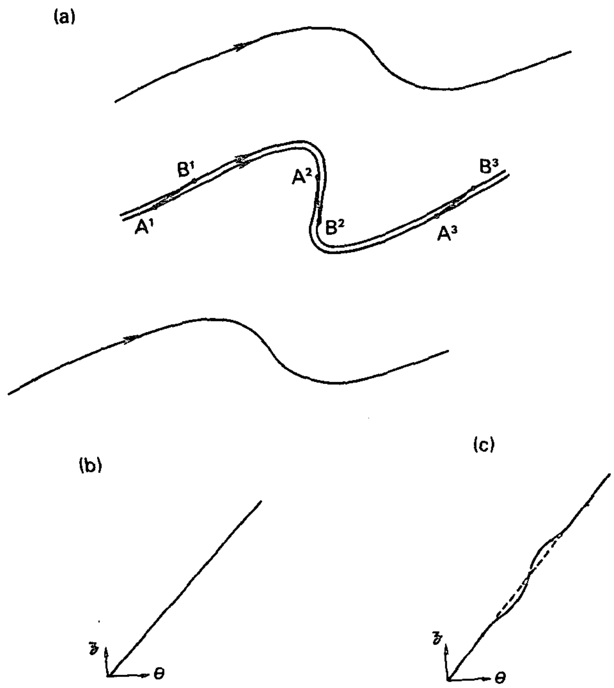

FIG. 1. A Lagrangian view of the gravity wave breaking process is shown in (a) where a wave packet is undergoing convective breakdown. The curves represent particle trajectories in a vertical plane. The line A^1B^1 connects particles on the trajectories shown. The potential temperature of B^1 is greater than A^1 (i.e., the fluid is stably stratified). The particles move into the vertical position A^2B^2 which is convectively unstable as particle B^2 has a greater potential temperature than A^2. The line A^3B^3 connects particles with very similar potential temperatures since convection has mixed the fluid. The net effect of this process on the vertical potential temperature gradient is shown in (b) and (c). The initial potential temperature distribution is shown in (b) and the final distribution is shown in (c). Because of convection, the potential temperature gradient is reduced. The net effect is downward heat transport.

In summary, the first fold takes a stably stratified fluid and makes it unstable, while the convective (second) stage neutralizes the unstable fluid. The third stage need not be considered (for fast convection) since the fluid is now nearly neutral and folding will produce a negligible heat transport. The net result is a downward heat transport (first stage minus second stage) manifested in the reduced θ gradient between A^3 and B^3. *Thus, the effect of the wave breaking on the basic mean* *flow is to induce mixing and produce a down gradient transport of the fluid properties* (Figs. 1b and 1c).

It should be recognized that wave breaking is a highly nonlinear process and that in a linear model one is forced to use a linear expression to represent the macroscopic effects of the nonlinear process. It remains doubtful that linearization of the wave breaking convection is possible as the stratified fluid in stage one is quite different from the mixed fluid in stage three. In this regard, the second-order turbulence closure model is, of course, a more precise approach.

c. The momentum equation

Since the diffusion formulation based on K-theory in the potential temperature equation appears to be incorrect, it is germane to ask if the momentum equation (1c) is consistent with the wave breaking processes. The perturbation winds u' and w' are correlated since

$$w' \approx -m\gamma^{-1}u' .$$

Likewise,

$$w' \approx -m\gamma^{-1}c\theta'_z/\bar{\theta}_z$$

so that u' and w' are both extreme where θ'_z minimizes. Thus, the turbulence modeled as eddy diffusion in (1c) has approximately the correct form.

d. Summary

The principal inconsistency with convective adjustment is the presence of eddy diffusion in the wave potential temperature equation. The simplest (but not the only) modification of Eqs. (1a)–(1e) that appears to be consistent with convective adjustment can be made by setting the turbulent Prandtl number to infinity. This modification gives a much smaller $(\overline{w'\theta'})$ and still provides for the turbulent diffusion of momentum in (1c). The change should be made only for the eddy equations describing *saturating* gravity waves. The turbulent Prandtl number should remain ~1 for gravity waves encountering a preexisting turbulent region and in the mean flow equations.

If eddy diffusion is eliminated from the thermodynamic equation then D will have to be twice as large to halt the wave growth with altitude. Thus, the D values estimated by Lindzen (1981) are half those required by this model. The downward transport of heat by eddies vanish to first order, but the turbulent heating, i.e., the conversion of wave energy to heat through turbulence, has the same magnitude.

As a final note, although the eddy heat transport by a breaking wave should be small, it will be enhanced in the presence of any radiative damping process (Schoeberl *et al.*, 1983). Furthermore, the turbulence induced by the breaking gravity wave on an otherwise stably stratified flow will generate a net downward diffusive heat transport even though the wave transport of heat is small.

TABLE 1. Observational estimates of the eddy diffusion coefficient.[a]

Reference	D ($m^2 s^{-1}$)
Kochanski (1964), Hines (1965)	~1300°
Cunnold (1975)	~100⁺
Rottger and Schmidt (1979)	4000⁺
Vincent and Ball (1981)	500°
Zimmerman and Murphy (1977)	1000–5000*

[a] Measurements were made by examining the average decay of gravity waves with altitude (°), direct measurement of backscattered power from turbulent zones (⁺) or *in situ* measurements of wind fluctuations by rocket grenade (*). Cunnold's value is probably in error because of assumptions about the basic state.

3. Discussion

Although observational evidence of gravity wave events in the mesosphere is scarce, it may still be possible to check some of the assumptions used in Lindzen's parameterization and to access the turbulent coefficients appropriate for the mean flow from observations.

Throughout the stratosphere and mesosphere, turbulent zones are observed to form as layers (Wand *et al.*, 1983; Zimmerman and Murphy, 1977; Rottger and Schmidt, 1979; etc.). These layers in the stratosphere appear to be very thin, roughly less than 1 km (Wand *et al.*, 1983; Gage *et al.*, 1981) while both thin and thick layers (1–5 km) appear in the mesosphere (Rottger and Schmidt, 1979). Although the evidence is not overwhelming, it does appear that these turbulent layers are associated with convective conditions (i.e., Philbrick, 1980; Clark and Morone, 1981, Figs. 9–10). The depth of a convective layer induced by the passage of a gravity wave will initially be very thin, expanding with altitude to some fraction of the gravity wave's vertical wavelength until at least enough turbulent mixing is present to halt the wave amplitude growth and altitude. The observed thin turbulent layers could be associated with the beginning of gravity wave convection, but more likely, the thin layers are associated with the breakdown of very short vertical wavelength gravity waves. These waves probably have encountered critical lines since it is very difficult to maintain short wavelength disturbances over large vertical distances against radiative damping (Schoeberl *et al.*, 1983).

Even though the thermal amplitude of the wave packet decreases as it approaches a critical line (CL) proportional to $\gamma^{-1/2}$ by conservation of wave action,[2]

[2] Wave action

$$A \equiv \frac{\overline{u'^2} + \overline{\phi_z'^2}/N^2}{2(c - \bar{u})}$$

the eddy lapse rate amplitude increases proportional to $\gamma^{1/2}$, so every wave will saturate before reaching the CL. The thicker turbulent layers may be due to the initial breakdown of the wave as it increases in amplitude with altitude.

The eddy diffusion coefficient D used in Lindzen's model is derived under the assumption that the wave amplitude would remain constant with height. Under such conditions, observations would show parallel turbulent layers or super adiabatic lapse rates at widely separated heights. Such structures are seen in the mesosphere associated with super adiabatic zones (Czechowsky *et al.*, 1979, Fig. 3; Fritts, 1984) which is in general agreement with Lindzen's (1981) theory.

Table 1 shows the wide variations in D estimated by experimental methods. Relating D values deduced from theory to those ascertained from observations may be difficult since the observing techniques vary widely. For example, the backscatter radars sense the turbulent fluctuations directly and are effectively peering into the strongly convective zone. On the other hand, gravity wave attenuation methods derive a D appropriate over the whole gravity wave depth, a technique more compatible with Lindzen's (1981) model. The latter methods suggest D values of 10–100 $\mathrm{m^2\,s^{-1}}$. These values are not inconsistent with values used in one-dimensional eddy diffusion mesospheric models of the nearly neutral tracer nitric oxide (Swider, 1979) and NLTE (nonlocal thermodynamic equilibrium) thermodynamic models (Apruzese *et al.*, 1984). This magnitude for the D coefficient is in the range predicted by Lindzen's (1981) model although it is generally agreed that D would increase or be constant with height rather than decrease as Lindzen suggested. The presence of additional waves with different phase speeds as well as wave intermittency can easily explain the discrepancy between observations, the nitric oxide models and Lindzen's predicted values for D.

4. Summary

The convective saturation of gravity waves assumed in Lindzen's (1981) model is shown to be incorrectly modeled by K-theory unless the turbulent Prandtl number is very large. With this modification the eddy heat transport vanishes and the turbulent diffusion coefficient becomes a factor of 2 larger to meet the constant amplitude requirement. Nevertheless, the net transport of heat must be down gradient since the breaking gravity waves simply induce turbulence on the stably stratified mean flow.

It is not clear that the larger eddy diffusion coefficient is appropriate for the mean flow computation. Indeed, the Lagrangian model presented here does not tell us what the appropriate mean flow diffusion rate should be. The appropriate rate could be computed from numerical models of gravity wave breaking incorporating convective adjustment.

After the original submission of this paper we developed a numerical model to test the ideas expressed above. Preliminary results suggest that fully developed breaking waves produce an eddy transport of heat equivalent to the use of a turbulent Prandtl number of 20. This large value appears to confirm our hypothesis. Further discussion of this model will be reported in a subsequent paper.

can be rewritten as

$$A = \frac{\overline{\phi_z'^2}}{N^2(c - \bar{u})}$$

when equipartition of wave energy is considered. Close to a critical level $c - \bar{u} = N\gamma^{-1}$. Thus the conservation of A gives $\phi_z' \propto \gamma^{-1/2}$, or $T' \propto \gamma^{-1/2}$.

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