CHAPTER 3

Multiple Projection Lines on Early Surveys

Before a topographic sheet (*see* 124) or a hydrographic sheet (*see* 125) was taken into the field to begin the survey of an area, a projection was first laid down on the sheet, that is, lines representing the parallels of latitude and meridians of longitude, and from these lines the triangulation control points plotted. On many of the early surveys of the Bureau two or more systems of latitude and longitude lines appear. These are traceable to various causes (*see* 35) some of which have been discussed in Chapter 2. In the engineering use of such surveys, it is important to keep this fact in mind. Two surveys cannot be properly evaluated, whether for studies of shoreline changes or for other purposes, unless comparable projection lines appear on the survey sheets. Before discussing the reasons for the existence of multiple projection lines on the early surveys, something of the nature of projections will be described, particularly as they relate to the type of projection ultimately adopted for the topographic and hydrographic surveys of the Bureau.

31. THE PROBLEM OF MAP MAKING

We have seen in Chapter 2 that for practical geodetic work, the earth may be taken to have the shape of a spheroid—a sphere flattened slightly at the poles.[1] It is axiomatic that the ideal method of studying the earth and its component relationships is by means of a globe. Being an accurate model of the earth the scale is constant, and areas, shapes, distances, and directions are preserved as on the earth. Because of this, a true idea of the fundamental principles of geography can be obtained—the general distribution of land and

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[1] The polar diameter of the earth is about 26 miles shorter than the equatorial diameter. For discussing projections the earth may be considered as a sphere, since the irregularities are very small when compared with the great size of the earth. If the earth were represented by a spheroid with an equatorial diameter of 50 feet, the polar diameter would be 49 feet 10 inches.

water areas, the relative sizes and shapes of continents and countries, and the correct direction and distance geographic features are from each other. But globes, in order to show any degree of detail, would have to be so large as to preclude their practical usefulness. For example, a globe on a scale of 1:80,000, the scale of the 1200 series Coast Survey charts along the Atlantic and Gulf coasts, would have a diameter of 523 feet. For detailed geographical information we are, therefore, dependent upon flat-surface maps.[2]

If it were possible to flatten a globe into a plane surface, without tearing or stretching, then the art of map projection would never have arisen. But we know this to be impossible from the simple attempt to flatten a hollow rubber ball. The problem of the map maker has therefore been to devise some means by which a portion or all of the curved surface of the earth can be represented on a plane with the least amount of distortion. The process by which this is accomplished is termed "map projection."

32. DEFINITION OF MAP PROJECTION

To Hipparchus—astronomer and mathematician, who lived in the 2d century B.C. and who is credited with having invented the trigonometry—we are indebted for the present-day method of dividing the earth by a system of parallels of latitude and meridians of longitude.[3] With this system, points on the earth's surface are located by their latitudes and longitudes, that is, their distance north or south of the equator and east or west of the meridian of Greenwich. Such points are said to be described by their geographic or geodetic coordinates. This is a universal system and is used exclusively in all the geodetic work of the Coast Survey and in all its cartographic work.[4] The problem of map projection has been to find some method of transferring the imaginary meridians and parallels on the earth to a flat map. They can be drawn in an arbitrary manner, but to avoid confusion and to be of scientific value they must follow an orderly correspondence. A map projection may

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[2] Most surveys, whatever their nature, have for their ultimate aim the making of a map. By its means can be visualized what has been surveyed. Until that is done there can be little or no conception of the nature or relation of the features located. In the simple survey of a city lot or farm, it is necessary to plot the courses and distances measured in order to determine the shape of the parcel of land. Even in a triangulation survey for the establishment of control points, some form of sketch must be prepared if the relationships of the points located are to be comprehended.

[3] 11 ENCYCLOPEDIA BRITANNICA 582 (1960). The meridians are formed by imaginary planes passing through the poles, and the parallels by planes passing through the earth parallel to the equator.

[4] The Bureau has also computed the plane coordinates on the State Coordinate Systems (*see* Part 1, 2113 B) of all triangulation points adjusted on the North American 1927 Datum (*see* 225).

therefore be defined as a systematic drawing of lines representing meridians and parallels on a plane surface, either for the whole earth or for some portion of it.[5] The number of ways in which this orderly arrangement can be determined is almost without limit, depending upon the conditions imposed.

33. TYPES OF PROJECTIONS

In strictness, the term "projection" is geometrical in concept and ought to be confined to representations obtained directly according to the laws of perspective, but geographers have borrowed it from geometers and have applied it to any method of representation of the surface of the earth upon a plane, whether it be by geometric construction, as in perspective projections, or by development.[6] For the field surveys and the nautical chart work of the Coast Survey, the latter type of projection is used exclusively. Of these, there is a large variety, each projection fulfilling a condition that exists on the sphere which it is desirable to preserve in the map, whether it be equivalence of area, right shape, true distances, or correct bearings. For example, in the "conformal" class of map projections the property of correct shape is preserved for geographic features, rather than correct size. In contrast, there is the "equal-area" type of map projection in which correct size is preserved, rather than correct shape. For mapping extensive portions of the world, it is mathematically impossible to preserve both properties in the same projection. Therefore, any projection is at best a compromise and the choice of projection usually depends upon the purpose which the map or chart is to serve.

It has already been noted that a sphere or spheroid is a nondevelopable surface and cannot be spread out in a plane without distortion. For that reason an ideal map is impossible of attainment.[7] However, there are some curved surfaces—the cone and the cylinder—that are developable and use is made of them as intermediate aids to projection. If the cone is cut from apex to base or the cylinder from base to base these surfaces can be spread out in a plane

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[5] DEETZ AND ADAMS, ELEMENTS OF MAP PROJECTION 22, SPECIAL PUBLICATION NO. 68, U.S. COAST AND GEODETIC SURVEY (1944).

[6] One objection to the perspective projections is that in their use one is limited to the properties which they already possess; they cannot be made to satisfy any special conditions which may be of importance in the particular mapping under consideration, or they may possess features which are not desirable on the map or chart. Perspective projections find application in astronomy and in the flat-surface representation of an extensive area, such as a hemisphere.

[7] Small areas can of course be represented with a high degree of fidelity because the earth is so large that for practical purposes any small portion can be considered a plane and no difficulty is encountered in reproducing one plane on another.

without stretching or tearing.[8] Projections that use the cone as the developable surface for determining their elements are called conical projections, while those that use the cylinder are called cylindrical projections.[9] In this chapter only the conical type will be considered, this being the type used for the field surveys of the Bureau.[10]

34. CONICAL PROJECTIONS

Conical projections employ a cone tangent generally at the middle parallel of the map to be constructed, but variations may be produced by altering the position of the place of tangency or by substituting for the tangent cone an intersecting cone which cuts the spheroid at two parallels. In the first, the scale is held true along one parallel, while in the second the scale is maintained true along two parallels. There is thus a wide variety of the conical type of projections. In this discussion, only those projections will be considered that lead up to the development of the polyconic projection, the projection that is used for all the topographic and hydrographic surveys of the Coast Survey.

341. SIMPLE CONIC PROJECTION

In the simple conic projection,[11] a right cone tangent to the sphere at the middle latitude (the standard parallel) of the area to be mapped is used. The apex of the cone will then be in the prolongation of the axis of the sphere. The slant height of the cone is the radius of the middle parallel and becomes the central meridian of the map. Distances corresponding to the latitude values on the surface of the earth are laid off on the central meridian and through these points circles concentric with the middle parallel are drawn using the apex of the cone as center. The meridians are determined by laying off on the middle parallel distances equal to the true longitude values and drawing straight lines from these subdivisions to the apex. The scale of the projection is therefore

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[8] Actually, the projection is not constructed upon the cone or cylinder, but the principles are derived from a consideration of these surfaces, and then the projection is drawn upon the plane just as it would be after development. DEETZ AND ADAMS (1944), *op. cit. supra* note 5, at 27.

[9] Both of these forms are generally included in the one class of conical projections, for the cylinder is really a special case of the cone with the apex at infinity.

[10] A conical projection was also used for a time as the base for the nautical charts of the Bureau (*see* 6411).

[11] This projection is credited to Ptolemy, the Greco-Egyptian astronomer, geographer, and geometer who lived in the 2d century.

true along the standard parallel and along all meridians, but above and below the standard parallel the scale along the parallels will not be true because these parallels when projected on to the tangent cone are greater than on the sphere. This makes the projection unsuitable for mapping areas with a considerable north-south extent. In this projection, parallels and meridians intersect at right angles as on the sphere.

This projection is, then, characterized by straight meridians and concentric arcs of circles for parallels, the two intersecting at right angles.

342. BONNE PROJECTION

An important modification of the simple conic projection was made by the French engineer Rigobert Bonne (1727–1795) in a projection which bears his name.[12] As in the simple conic a cone tangent to a selected standard parallel is used. The central meridian is subdivided true to scale and concentric circles drawn to represent the parallels of latitude. But instead of subdividing only the standard parallel true to scale, as in the simple conic, Bonne subdivided all the parallels according to their values on the earth. Through the corresponding points on each parallel he drew smooth curves for the meridians. Thus, the scale along all the parallels is correct by construction, and the shortcoming of the simple conic projection in this respect is remedied, making the Bonne projection more suitable for maps having considerable north-south extent. But with the exception of the central meridian no other meridian intersects the parallels at right angles, the departure from perpendicularity increasing with the distance from the central meridian and the approach to the polar regions where they intersect quite obliquely. The scale along the meridians is therefore too great for all except the central one, and this defect makes it unsuitable for projecting extensive east-west areas.

The characteristics of this projection are, then, curved meridians except the central one, concentric arcs of circles for all the parallels, and nonorthogonality of intersections.[13]

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[12] The Bonne projection was adopted in France about 1803. Annual Report, U.S. Coast and Geodetic Survey 292 (1880).

[13] There is some evidence that the Bonne projection was used for some of the nautical charts of the Bureau. In the 1880 annual report, it is stated with reference to the 1844–1845 charts of Delaware Bay and River (scale 1: 80,000) that they have conic projections and that the "parallels of latitude are sensibly curved, but owing to their small latitudinal extent the meridians appear necessarily as straight lines and consequently the character of the particular conic projection is not revealed. There is, however, reason to suppose that it was Bonne's." *Id.* at 293.

343. POLYCONIC PROJECTION

The advent of the Bonne projection was a distinct improvement over the simple conic, but its limitations indicated the propriety of still another modification, namely, developing each parallel of latitude as the circumference of the base of a right cone tangent to the sphere along that parallel. The result was termed a "polyconic projection." Ferdinand Hassler, the first Superintendent of the Coast Survey, appears to have been the one to originate, at least in concept, this type of projection. This may be inferred from Hassler's statement that "This distribution of the projection, in an assemblage of sections of surfaces of successive cones, tangents to or cutting a regular succession of parallels, and upon regularly changing central meridians, appeared to me the only one applicable to the coast of the United States. Its direction, nearly diagonal through meridian and parallel, would not admit any other mode founded upon a single meridian and parallel, without great deviations from the actual magnitudes and shape, which would have considerable disadvantage in use."[14]

In the polyconic projection, the central meridian appears as a straight line while all other meridians appear concave toward it. The parallels appear as arcs of nonconcentric circles of different radii, the centers of which lie on the central meridian (produced), the equator alone being represented by a straight line, and all parallels have their convexity turned toward it. Near the middle portion of a map, or for limited charts of large scale, the intersections of meridians and parallels do not differ much from a right angle but the departure from orthogonality increases markedly as extensive east-west areas are mapped. Developed arcs of the parallels appear in their true length according to the scale of the map, as also do the differences of latitude on the central meridian; hence, for equal differences of longitude the corresponding parts on any parallel are equal, whereas the meridional differences widen out as we recede from the central meridian. These characteristics of the polyconic projection are readily observable in a development of the sphere (*see* fig. 32).

The polyconic projection belongs neither to the conformal nor the equal-area class of projection (*see* 33), but may be considered as in a measure *only compromising* various conditions impossible to be represented on any one map or chart.[15] Its great popularity rests on its mechanical ease of construction

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[14] TRANSACTIONS OF THE AMERICAN PHILOSOPHICAL SOCIETY (NEW SERIES) 407–408 (1825). In a dissertation on the relative value of the polyconic projection with other projections, Charles Schott—a former chief of the Computing Division of the Survey—credits E. B. Hunt—an Assistant in the Survey—with having named this projection the "polyconic projection." Annual Report (1880), *supra* note 12, at 292.

[15] DEETZ AND ADAMS (1944), *op. cit. supra* note 5, at 62. It has been stated that the polyconic projection is a link between those projections which have some definite scientific value and those generally called conventional, but possessing properties of convenience and use. HINKS, MAP PROJECTIONS 52 (1921).

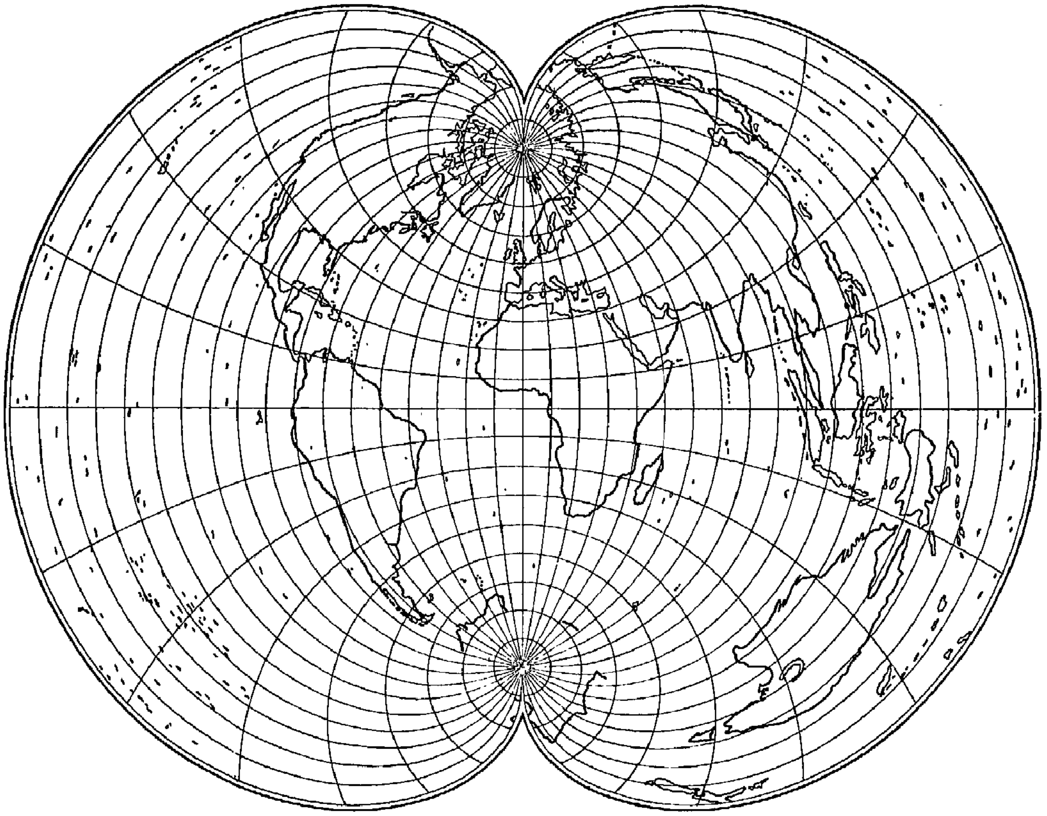

Figure 32.—Development of the world on the ordinary polyconic projection.

and on the fact that a general table can be computed for use in any part of the world.[16] In most other projections there are certain elements that have to be determined for the region to be mapped, and therefore a separate table must be

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[16] Projection tables for the construction of a polyconic projection appeared in the Annual Reports of the Coast Survey for 1853, 1856, 1859, and 1865, and were based on the Bessel spheroid of 1841 (*see* 211). The 1856 tables specially provided for projecting maps of large extent. These received some further extension in 1859. The 1865 tables provided for a special case. The first polyconic tables based on the Clarke spheroid of 1866 (*see* 211) were published as Appendix 6 to the Annual Report for 1884, and as a special publication (No. 5) in 1900. The latter has gone through several editions, the latest being the sixth (1946 reprint) and is titled Tables for a Polyconic Projection of Maps and Lengths of Terrestrial Arcs of Meridian and Parallels, Special Publication No. 5, U.S. Coast and Geodetic Survey (1946). This title for the publication first appeared in 1930. Prior thereto it was called simply "Tables for a Polyconic Projection of Maps." The reason for the change was that only the last two columns of each right-hand page are strictly applicable to the polyconic projection. These give the X and Y coordinates for plotting the intersection of parallels and meridians. The remaining columns of the tables are true lengths in meters of meridional arcs and arcs of the parallels as they appear on the spheroid, and are used in the construction of other projections. The tables are based on the legal value of the meter in the United States which is 39.3700 inches and corresponds to 1 meter=3.2808333 feet and 1 foot=0.3048006 meter. *Id.* at 3.

computed for each region under consideration; the universal table for the polyconic projection is, therefore, a great point in its favor.[17]

The projection described above is the one generally referred to in this country as the polyconic projection, but actually it is an exceedingly broad class and contains examples of many kinds of projections.[18] It has also been referred to as "ordinary," "simple," "Coast Survey," and "American" polyconic, the last designation having been given it by European writers. It is the projection that is used on all the present field surveys of the Bureau.[19]

This form of the polyconic projection is, then, characterized by curved meridians except the central one, nonconcentric arcs of circles for the parallels, and nonorthogonality of intersections.

344. RECTANGULAR POLYCONIC PROJECTION

As a matter of historical interest, mention should be made of the *rectangular* polyconic projection (*see* fig. 33), which is a modification of the ordinary polyconic (*see* text following note 18 *supra*). In this projection, the parallels are constructed as in the ordinary polyconic but they are not all divided truly. Only a selected parallel is truly divided and through the points of division the meridian lines are drawn so as to cut all the parallels at right angles—hence, the name rectangular polyconic.

The first reference to the use of this projection in Coast Survey work is found in the Annual Report of 1853, where a detailed description is given of two forms of the polyconic projection—the rectangular polyconic, and the equidistant polyconic (*see* 345). It is there stated: "By the aid of the subjoined tables a *rectangular polyconic* projection can at once be made for each locality or subdivision of the United States, or for the United States as a whole." It is then stated that beyond the limit of the equidistant polyconic (a square degree on a scale of 1:10,000) "the *rectangular-polyconic* method should be employed, at least in all Coast Survey projections." Finally, the report states: "The polyconic method of projection has been developed in the Coast Survey office, and the

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[17] This advantage is shared by the Mercator projection (*see* 613 note 11) for which a general table has also been computed and can be at once applied to the construction of a sailing chart for any part of the world.

[18] ADAMS, GENERAL THEORY OF POLYCONIC PROJECTION 13, SPECIAL PUBLICATION NO. 57, U.S. COAST AND GEODETIC SURVEY (1934).

[19] Detailed instructions for the construction of small-scale and large-scale polyconic projections are given in DEETZ AND ADAMS (1944), *op. cit. supra* note 5, at 63–65, and in TABLES FOR A POLYCONIC PROJECTION OF MAPS (1946), *op. cit. supra* note 16, at 8–9.

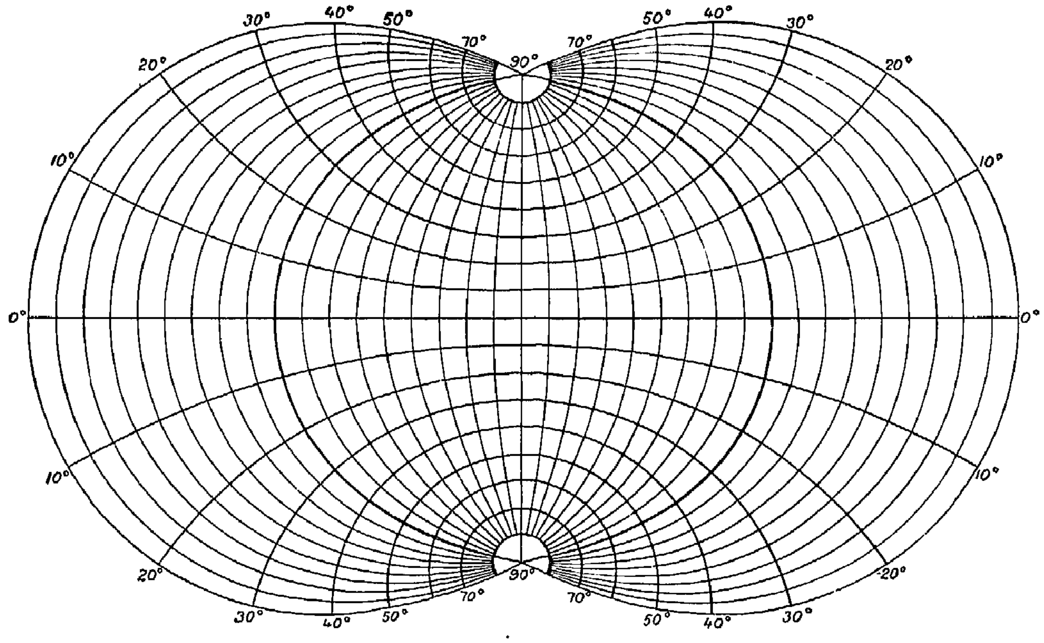

Figure 33.—Development of the sphere on the rectangular polyconic projection.

subjoined tables, prepared for facilitating its use, were there computed, and are now first published." [20]

From these extracts, there can be no doubt that at this period in the history of the Survey the rectangular polyconic projection was being used, and very likely for all work other than for small areas such as those covered by planetable and hydrographic sheets (*see* 345).[21]

345. Equidistant Polyconic Projection

The equidistant polyconic projection—so-called in the annual report of the Survey for 1853—appears to have been used for the field sheets of the Coast

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[20] Annual Report, U.S. Coast Survey (Appendix 39) 99, 100 (1853). The tables included in the report gave values for the construction of the projection by means of rectangular coordinates and were based on the constants of the Bessel spheroid of 1841 (*see* 211). For a mathematical development of this projection, *see* Adams (1934), *op. cit. supra* note 18, at 13–23. This projection has been used by the British War Office for the construction of maps. *Id.* at 18.

[21] For the area covered by the chart and map work of the Survey there is little difference between the ordinary polyconic and the rectangular polyconic. Thus, for a chart representing the whole of North America, the unaided eye can barely perceive any departure from orthogonality in the intersections of meridians and parallels, which is the chief distinction between the two projections. It is only in the development of the whole sphere or of a hemisphere that departures from right angles become apparent.

Survey and perhaps some charts during the early years,[22] and possibly as late as 1882.[23]

In this projection, a central meridian and a central parallel are constructed as in the ordinary and the rectangular polyconic. Additional parallels are then constructed (for the average sheet, the top and bottom parallels) from the tabular values, and the points of intersection of the different meridians with these parallels are then found and the meridians drawn. On the meridians thus determined, distances are laid off from the central parallel equal to the distances between corresponding parallels on the central meridian, and the other parallels are drawn (the tabular auxiliary parallels except the central one are then removed from the sheet). From this process of construction, a projection results in which equal meridian distances are intercepted everywhere between the same parallels—hence, the name equidistant. This should be regarded as a convenient graphic approximation, admissible within certain limits, rather than as a distinct projection, although it is capable of being extended to the largest areas with results quite peculiar to itself. If extended to include the entire earth, with the equator as the central parallel, all the parallels would become concave toward this line, for the distance between parallels measured along the curved meridians being constructed equal to that along the central straight meridian, it necessarily follows that the parallels must converge in receding from the central meridian. This is exactly the reverse situation that exists in the ordinary and the rectangular polyconic projections (*see* figs. 32 and 33).[24]

346. IDENTIFICATION OF PROJECTIONS ON EARLY SURVEYS

Because of the limited area covered by the field surveys of the Bureau, the type of projection used is not readily identifiable. Thus, on the average topographic and hydrographic surveys, at scales 1:10,000 or 1:20,000, the curvature of the meridians never becomes sensible and the parallels only rarely so.

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[22] This is evident from the statement in the 1853 report that "The method of projection in common use in the Coast Survey office for small areas, such as those of plane-table and hydrographic sheets, may be called the *equidistant polyconic.*" Annual Report (1853), *supra* note 20, at 100. *See also* text at note 20 *supra.*

[23] CRAIG, A TREATISE ON PROJECTIONS 208, TREASURY DOCUMENT NO. 61, U.S. COAST AND GEODETIC SURVEY (1882).

[24] Annual Report (1853), *supra* note 20, at 100. To have called this a *polyconic* projection was particularly faulty, since it does not possess the basic requirements of such a projection, namely, parallels of latitude formed by a system of nonconcentric arcs of circles whose centers lie on the central meridian (*see* 343). The only similarity to a polyconic projection is the fact that the auxiliary parallels used to develop the meridians are conic developments. The later works on projections do not include the equidistant polyconic in the classification of projections nor in the polyconic category. *See* ADAMS (1934), *op. cit. supra* note 18, and DEETZ AND ADAMS (1944), *op. cit. supra* note 5.

And even where a difference between two types of projections is perceptible, the difference may be obscured by unequal distortion of the paper, and identification by measurements becomes impossible.

While there may be some doubt as to the identity of the projection used prior to 1853, when the first polyconic tables were published (*see* note 16 *supra*), as a practical matter there is no difference whether the survey was projected on one of the forms of the polyconic, heretofore discussed, or on the Bonne projection (*see* 342). The important thing to remember is that because of the limited extent of these surveys, whatever the projection used, the shapes, areas, distances, and azimuths are preserved as they are on the surface of the earth. The plotting of control points, the determination of geographic positions of plotted points, or corrections to the projection lines may be accomplished by the use of the meridional distances and arcs of the parallel given in Special Publication No. 5 (*see* note 16 *supra*), the scale being applicable throughout the projection.

It is also important to bear in mind, when comparing surveys, maps, or charts on different projections, that while the type of projection used determines the geometrical characteristics of the map or chart, the geographic positions of points (their latitudes and longitudes) are unaffected, if the maps or charts are on the same datum (*see* 22). In other words, while the scale on one map may vary because of the projection used, features may be compared with other maps on a different projection provided comparisons are made by latitudes and longitudes and provided that the datums are the same, because a point on the surface of the earth can have one and only one latitude and longitude.

35. REASONS FOR MULTIPLE PROJECTION LINES

The reasons for multiple projection lines on the early surveys can usually be traced to one or more of the following:

1. A change in the spheroid of reference for the computations.
2. A change in longitude values.
3. A change in the horizontal datum for the triangulation.

These changes resulted in a change in the geographic values of control points. To take them into account, the survey could of course be replotted on a new projection using the new values of the control points. Expediency, however, has dictated against so time-consuming a procedure. Instead, the practice was adopted of applying a differential displacement to the projection lines to take into account the new geographic values of the control points. This gives acceptable accuracies (*see* 38).

While all of these changes may have contributed toward changing the geographic values of control points in an area, the projection lines on a survey sheet were not always corrected immediately upon a change. Corrections were generally applied when it became necessary to use the survey for charting or for other purposes, so that one actual change might be the result of more than one cause. For example, one correction might reflect a change in the spheroid and a change in the horizontal datum.[25] This frequently makes it difficult to segregate and allocate the various corrections applied.[26]

351. A CHANGE IN THE SPHEROID OF REFERENCE

The use of a new spheroid meant new factors for the computations, which in turn meant new values for the geographic positions of the control points. Therefore, on the early surveys, where the original triangulation positions were based either on the spheroid prior to the Bessel or on the Bessel spheroid of 1841 (*see* 211), it was necessary to apply a correction to the projection lines to bring the work to the new spheroid.

When the triangulation consisted of small detached portions in various parts of the country (*see* 221), it made relatively little difference what spheroid was used (provided of course it was not extremely erroneous) and the change from one spheroid to another produced little effect on the geographic values of the triangulation points. As the work expanded and greater distances became involved, the corrections became more significant. These corrections are not constant, except for small local areas, but are dependent upon the distance of the points from the origin of the triangulation.[27]

3511. *Magnitude of Corrections*

Some idea of the magnitude of the corrections necessary for a change from the Clarke spheroid of 1866 to the Bessel spheroid of 1841 for an area comparable

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[25] This might also apply to the computations. The recomputation of points in an arc of triangulation may not have been made immediately upon the introduction of telegraphic longitude, but may have been made after the triangulation was adjusted or after a new spheroid was adopted, in which case one correction would include changes due to both causes.

[26] The independent datum used for the early triangulation in the New England area later became the basis for the United States Standard Datum (*see* 223). The change to the new datum in this area was therefore merely a change from the Bessel to the Clarke spheroid and the corrections to the triangulation would indicate the magnitude of the change due to this cause (*see* 3511).

[27] This problem has been studied in the Coast Survey and the results are given in LAMBERT, EFFECT OF VARIATIONS OF ASSUMED FIGURE OF THE EARTH ON THE MAPPING OF A LARGE AREA 4–15, SPECIAL PUBLICATION NO. 100, U.S. COAST AND GEODETIC SURVEY (1924).

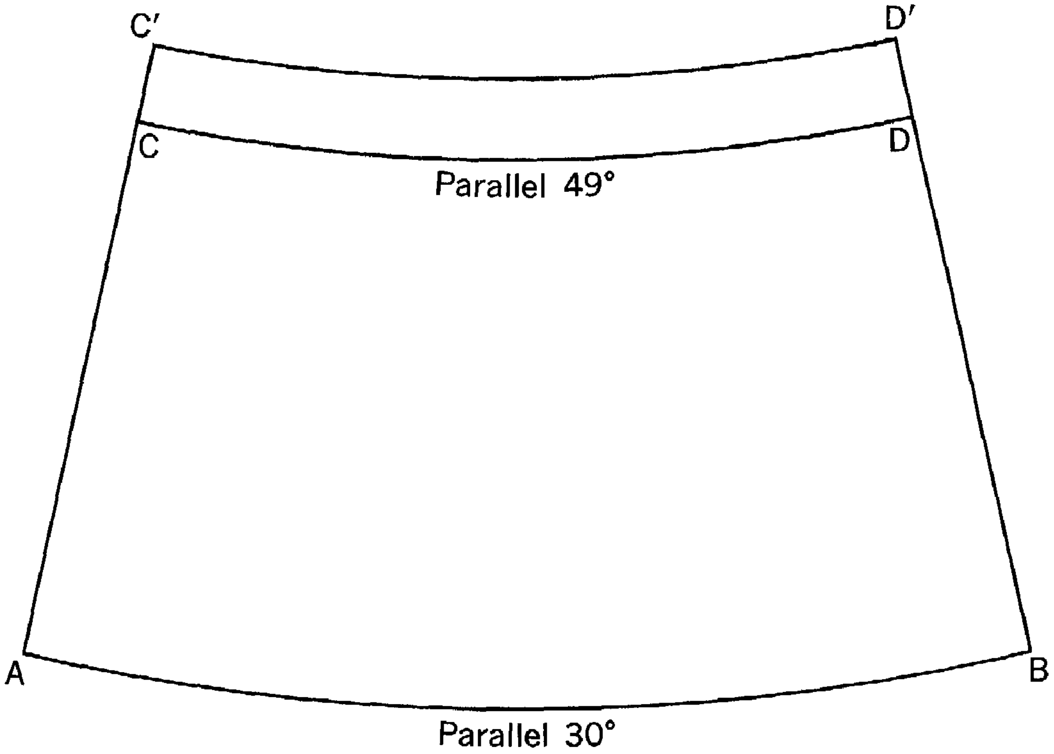

FIGURE 34.—Effect of change of spheroid on the triangulation of a country.

to the United States can be had from the following example, which is paraphrased from LAMBERT (1924), *op. cit. supra* note 27, at 10–11:

> Assume two points, *A* and *B* (*see* fig. 34), in latitude 30°, and 40° apart in longitude. For the Clarke spheroid, the distance *AB* measured along the 30th parallel is 3,859,529 meters, and the distance *CD* along the 49th parallel is 2,926,965 meters. The length of the arc of the meridian *AC* or *BD* between the parallels of 30° and 49° is 2,109,475 meters.
>
> If we inquire merely what the distances are between the same meridians and parallels on the Bessel spheroid, which has a semiminor axis 505 meters less and a semimajor axis 809 meters less than the Clarke spheroid (*see* 211), we obtain the following: The arc of the meridian between 30° and 49° is 2,109,286 meters, or 189 meters less than on the Clarke spheroid. The arc of 40° on the 30th parallel is 3,858,994 meters, or 535 meters shorter than on the Clarke spheroid, and the arc of 40° on the 49th parallel is 2,926,515 meters, or 450 meters shorter than the corresponding arc on the Clarke spheroid.
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> A length of 31 meters along the meridian represents 1″ of latitude while 27 meters and 20 meters represent 1″ of longitude on the 30th and 49th parallels, respectively. Therefore the differences in distances found correspond to 6″.1 in latitude and 20″ and 22″ in longitude.
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> If now we suppose the distance $AB = 3{,}859{,}529$ meters to have been obtained from the Clarke spheroid and then laid off on the 30th parallel of the Bessel spheroid, it would correspond on the latter to a difference of longitude of not exactly 40° but of 40°00′20″.0.

Similarly, the meridian distances $AC=BD=2{,}109{,}475$ meters, when laid off from latitude 30° northward, puts the end points C' and D' not in latitude 49° even, but in latitude 49°00′06″.1. (The distances CC' and DD' are greatly exaggerated in the figure.) Along this parallel, the distance $C'D'$ is 2,926,822 meters, or 143 meters less than the distance between corresponding points on the Clarke spheroid.[28]

352. A CHANGE IN LONGITUDE VALUES

One of the primary causes of corrections to the projection lines on early surveys was a change in the longitude values of the triangulation stations.

The determination of longitude (the difference between the local time at a given place and the corresponding Greenwich time) was one of the important problems that the Coast Survey had to deal with in the early years of its history. The problem engaged the attention of the most able and distinguished astronomers, mathematicians, and other scientists of the time, some of whom were associated with the Survey.[29]

3521. *Methods of Longitude Determination*

Various methods of longitude determination have been used by the Coast Survey at different periods in its history. Prior to the advent of the radio method, the principal methods employed were the lunar, the chronometric, and the telegraphic.[30] Both the lunar and the chronometric methods were subject to considerable error as evidenced by later telegraphic determinations.

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[28] For additional interpretations of the problem posed, *see id.* at 11–13.

[29] In different periods of history, various meridians have been used as the initial or zero of longitudes from which other longitudes were reckoned. Hipparchus, the earliest astronomer to determine longitude by astronomical observations, used the meridian of Rhodes. Eventually, each country came to have its own initial meridian, usually the meridian of its capital. In the United States, the center of the dome of the old Naval Observatory at Washington, D.C., was the initial meridian—sometimes called the American meridian. Many of the early state surveys are referred to this meridian. Congress, by the Act of Sept. 28, 1850 (9 Stat. 515), ordered "that hereafter the meridian of the observatory at Washington shall be adopted and used as the American meridian for all astronomical purposes, and that the meridian of Greenwich shall be adopted for all nautical purposes." This act was repealed Aug. 22, 1912 (37 Stat. 342). In 1884, a conference of nations, called at the instance of the President of the United States, recommended the adoption of the meridian of the Royal Observatory, Greenwich, England, as the common initial or zero of longitude for all nations. BEALL, ASTRONOMIC DETERMINATIONS BY U.S. COAST AND GEODETIC SURVEY AND OTHER ORGANIZATIONS 3, SPECIAL PUBLICATION NO. 110, U.S. COAST AND GEODETIC SURVEY (1925). In the Coast Survey, longitudes were always reckoned from Greenwich, although on some of the early charts of New York Harbor (1845) longitudes based on the City Hall of New York were also shown, and on the early Boston Harbor chart (1856) the charted longitudes are reckoned from the Boston State House, while the positions of lighthouses and light vessels are based on Greenwich.

[30] Solar eclipses seem also to have been used in the early days. In one of Hassler's early reports, the following statement appears: "A special station was made in 1838 upon Weazel mountain near Paterson, N.J., for latitudes and azimuths; and a solar eclipse occurring just at that time, gave occasion for an observation of longitude at the same place. Every solar eclipse that has occurred during the time the work has as yet lasted, has been observed at some one of the survey stations." Annual Report, U.S. Coast Survey 5 (1841) (H. Doc. 28, 27th Cong., 2d sess.).

A. LUNAR METHOD

One of the first methods used by the Coast Survey for the determination of longitude was the lunar method. The observer at a station of which the longitude was required observed the position of the moon and noted the *local* time of observation. From the Ephemeris, he could find at what instant of Greenwich time the moon was actually in the position in which he observed it. The difference between this time and the local time of observation was his longitude reckoned from Greenwich. The moon's position could be fixed by measuring the angular distance between the moon and the sun or one of the four larger planets, or between the moon and one of the brighter stars, or by occultation of a star, or a group of stars such as the Pleiades, or by moon culminations. In each case, the Greenwich time at which the moon occupied the position in which it was observed was obtained either from the Ephemeris, from observations at Greenwich at about the time in question, or from similar observations at some station of known longitude.[31]

The lunar method required extreme accuracy of observation because the longitude error was many times the observational error. Also, due to the nearness of the moon to the earth, long, complex, and difficult computations were required for reducing the observations to the center of the earth.

B. CHRONOMETRIC METHOD

Another method used in the early work was by transporting chronometers between stations whose difference in longitude was to be ascertained. This was known as the chronometric method. The difference in time was obtained by comparing the local chronometer times at the two stations by means of other chronometers transported between the stations, the rates of all chronometers having previously been determined. Since it was necessary to travel back and forth between the stations, the cost of such a longitude determination increased with increased cost of travel and its accuracy decreased as the time required to make a round trip increased. This caused the chronometric method to give way to lunar methods in certain situations.[32] The method was also used in later years at many stations not reached by the telegraph.

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[31] BOWIE, DETERMINATION OF TIME, LONGITUDE, LATITUDE, AND AZIMUTH 78, SPECIAL PUBLICATION No. 14, U.S. COAST AND GEODETIC SURVEY (1917), and BEALL (1925), *op. cit. supra* note 29, at 5.

[32] The points at which the boundary between the United States and Canada along the 141st meridian crosses the Yukon and Porcupine Rivers were originally determined by lunar methods. (The final demarcation of the boundary was determined telegraphically.) BOWIE (1917), *op. cit. supra* note 31, at 79.

The chronometric method required precision in the chronometer and the accurate determination of its rate. An error of 1 second in the chronometer time meant an error of 15″ in longitude, which for the average latitude of the United States (40°) was equivalent to an error of 356 meters.

C. TELEGRAPHIC METHOD

When, in May 1844, Morse flashed his first telegraphic messages over the wires between Washington and Baltimore, with a transmission time of his signals so short as to be barely perceptible, a new era in longitude determination was foreshadowed.[33] Two years after this epoch-making event, on October 10, 1846, the telegraphic method was put into successful operation by the Coast Survey and time signals were exchanged between the United States Naval Observatory (old site) at Washington, D.C., and the Central High School at Philadelphia, Pa.

In the telegraphic method, the error of the local chronometer was determined at each of the two stations by the ordinary time observations, and the two chronometer times were then compared by telegraphic signals sent between the two stations.

During the long interval since its introduction and until the adoption of the radio method in 1922, the telegraphic method was gradually brought to a high state of perfection and represented the most accurate known method of determining differences of longitude.[34]

3522. *Magnitude of Corrections*

When the methods prior to the telegraphic were used for longitude determinations, there was considerable variation in the results obtained. For example, with the same method, the longitude determinations at the principal observatories, such as Cambridge, Mass., where many chronometers were used and numerous observations extended over a long period, were of a much higher order of accuracy than the determinations at some of the triangulation stations where conditions did not permit the same degree of precision. Also, the

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[33] Annual Report, U.S. Coast and Geodetic Survey 202 (1897).

[34] In the Annual Report for 1897, it is stated: "It will be seen from these results that we are justified in assigning to any of our American longitude results a probable error of about . . . ±0″.78, which, in latitude 39°, represents a linear extension of but 18.8 metres, or 61.7 feet nearly." Annual Report, U.S. Coast and Geodetic Survey 256 (1897).

instruments used in the early work had not reached the stage of refinement they later did, and this introduced an additional error in the early determinations.

Some idea of the magnitude of the corrections due to changes in longitude values can be had from an examination of one of the early topographic surveys covering the western portion of Long Island Sound, N.Y. (Register No. T-14 (1837)). The survey was based on the Hassler triangulation of 1834. The original projection on this survey shows the longitude of station *Rikers Island* to be 1,273 meters westward of meridian 73°55′.[35] The first correction to the longitude value was that derived from the connection by telegraph of this area with Harvard College Observatory at Cambridge, Mass. This amounted to over 3′ of arc and brought the longitude of station *Rikers Island* to 1,224 meters west of meridian 73°52′.[36]

Until the year 1866, the longitude of Cambridge Observatory from Greenwich, England, depended upon moon culminations, eclipses, transits, occultations, and chronometer transportation. The adopted value was 71°07′22″.50, and this served as the standard for all the work connected with that observatory. With the completion of the electric cable across the Atlantic in 1866, the longi-

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[35] According to an unpublished 1912 report by the Geodesy Division of the Bureau on "Old longitudes in the vicinity of New York City," these early longitude values (the Hassler values) were computed from independent astronomic data. The report centers around the various longitude values of the New York City Hall, beginning with the value of 74°03′05″.2 in the Hassler computations. The earliest published position is given as 74°00′56″.7 (from a circular to the Collector of Customs, dated May 27, 1843, a copy of which is filed in Vol. 1 of "Miscellaneous Scientific and Business Papers" (Acc. No. 5), at page 63), or a difference of 2′08″.5 from the Hassler value. The report states that on the 1844 and 1845 charts of New York and vicinity (the first charts published of the area), the scaled position of the City Hall is 74°00′41″, or a difference of approximately 2′24″ from the Hassler value, the difference being "due to the fact that the Hassler positions are based on independent astronomic data, the best that could be obtained in 1837." The report states further that the first positions published in the Bessel registers (*see* note 39 *infra*) gave the longitude of the City Hall as 74°00′44″.587, or a change of approximately 3″.6 in longitude from the position on the first published charts, and a difference of approximately 2′21″ from the Hassler value. The second positions appearing in the Bessel register give the longitude of the City Hall as 74°00′03″.09, a change of approximately 41″.5 from the first Bessel longitude, and a difference of 3′02″ from the Hassler value. The positions in red in the Bessel register made in 1869 show a change of +20″.6 in longitude from the second Bessel value, due to the telegraphic determination of longitude (*see* text at note 37 *infra*). The longitude of the City Hall is given as 74°00′23″.71, which differs from the Hassler value by 2′41″. The positions based on the telegraphic determination of longitude were first published in the Annual Report of the Survey for 1851. The final positions on the United States Standard Datum give the longitude of the City Hall as 74°00′23″.017, a difference of approximately 2′42″ from the Hassler value. The report concludes with the statement that after a consideration of all the facts gathered "we are led to believe that the longitudes of the Hassler computations were based on independent astronomic data . . . and that they were never intended to be used in making the final published charts. They were simply used for making the field sheets of a particular detached locality, and that they were correct as far as relative positions were concerned . . . . The various changes in longitude that were made between 1845 and the time of the U.S. Standard Datum were due to various causes such as moon culminations, parallax, chronometers, etc., and to the telegraphic determination of longitude."

[36] A small part of this correction may have been due to the adoption around this time of the Bessel spheroid of 1841 for the computations, but the major correction was due to the introduction of telegraphic longitudes. Annual Report, U.S. Coast Survey 163, 164, 278 (1851).

tude of Cambridge Observatory was found to be 71°07′42.″75. In consequence, in April 1869, the longitudes of the surveys on the Atlantic coast were increased by 20.″25 of arc, which in the latitude of station *Rikers Island* amounted to 475 meters.[37]

A similar increase in longitudes, or pushing westward of the coastline, was found necessary on the Pacific coast, only the amount of change was even greater. In the spring of 1869, San Francisco and Cambridge Observatory were directly connected by wire, and the longitude of station *Telegraph Hill,* which had previously been determined from 206 moon culminations observed at 7 different places and reduced to the station by means of chronometer transportations, was found to be greater by 46.″5 of arc, which in the latitude of San Francisco amounted to 1,138 meters.

353. A CHANGE IN THE HORIZONTAL DATUM

Under this category are included only those changes that result from the adoption of a standard horizontal or geographic datum. The subject of datums in the United States and Alaska and the development of the final North American 1927 Datum have been discussed in Chapter 2. It was there pointed out that a number of independent datums existed in the United States prior to the adoption of the United States Standard Datum (*see* 22). This brought about the major changes in the latitudes and longitudes of triangulation stations, resulting in a shift in the projection lines. The change to the North American 1927 Datum was of considerably less magnitude (*see* 37).

36. SUMMARY OF CORRECTIONS TO PROJECTION LINES

As a result of the three items considered in 351, 352, and 353, it is possible to encounter on the early surveys corrections to the projection lines corresponding to the following situations:[38]

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[37] Annual Report (1897), *supra* note 33, at 201. The value of 71°07′42.″75 was later adjusted to 71°07′44.″85 from additional determinations. *Ibid.* For a historical account of the trans-Atlantic longitude program of the Coast Survey beginning with the initial project in 1843, *see* Knox, *Precise Determination of Longitude in the United States,* 47 THE GEOGRAPHICAL REVIEW 555 (1957).

[38] There is an additional projection-line correction sometimes encountered on survey sheets—the correction due to office adjustment of the triangulation. If the survey is based on field computations of the control points, their geographic positions will not correspond to the adjusted values. The correction due to this cause, however, is usually of the order of 5 to 10 feet, which would be of little significance cartographically. On many of the early surveys this correction was probably absorbed in other changes.

(*a*) *Atlantic and Gulf Coasts*

(1) Unknown spheroid and chronometric and lunar longitudes—independent datum.

(2) Bessel spheroid of 1841 with old data (chronometric and lunar longitudes from Greenwich and telegraphic longitude within network)—independent datum.

(3) Bessel spheroid of 1841 with new data (telegraphic longitude from Greenwich)—independent datum.

(4) Clarke spheroid of 1866—independent datum.

(5) United States Standard Datum (same as North American Datum).

(6) North American 1927 Datum.

(*b*) *Pacific Coast*

The work on the Pacific coast was started in 1850, several years after the Bessel spheroid of 1841 was adopted by the Bureau (*see* 211); hence, item (1), above, never entered into the work on that coast. Items (2) to (6), however, are equally applicable.

37. EXAMPLE OF CORRECTIONS TO A SURVEY SHEET

Figure 35 is a section of a topographic survey of the Mississippi River area made in 1867 (Register No. T–1037) and shows the number and magnitudes of the several corrections applied to the original projection. Since the survey was made after the Bessel spheroid of 1841 was adopted (*see* 211), only the last five of the corrections listed under 36(*a*) had to be applied at different times, as the survey was needed for charting or for other use.

The successive values for station *South Base* are as follows:

	Latitude	*Longitude*
(1)	29°09′25″.82 (794.8 m.)	89°13′19″.81 (535.3 m.)
(2)	29°09′25″.82 (794.8 m.)	89°14′18″.12 (489.6 m.)
(3)	29°09′24″.18 (744.6 m.)	89°14′17″.10 (462.2 m.)
(4)	29°09′19″.65 (605.3 m.)	89°14′17″.29 (467.6 m.)
(5)	29°09′19″.546 (601.8 m.)	89°14′18″.121 (489.7 m.)

The first value is based on the Bessel spheroid of 1841 with astronomic data. It corresponds to the original projection shown on the survey sheet with black solid lines.

The second value is based on the same spheroid but with the introduction of telegraphic longitude. This made no change in the latitude value, but increased the longitude by 58″.31, or 1,576.0 meters. This change includes the 20″.25 due to cable connection between Cambridge Observatory and Greenwich, England (*see* text at note 37 *supra*).

The third value is due to a change from the Bessel to the Clarke spheroid. The triangulation was still on an independent datum. This effected a reduction in latitude of 1″.64 or 50.2 meters, and a reduction in the longitude value of 1″.02 or 27.4 meters.

The fourth value resulted from the adoption of the United States Standard Datum (N.A. Datum) and reduced the latitude value by 4″.53 or 139.3 meters. The longitude value, however, was increased by 0″.19 or 5.4 meters.

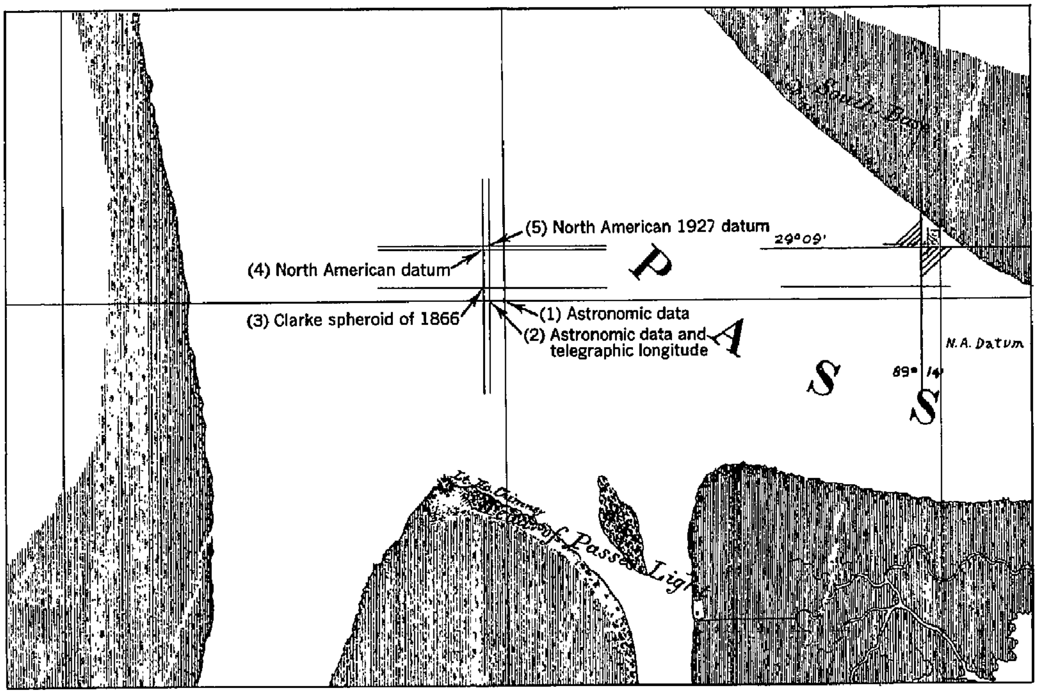

Figure 35.—Portion of early topographic survey (on reduced scale) showing corrections to the projection lines as a result of changes in the geodetic data.

The fifth and final value brought the work to the North American 1927 Datum. The latitude was reduced by 0″.10 or 3.5 meters and the longitude was further increased by 0″.83 or 22.1 meters.[39]

From the example given, it should be obvious that before features on successive surveys are studied and conclusions drawn, it is of the utmost importance to first bring the surveys to a common datum, if comparisons are to be made by superposing the projection lines. This does not necessarily mean the latest datum unless the comparison is to be made with a survey that is based on the latest datum. When requests are made for copies of early surveys, it is customary to place them all on a common datum—generally the latest—before they are sent from the Washington Office. The user should make certain this is the case before using them.

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39. The early values of the triangulation stations were recorded in a series of volumes or registers and were maintained in the Chart Division of the Bureau. These registers are still referred to when it becomes necessary to identify a station on a survey sheet that has not been brought to the latest datum (*see* 382). Triangulation stations that have been reduced to the North American 1927 Datum are maintained in a card file in the Geodesy Division of the Bureau and copies may be obtained on request.

38. CHANGING THE DATUM OF A SURVEY SHEET

Practically all of the recent surveys of the Bureau in continental United States are based on the North American 1927 Datum, but many of the early surveys are still on the North American Datum or some independent datum (*see* 224 and 23). To change the datum of a survey sheet, the usual practice is to apply corrections to the projection lines on the sheet.

There are two methods of applying a datum correction—a numerical one and a graphic one. The method selected is usually dictated by the presence of certain conditions which will be described below.

381. DISTORTION FACTOR

Before datum corrections can be applied, the distortion factor of the survey sheet must be determined in a north-south and an east-west direction. This is done by comparing the scaled distances between projection lines on the sheet with the corresponding values given in Special Publication No. 5 (*see* note 16 *supra*). The distortion factor is determined from the relationship

$$\frac{\text{Tabular value} - \text{Survey value}}{\text{Tabular value}} = \pm \text{ Distortion factor.}$$

Several distances in each direction should be measured in order to obtain a mean factor and to ensure against errors in the original projection. The distortion factor should be applied to every distance that is to be plotted on the sheet.[40]

382. NUMERICAL METHOD

In the numerical method, three widely distributed triangulation stations on the sheet are selected whose geographic positions on the North American 1927 Datum are available. Identify these stations on the datum of the projection in the old registers (*see* note 39 *supra*), or in any of the publications of the Bureau, and check their geographic positions with the positions plotted on the sheet. The mean of the differences between the values on the two datums is the correction to be applied to the projection on the sheet. The differences for the three stations should nearly equal each other. If a wide variance is found, an investigation should be made for possible errors in the computations or for failure to identify common stations on the two datums.

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[40] To reduce measurements on a distorted sheet to true values the formula for "correction factor" should be used (*see* 1321).

(*a*) *Applying the Correction.*—Great care must be taken to see that the correction is applied in the proper direction. The following rule will be found helpful in determining the direction:

> If the latitude (N.) and longitude (W.) on the old datum are greater than the corresponding values on the new datum, then the new projection will be to the north and west, respectively, of the old projection; if the old values are smaller than the new ones, it will be to the south and east.

From this, the direction of the correction for other relationships can be readily determined. To avoid errors of application in plotting, a sketch should be made showing the relationship of the two datums to one of the selected triangulation stations, with the corresponding latitudes and longitudes indicated.

The new datum is usually marked in colored ink by short intersections at two or more projection intersections. At one of the intersections the name of the datum, the latitude and longitude, the initials of the cartographers who made the correction and the verification, and the date the correction was made, are all shown in colored ink. (*See* fig. 36.)

Example: In figure 36, the latitudes and longitudes of triangulation station *Front* (1918) on the old and new datums are as follows:

North Latitude	*West Longitude*
47°25′ 450 meters (old datum)	124°10′ 900 meters (old datum)
47°25′ 300 meters (new datum)	124°10′ 750 meters (new datum)
150 meters difference	150 meters difference

Applying the rule given above, the new projection is found to be to the north and west of the old projection. Therefore, the intersection of latitude 47°25′ N. and longitude 124°10′ W. on the new datum is obtained by laying off the distance 150 meters (corrected for distortion) in a north and in a west direction from the old datum.

(*b*) *Reference Station on the Sheet.*—If the value of the latitude and longitude of one of the plotted stations is recorded on the sheet, the correction may be obtained by subtracting from it the corresponding value on the new datum.[41] Where photographic copies of old surveys are used, the single correction thus obtained is sufficiently accurate for use on the entire sheet. In the Washington Office, where the original survey sheet is available, additional stations are used and a mean correction obtained in accordance with the procedure outlined above.

The numerical value shown on the sheet should always be checked against the plotted position of the station before using it for determining a datum correction.

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[41] As a rule the early surveys did not include such a reference station. Since 1942, this (together with the geographic datum) has been a specific requirement. ADAMS, HYDROGRAPHIC MANUAL 691, SPECIAL PUBLICATION NO. 143, U.S. COAST AND GEODETIC SURVEY (1942).

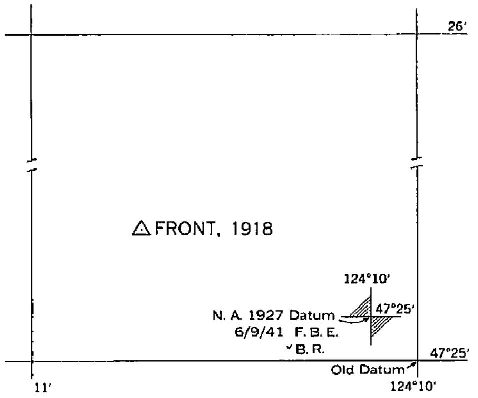

FIGURE 36.—Change of datum of survey sheet—by numerical method.

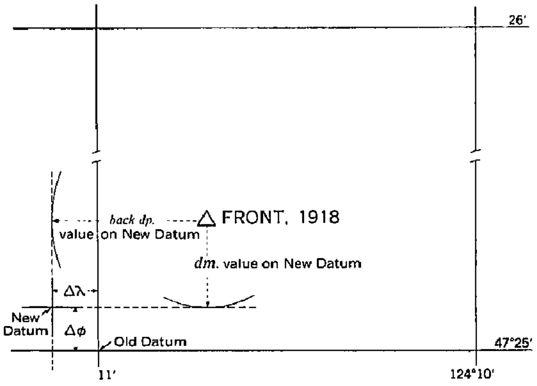

FIGURE 37.—Change of datum of survey sheet—by graphic method.

383. GRAPHIC METHOD

This method of correcting the datum of a survey sheet is applicable under three conditions: (1) as an alternative to the numerical method, (2) where geographic positions of the stations on the datum of the sheet can be found neither in the old registers (*see* note 39 *supra*) nor in any of the publications of the Bureau, and (3) where the old registers are not available.

Under these conditions, three well-distributed stations on the sheet are selected the geographic positions of which are available on the new datum. Determine the *dm.* and *dp.*[42] of each station from the projection line nearest to it (this will mean using the back *dms.* and *dps.* in some cases), and with these values as radii, short pencil arcs are swung in the proper directions from the plotted position of the station as a center. Pencil lines are then drawn tangent to these arcs and parallel to the latitude and longitude lines on the sheet. The intersection of these lines will give the position of the new datum. The offsets (Δφ) and (Δλ) are then carefully scaled from the old datum on the sheet. Figure 37 illustrates the application of this method to the example given in 382.

The same procedure is followed with the other selected stations. The offset values obtained in each case are then compared. If they do not differ by a

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[42] The equivalents in meters of the seconds of latitude and longitude of triangulation stations are known as the *dms.* (meridional differences) and *dps.* (parallel differences), respectively, of the stations. Thus, if the position of a station is given as latitude 54°44′34″.189 N. (1,057.2 meters), longitude 130°56′42″.362 W. (756.5 meters), its *dm.* is 1,057.2 meters north of the 44′ parallel, and its *dp.* is 756.5 meters west of the 56′ meridian. The back *dm.* is the distance in meters from the next minute of latitude (45′ in the case cited), and the black *dp.* is the distance in meters from the next minute of longitude (57′ in the case cited).

plottable amount, the intersections as determined are accepted; otherwise a mean value is used. Any appreciable differences are investigated. The method of designating the new datum on the sheet is the same as for the numerical method.

384. UNRECOVERABLE STATIONS

Stations sometimes become unrecoverable and cannot be included in the triangulation to connect them with the new datum. Where a survey sheet contains such stations only, then the relationship between the old and new datums must be obtained from stations on adjoining sheets for which the new datum values are available. A mean of the corrections derived from the adjoining sheets should be used.

In some of the published volumes of triangulation data for the coastal states a list of "lost" (unrecoverable) stations, with their geographic positions (unadjusted) on the North American Datum, was also included.[43] While such values would not be used for extending triangulation, they are adequate for coordinating old and new surveys.

39. PROJECTION CONSTRUCTED AFTER SURVEY

Some of the early surveys (hydrographic and topographic) were executed prior to the determination of the geographic positions of the control points, as where the local triangulation had not been connected to the main net of triangulation or where astronomic observations had not been made. In such cases, the only elements given were the distances from the points to the two axes of a system of rectangular coordinates (which is assumed) and the distances between the points. The projection for plotting these consisted simply of axes of ordinates and abscissas so laid on the sheet as to embrace all the points that were required by the surveyor in his work. The points were plotted from these by the intersection of two arcs with the distances of the points from the axes as radii, either north or south and east or west of the axes, as the plus or minus sign may indicate. In some cases, where it became necessary to undertake the topography when neither the data for constructing a projection nor for plotting the coordinates were available, the points were plotted by distances. Where a survey sheet had no projection, it was customary to draw squares of

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[43] *See,* for example, MITCHELL, TRIANGULATION IN MARYLAND 234–255, SPECIAL PUBLICATION NO. 114, U.S. COAST AND GEODETIC SURVEY (1925).

1,000 meters, or some other specified number, on it. This facilitated the construction of a projection on the sheet at a later date.[44]

In constructing a projection on such a sheet, the distortion factor is one of the important elements that has to be considered. Upon the accuracy of its determination depended the accuracy of the projection. Paper does not always distort uniformly and the shrinkage or expansion should be determined in both a north-south and an east-west direction and a factor applied to all measurements to be laid down on the sheet (*see* 381). The smaller the distance to be plotted the less will be the error of distortion so that in laying down projection lines from plotted triangulation stations, corrections are avoided by selecting stations close to the lines to be constructed.

There are two methods of reconstructing a projection on a survey sheet—a rigid one applicable to small-scale surveys, and a graphic one applicable to large-scale surveys. In both methods the essential problem is the determination of the cardinal lines of the projection, namely, the central meridian and the central construction line.

391. ON SMALL-SCALE SURVEYS

For surveys on scales smaller than 1:10,000, the rigid method is used and comprises the following steps:

(1) Three triangulation stations, *A, B,* and *C,* are selected (*see* fig. 38), so situated with respect to the central meridian that the distance *de* will cover more than half the latitude extent of the survey sheet. Two of the stations should be selected, if possible, in about the same latitude (*see* (6) below). From the scaled lengths of *BA* and *CA* and the corresponding computed lengths (make inverse computations for these if they are not in the triangulation data), determine the distortion factor along each line (*see* 381).

(2) Select the central meridian (as near to the middle of the sheet as possible) and compute the distances *Bd* and *Ce* and the latitudes of the points *d* and *e*.[45]

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[44] WAINWRIGHT, PLANE TABLE MANUAL 28, SPECIAL PUBLICATION NO. 85, U.S. COAST AND GEODETIC SURVEY (1922). Sometimes it was necessary to make a topographic survey in advance of the triangulation work. In such case, a base line was measured and laid down on the sheet. The ends of the base were occupied with the planetable and additional points located by intersection and directions taken to all visible points. The survey then proceeded as usual (*see* 4111). Lengths of 1,000 meters were usually laid down on the sheet in several places for the purpose of determining the true scale at any time in the future (*see* 131). *Id.* at 64.

[45] This is accomplished on Form 27, Position Computation, Third-Order Triangulation. Since both the distance and latitude are involved in the position computation formulas, a trial-and-error method is used. A value is assumed for the required latitude (ϕ') and from the known longitudes (λ) and (λ') of station *C* and the central meridian, respectively, and the known azimuth (α) of the line *Ce* (obtained either directly from the triangulation data or from the inverse computation), the distance (s) is computed from the longitude formula. This distance is then used in the latitude formula and the latitude increment or decrement ($\Delta\phi$) is obtained. This computed $\Delta\phi$ may not agree with that derived from the assumed latitude (ϕ'), but by repeated assumptions a value for $\Delta\phi$ will be obtained that is in absolute agreement with the assumed latitude. (To obtain a close first approximation, a sketch should be drawn at about one-fifth the scale of the sheet, using a rectangular grid and using as units of measurement the values of a minute of latitude and longitude at the center of the sheet. From this sketch the latitude where a given line crosses the central meridian can be scaled and this value used as the first assumed value in the computation.)

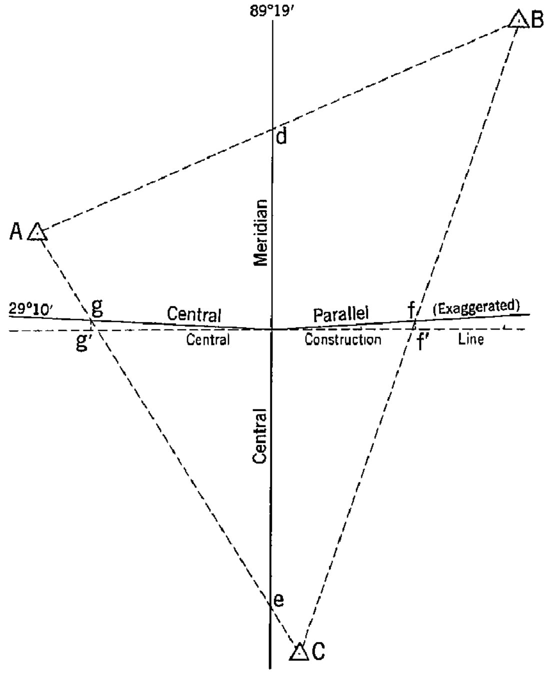

FIGURE 38.—Construction of a polyconic projection on a completed survey sheet—for small-scale surveys.

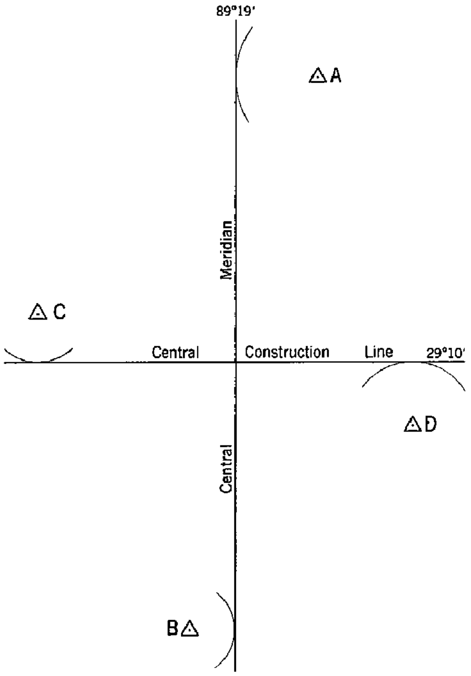

FIGURE 39.—Construction of a polyconic projection on a completed survey sheet—for large-scale surveys.

(3) Plot points *d* and *e* along lines *BA* and *CA* correcting each computed distance for the distortion determined along the respective lines. Through *d* and *e* draw a straight line for the full length of the sheet. This is the *central meridian* of the projection.

(4) Scale the distance *de* on the sheet and from the computed latitudes obtain from Special Publication No. 5 (*see* note 16 *supra*) the true distance on the earth. From these two values the distortion in a north-south direction is determined (*see* 381).

(5) On the central meridian lay off the true distance from point *d* or *e* to the central parallel and at this point construct a perpendicular to the central meridian for the full east-west length of the sheet. This is the *central construction line.*

(6) From here the problem is the ordinary one of constructing a polyconic projection (*see* note 19 *supra*). The distortion factor to be applied to east-west measurements may sometimes be obtained from the disposition of the selected triangulation stations (*see* (1) above), otherwise follow instruction below.

(7) The projection should be checked against every triangulation station on the sheet. Small differences in latitude or longitude may be due to unequal distortion of the sheet. In such case the projection should be made consistent with the triangulation even though this results in a slightly skewed projection.

Determining East-West Distortion.—If the east-west distortion of the sheet is not available from the disposition of the triangulation stations, it can be determined in the following manner:

(1) Compute the longitude crossings of the lines *CB* and *CA* (*see* fig. 38) on the central parallel or any other parallel that will give a distance long enough to determine a good distortion factor.[46]

(2) Plot the computed distances *Cf* and *Cg* (corrected for distortion) along lines *CB* and *CA*, and at points *f* and *g* draw short lines parallel to the central meridian. Lay off to the south (in north latitude) on these lines the *Y*-coordinates from Special Publication No. 5 (*see* note 16 *supra*) for the appropriate longitude distance and obtain points *f′* and *g′*. The scaled distance between these points compared with the tabular distance as determined from their *X*-coordinates will give the distortion factor in an east-west direction (*see* 381).

(3) Wherever possible, advantage should be taken of the location of some of the triangulation stations to reduce the amount of computation for determining this distortion. For example, in figure 38, the parallel through station *A* can be used instead of the parallel through *g*, thereby making it unnecessary to compute the longitude of *A*.

392. ON LARGE-SCALE SURVEYS

For small areas such as those covered by the large-scale topographic and hydrographic surveys of the Bureau, the polyconic projection is practically identical with the rectangular projection or a modification thereof (projection with converging meridians); therefore, in reconstructing a projection on surveys of scale 1:10,000 or larger, the following graphic method can be substituted for the more rigid method described above:

(1) After the distortion of the sheet has been determined from comparisons between scaled and computed distances, two triangulation stations *A* and *B* are selected near the north and south extremities of the sheet and as close to the center of the sheet as possible (*see* fig. 39). From the "Arcs of the parallel" in Special Publication No. 5 (*see* note 16 *supra*), the distances from each station to the central meridian are obtained and with these distances (corrected for distortion) as radii and the stations as centers, arcs are swung toward the center of the sheet. A line is then drawn tangent to these arcs for the entire length of the sheet. This line is the *central meridian* of the projection.

(2) Two other triangulation stations *C* and *D* are selected near the east and west extremities of the sheet and as close to the central parallel as possible. From the "Meridional arcs" in Special Publication No. 5 (*see* note 16 *supra*), the distance in meters is obtained from each station to the central parallel. To these distances are added or subtracted the *Y*-coordinates from the tables in Special Publication No. 5 corresponding to the difference in longitude between the central meridian and each station. (For north latitude, *add* if the station is above the central parallel and *subtract* if below. For south latitude the reverse is true.)

[46] This is accomplished on Form 27 in a manner similar to that described in (2) above for determining the latitude intersection. With the azimuth of the line *CB* as previously determined and the known latitude increment or decrement ($\Delta\phi$), a value for s is found by trial and error that will make the sum of the latitude terms in the computation formula equal to $\Delta\phi$. From this value of s the required longitude (λ') is computed. A close first approximation for the distance (s) can be obtained by making $\Delta\phi$ equal to the 1st term in the latitude formula (neglecting the 2d term) and with the s value thus found, the 2d term is computed. A new value for the 1st term is then found that will make the sum of the two terms equal to $\Delta\phi$. The resulting value of s is then used in the longitude formula to obtain λ'. Because of the distances usually involved, it will seldom be necessary to carry the computation beyond the 2d term in the latitude formula.

(3) With the distances thus obtained (corrected for distortion) as radii, arcs are swung from stations *C* and *D* in a direction toward the central parallel. A line is then drawn tangent to these arcs for the entire width of the sheet. This line will be perpendicular to the central meridian and will be the *central construction line* of the projection.

(4) The remainder of the projection is constructed according to the method given in Special Publications Nos. 5 and 68 for large-scale projections (*see* note 19 *supra*).

393. MODIFIED METHODS

It is sometimes necessary to modify the above methods because of special conditions encountered. The rigid method may not always be applicable in its entirety, as where all the triangulation stations are on one side of the central meridian. In such cases, a combination of the two methods is used.[47] Even in a small-scale survey, if the longitude extent is small enough, the curvature of the parallels may be neglected and the graphic method used.

There are also cases where the survey is a planetable traverse or a running ship survey in which the azimuths and distances at one end are comparatively accurate but which decrease progressively in accuracy toward the other end. Where such a survey is plotted without a projection and an attempt is later made to add a projection adjusted to selected stations subsequently located by triangulation, it will be found that a single harmonious projection cannot be constructed. If the work is not to be replotted, then the only way to bring the entire survey into harmony geographically is to place a discontinuous projection on it, each portion being based on the triangulation in its vicinity.

In adapting any of the methods to a particular situation, the two considerations to be kept in mind are the theory of the polyconic projection and the means available for determining the distortion of the sheet.

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47. The central meridian is determined graphically and the intersection of the central parallel with a line between two of the triangulation stations is computed. The Y-coordinate of the intersection is plotted and a perpendicular erected to the central meridian, the intersection of the two being the center of the projection.