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by

**J.J. Filliben**  
Statistical Engineering Division  
National Institute of Standards and Technology  
Gaithersburg, MD 20899 USA

**K. Gurley**  
University of Florida USA

**J.-P. Pinelli**  
Florida Institute of Technology USA

and

**E. Simiu**  
Building and Fire Research Laboratory  
National Institute of Standards and Technology  
Gaithersburg, MD 20899 USA

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# **Fragility curves, damage matrices, and wind induced loss estimation**

J. J. Filliben<sup>1</sup>, K. Gurley<sup>2</sup>, J.-P. Pinelli<sup>3</sup>, E. Simiu<sup>4</sup>  
& C. Subramanian<sup>5</sup>

<sup>1,4</sup> *National Institute of Standards and Technology, USA*

<sup>2</sup> *University of Florida USA*

<sup>3,5</sup> *Florida Institute of Technology, USA*

## **Abstract**

This note presents a conceptual framework for the definition of basic damage states and of the corresponding fragility curves and conditional probabilities, and its use for the estimation of damage matrices. The framework is designed with two considerations in mind. First, losses due to multiple types of damage are calculated so that no type of damage is counted more than once, no type of possible damage is omitted from the calculations, and all interactions between various types of damage are accounted for. Damage is included that may vary continuously as a function of wind speed but is discretized for computational purposes. Second, the losses are calculated by correctly accounting for the dependence between various damage states (e.g., window breakage and roof uplift). The note also discusses the use of damage matrices for the estimation of expected losses due to a windstorm event, of expected annual losses, and of measures of uncertainty pertaining to expected losses, both at a specified location and over a larger geographical area. The framework developed in the paper is illustrated for the conceptually simple case of two basic damage states. Work is in progress on the application of the framework to various types of structures involving larger numbers of basic damage states with various mutual dependence and damage sequence scenarios. Work is also in progress for the estimation of uncertainties in loss calculations, based on uncertainties in the estimation of fragility curves, associated conditional probabilities, and hurricane wind speeds. One of the applications of our work is the development of vulnerability curves and associated uncertainty measures for cases

where comprehensive loss data from which such curves may be developed are not available.

## 1 Introduction

We consider a group of  $n$  buildings belonging to a specified building class (e.g., manufactured homes). The buildings are subjected to wind loading and located within a relatively small area (e.g., within a sub-zone with uniform terrain roughness belonging to a zip code zone), so that all the structures in the group may be assumed to experience approximately the same maximum wind speed during a storm. For any given wind speed  $v$  each of the buildings will either not experience damage or experience damage of one or several types (e.g., loss of part of the girt, loss of the entire roof, breakage of windows, building collapse). Some of the types of damage are independent of each other (e.g., loss of part of the girt and loss of shingles), while others are dependent on others (e.g., given that the building has experienced window breakage, the probability of its losing the roof increases).

Unless vulnerability curves can be developed from comprehensive sets of loss data, the estimation of building losses induced by a windstorm requires the development of damage matrices, that is, matrices whose entries are probabilities that, at wind speed  $v$ , a building will experience damages of various types. These probabilities are calculated from probabilities of basic damage states, commonly referred to as fragility curves. Associated with each damage type is a repair/replacement cost. If a damage matrix and the corresponding set of repair/replacement costs are available it is possible to estimate the expected repair/replacement costs (i.e., the total expected loss) per building due to the effects of a windstorm with wind speed  $v$ . For the entire group of  $n$  buildings belonging to the specified building class at the location of concern the total loss due to the windstorm is  $n$  times the expected loss per building. The corresponding expected annual loss is obtained by integrating over all wind speeds  $v$  the losses corresponding to  $v$  weighted by the respective ordinates of the probability density function of  $v$ . Losses for groups of buildings belonging to various building classes are additive, as are losses at various geographical locations. Using elementary methods it is similarly possible to estimate measures of uncertainty in the estimation of the losses, both at a specified location and over wider geographical areas.

In this note we focus on the calculation of damage matrices from information embodied in fragility curves and measures of dependence between basic damage states. Our main objective is to define a conceptual framework suitable for wind engineering applications. For the sake of simplicity we illustrate our approach for the case where the structure can experience two basic damage states: loss of roof, and building collapse. However, the development of damage matrices can be generalized for larger numbers of types of damage. We first discuss the definition of fragility curves. We then examine the issue of the sets of damage states that need to be included in the damage matrix. For these sets we discuss the construction of the corresponding damage matrices. Finally, we briefly

outline the use of damage matrices for the estimation of repair/replacement costs corresponding to a storm event and of expected annual repair/replacement costs.

## 2 Fragility curves

For a class of structures with two basic damage states we consider the following matrix for the basic damage state R (partial loss of roof) and the basic damage state C (building collapse).

Table 1: Complements of fragility curves basic state probability

$v$ (mph)		80	90	100	110	120	130	140
R	$P(R v)$	0	0.03	0.20	0.5	0.8	1.0	1.0
C	$P(C v)$	0	0	0	0.02	0.05	0.2	1.0

Numbers in row R are probabilities  $P(R|v)$  that the basic state R occurs given that the wind speed is  $v$ . Numbers in row C are probabilities  $P(C|v)$  that state C occurs given that the wind speed is  $v$ .  $P(R|v)$  and  $P(C|v)$  are cumulative distribution functions of the wind speed  $v$ . This means, for example, that the wind speeds that will cause a building to experience with probability  $P(R|v)$  the basic state R (loss of its roof) are less than or equal to  $v$ .

For example, Table 1 states that, given that  $v = 120$  mph,  $P(R|v)=0.8$  is the probability that a building will experience partial loss of its roof, and  $P(C|v)=0.05$  is the probability that the building will collapse. Note that the basic state "partial loss of roof" for which probabilities  $P(R)$  may be obtained in laboratory tests, does not necessarily exclude total building collapse, which may, for example, occur owing to overturning.

To simplify the notation we will omit the notation " $v$ " in all subsequent developments, that is, we will use the shorthand notation  $P(x)$  in lieu of  $P(x|v)$ . *It should be recalled, however, that unless otherwise indicated, in this note all probabilities represent probabilities of damage states given that the wind speed is  $v$ .*

Fragility curves have historically been defined as the complements of cumulative distribution functions such as those listed in Table 1. The fragility curves corresponding to Table 1 are therefore defined by the expressions  $1 - P(R)$  and  $1 - P(C)$ . Probabilities  $P(R)$  and  $P(C)$  can in principle be obtained directly from laboratory tests, analytical studies entailing simulations, and post-disaster observations of damage. In some instances it may be convenient to define fragility curves for basic damage states other than R and C. For example, the damage state (R and not C), rather than for the state R, may in some instances be chosen to be a basic state. The choice of basic damage states is determined by practical considerations such as the format in which the requisite probabilistic information is available. For example, information on partial roof loss could in principle be available from laboratory tests in which the structure is

strong and will not collapse before partial loss of the roof. In this case fragility curves would be available for the damage state (R and no C), which could be considered a basic state, as noted earlier. Alternatively, the information could be available from the observation of states R where partial loss of the roof may occur either without the structure collapsing or with the structure collapsing (this is the case for Table 1). The estimate of the total damage induced by the wind speed  $v$  is independent of the basic states for which the fragility curves are defined, provided that the construction of the damage matrix from the fragility curves is consistent from a set theoretical and probabilistic viewpoint.

### 3 Damage matrix

A damage matrix is a matrix of those probabilities for damage states that need to be accounted for in the estimation of wind-induced losses. In this section we discuss states that are appropriate for inclusion in the damage matrix. These states must satisfy two requirements:

The states must be functions of basic states.

The states must be chosen with a view to enabling loss estimates to be made correctly, in the sense that no possible damage state is omitted, and no double or multiple counting of damage states occurs.

Associated with the basic damage states R and C are the following theoretically possible intersection events (states):

*Combinations of 4 taken by 0:  
(This set is empty)*

*Combination of 4 taken by 1:  
R; C; not R; not C*

*Combination of 4 taken by 2:  
R and C; R and not R (this set is empty); R and not C; C and not R; C and not C  
(this set is empty); not R and not C*

*Combination of 4 taken by 3:  
R and C and not R (this set is equivalent to C); R and C and not C (this set is  
equivalent to R); R and not R and not C (this set is equivalent to not C); C and  
not C and not R (this set is equivalent to not R)*

*Combination of 4 taken by 4:  
R and C and not R and not C (this set is empty).*

The total number of states listed above is  $2^4 = 16$ . Similar sets of  $2^4 = 16$  "or" states also exist. In general not all possible states are of interest from a loss estimation point of view. In this note we focus on the following damage states:

Case 1. *R and not C* (partial roof loss but no collapse). The probability of this state (i.e., of the intersection of R and C) will help to estimate the cost of repair of roofs for the structures that have not collapsed and need roof repair.

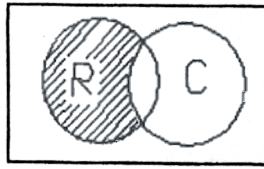


Figure 1: Venn diagram for R and not C

Case 2. *C and not R* (structure collapsed but roof intact). Assuming that a structure collapses with the roof being intact (as might be the case if the collapse is due to overturning), it could in principle be rebuilt at a lower cost than a structure that collapsed and whose roof was damaged. While this scenario is likely not realistic for our example, it is included here for illustrative purposes, since the approach we use may be applicable for similar, but realistic, situations involving other sets of basic damage states and types of failure mechanism or sequence.

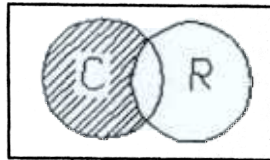


Figure 2: Venn diagram for C and not R

Case 3. *R and C* (roof damaged and structure collapsed). The probability of this state will help to estimate the cost of replacement of the entire structure, including the roof.

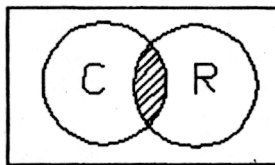


Figure 3: Venn diagram for R and C

Case 4. *not R and not C*. This state designates “no damage.”

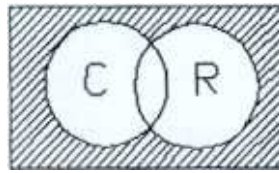


Figure 4: Venn diagram for not R and not C

These four cases correspond in Venn diagrams to the hatched areas of Fig. 1 to 4.

#### 4 Calculation of damage matrix probabilities

The probabilities are calculated from (a) the fragility curves and (b) probabilistic information on dependence between elementary damage states.

Case 1 (*R and not C*). To calculate the probabilities for the state (*R and not C*) we make use of the fact that the probabilities of R in the complements to the fragility curves (Table 1) are *marginal probabilities*, that is,

$$P(R) = P(R \text{ and } C) + P(R \text{ and not } C) \quad (1)$$

so

$$P(R \text{ and not } C) = P(R) - P(R \text{ and } C). \quad (2)$$

Case 2 (*C and not R*). Similarly,

$$P(C) = P(R \text{ and } C) + P(C \text{ and not } R), \quad (3)$$

$$P(C \text{ and not } R) = P(C) - P(R \text{ and } C). \quad (4)$$

Case 3 (*R and C*). From conditional probability theory, the joint probability of (R and C) is given by

$$P(R \text{ and } C) = P(R|C) P(C) \quad (5)$$

$$P(R \text{ and } C) = P(C|R) P(R) \quad (6)$$

where  $P(R|C)$  is the probability of state R, given that state C has occurred (a similar definition holds for  $P(C|R)$ ).

The conditional probabilities are functions of the engineering dependence between the states R and C. The specification of this dependence is an

engineering input to the problem. It is a function of the damage mechanisms as determined by the engineering analyst.

Case 4 (*not R and not C*). The probability of no damage is

$$P(\text{not } R \text{ and not } C) = 1 - P(R \text{ or } C) = 1 - [P(R) + P(C) - P(R,C)] \quad (7)$$

We consider the following sub-cases:

A. *100% dependence*. The probability of R given that C has occurred is unity (i.e.,  $P(R|C)=1$ ). Physically, for our problem, this means that if the structure has collapsed then the roof has necessarily been damaged. For this case it follows from Eq. 5

$$P(R \text{ and } C) = P(C) \quad (8)$$

B. *Independence (0% Dependence)*. Physically, for our problem independence means that collapse does not necessarily entail damage to the roof (collapse may be due to overturning while the roof remains intact). In this case  $P(R|C) = P(R)$ , that is, from Eq. 5,

$$P(R \text{ and } C) = P(R) P(C). \quad (9)$$

For example, for  $v = 120$  mph,  $P(R \text{ and } C) = 0.8 \times 0.05 = 0.04$ ,  $P(R \text{ and not } C) = 0.8 - 0.04 = 0.76$ ,  $P(C \text{ and not } R) = 0.05 - 0.8 \times 0.05 = 0.01$ . The damage matrices for case 1 and case 2 are given in Tables 2 and 3.

C. *Partial Dependence*. For this sub-case it is required to specify the conditional probability on the basis of structural engineering considerations. (This would apply, for example, to roof loss given window breakage.)

D. *Mutually Exclusive States*. For mutually exclusive states  $P(R|C)=0$ , so  $P(R,C) = 0$ .

For  $v = 120$  mph, Table 2 shows for the two fragility curves of Table 1 and various values of the conditional distribution  $P(R|C)$  the corresponding values of the joint distribution  $P(R,C)$  and the probability of no damage  $P(\text{not } R \text{ and not } C)$ .

Table 2. Probabilities  $P(R,C)$  and  $P(\text{not } R, \text{not } C)$  corresponding to  $P(R)=0.80$  and  $P(C)=0.05$  and various values of  $P(R|C)$ .

P (R)	P (C)	P (R/C)	P(R,C)	P(not R,not C)
0.80	0.05	0 <sup>a</sup>	0.0	0.15
0.80	0.05	0.2	0.01	0.16
0.80	0.05	0.4	0.02	0.17
0.80	0.05	0.6	0.03	0.18
0.80	0.05	0.8= $P(R)$ <sup>b</sup>	0.04	0.19
0.80	0.05	1.0 <sup>c</sup>	0.05	0.20

<sup>a</sup> R and C are mutually exclusive states

<sup>b</sup> R and C are independent states



<sup>c</sup> 100% dependence of R on C

Tables 3 and 4 show calculated damage matrices corresponding to sub-cases A and B.

Table 3: Damage matrix, Case A (100% Dependence)

v (mph)	80	90	100	110	120	130	140
R and not C	0	0.03	0.2	0.48	0.75	0.80	0
R and C	0	0	0	0.02	0.05	0.20	1.0
C and not R	0	0	0	0	0	0	0
No Damage( not R and not C)	1	0.97	0.8	0.50	0.20	0	0

Table 4: Damage matrix, Case B (Independence)

v mph	80	90	100	110	120	130	140
R and not C	0	0.03	0.2	0.49	0.76	0.80	0
R and C	0	0	0	0.01	0.04	0.20	1.0
C and not R	0	0	0	0.01	0.01	0	0
No damage ( not C and not R)	1	0.97	0.8	0.49	0.19	0	0

## 5 Estimates of repair/replacement costs

Assume, for example, that the repair/replacement costs per building are the following:

R and not C	\$30,000
R and C	\$100,000
not R and C	\$90,000.

For case B (independence between R and C), the expected cost of repair/replacement per building for damage induced by a windstorm with wind speed 120 mph is  $0.76 \times 30,000 + 0.04 \times 100,000 + 0.01 \times 90,000 = 22,800 + 4,000 + 900 = \$27,200$ .

Estimates of the type illustrated in this example are building blocks for estimating costs for building inventories over a geographical area comprising a large number of communities, of which each may comprise several building types in terrains with different terrain roughness. A procedure for estimating such costs is described below.

We recall that the choice of two damage states was made for purposes of illustration of our conceptual framework. The authors are currently applying that framework for actual situations entailing more complex, realistic damage scenarios.