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Optimal Weighting of Data to Detect Climatic Change: Application to the Carbon Dioxide Problem

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Increasing carbon dioxide concentration in the atmosphere is expected to lead to warming of the surface of the earth. Detection of the warming is difficult because it must be distinguished from natural variability of temperatures due to daily weather changes. It is shown that a weighting of surface temperature data using information about the expected level of warming in different seasons and geographical regions and statistical information about the amount of natural variability in surface temperature can improve the chances of early detection of the warming. Surface temperature data are conventionally averaged over the surface of the earth, weighted according to the geographical area represented by the data. A preliminary analysis of the optimal weighting method suggests that it is 25% more effective in revealing a surface warming than the conventional weighting method, in the sense that 25% more data analyzed in the conventional way are needed to have the same chance of detecting the climatic warming. The possibility of detecting the warming in data already available is examined. A rough calculation suggests that the warming ought to have already been detected if the only sources of significant variability in surface temperature had time scales less than 1 year.

1. Introduction

It is the unpredictability of atmospheric behavior that makes the discussion of climatic change both interesting and difficult. Although the laws governing the behavior of the atmosphere may by their very nature preclude detailed predictions of the state of the atmosphere more than a few weeks into the future, it may still be possible to say something useful about probabilities of various future states, that is, to predict climatic evolution. Predictions of climatic change are usually for the evolution of climatic means. The warming trend associated with increasing atmospheric CO_2 is probably the best known example of such predictions. Verification of predictions of climatic change is, however, a more subtle problem than verifying weather forecasts. As *Leith* [1973] points out, unambiguous detection of climatic warming is hampered by the unpredictable variations in temperature that would occur even in the absence of changing CO_2 levels. Only if the warming is large in relation to this natural variability, or 'climatic noise,' can one feel some confidence that a true climatic change has occurred.

The warming due to the CO_2 increase may be difficult to detect during this century at any one location on the earth because the natural variability of temperature at a given location tends to be high in comparison to the warming so far expected there. However, the climatic noise in temperature averaged over large geographical areas is much lower, and the warming trend should be detectable much earlier in such averages than in local temperatures. A pioneering study by *Madden and Ramanathan* [1980] showed that by averaging temperature data from many stations around latitude 60°N the climatic noise ought to be lowered enough to make the warming detectable very soon.

A number of different averaging schemes can be imagined for computing globally averaged temperature. The most straightforward is a simple arithmetical average of the data from all reporting stations. The most natural is probably an average in which data from each area of the globe contribute to the average in proportion to the geographical area they represent. *Hasselmann* [1979] and *Stefanick* [1981] have suggested constructing weighted averages designed to have the best chance of revealing the climatic change if the change occurs as predicted. The concept is similar to that of designing a filter optimally tailored for detecting a signal in radio or radar transmissions [*Wainstein and Zubakov*, 1962]. It can also be related to the problem of classification as described, for example, by *Anderson* [1958]. We pursue this idea here, concentrating on the problem of detecting the warming due to increasing CO_2, though the method is clearly applicable to detecting climatic change of any sort.

In section 2 we derive the optimal weighting of data to detect a predicted climatic change, and give a useful approximation for the weights. The weights are functions of the predicted warming and of the covariance statistics of the data. Data from a number of stations are assumed to be available, but their geographical distribution need not be particularly uniform. The weighting varies with season as well as geographical location. The weighting derived is intuitively plausible: areas and seasons where the warming is expected to be large are weighted more. Areas and seasons where there is a large amount of climatic noise are weighted less. Correlations between stations are taken into account.

In section 3 we estimate the improvement that might be possible using this scheme to detect the surface warming predicted for the northern hemisphere by the climate model of *Manabe and Stouffer* [1980]. The estimated improvement is about 25%, in the sense that one would need 25% more data analyzed in the conventional way, using annual, hemispherical averages, to have an equal chance of detecting the climatic warming. The analysis of section 3 suggests that the climatic warming ought already to be detectable. This is discussed further in section 4. More general discussion of the method and conclusions appear in section 5.

2. Optimal Weighting

Derivation

To derive the weighting of data that has the best chance of showing a global climatic warming if the warming is actually

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occurring as predicted, let us suppose that a surface temperature data set has been assembled that is as nearly global as possible and goes sufficiently far back in time that the statistical characteristics of the surface temperature before the CO_2 warming has become significant can be established. In practice, of course, few regions on the earth can boast of temperature records going back more than a hundred years. The procedure outlined here, when applied to an actual data set to test for climatic change, will require additional assumptions about the nature and magnitude of fluctuations in the atmospheric state with time scales longer than a decade. Such assumptions, which must derive from a physical understanding of the climate system and from proxy data sets extending over longer periods of time, are inevitable in any search for climatic change. We shall return to the problems of dealing with a limited data set at the end of this section.

The earlier part of the temperature record, where the CO_2 warming is supposed small, is used to establish the climatic mean at each station, the level of variability to be expected, and the amount of correlation between temperature fluctuations at the various stations. Based on this information, a weighting scheme is designed that has the best chance of revealing a climatic shift when applied to the portion of the temperature record where a climatic change is predicted.

The derivation of the optimal weighting is considerably simplified by using the concept of an ensemble, which was introduced as a tool for discussing climatic change by *Leith* [1973]. One imagines an ensemble of planets, identical to the earth, but differing in the initial conditions for the atmospheres. The ensemble is constructed so that an average over the ensemble would give the same statistics for the atmosphere as would a time average over a single planet, in the absence of changing boundary conditions such as the CO_2 increase. A climatic shift due to an external disturbance such as increasing CO_2 will appear as an increase, for example, in the ensemble-averaged temperature. We denote averages over the ensemble by angular brackets.

Suppose that data from S stations are used, in the form of monthly averaged temperature $T_s(t)$ at each station s, $s = 1, \cdots, S$, with t labeling month and year. We suppose that earlier than $t = t_*$ ($t_* \sim 1950$, say), the CO_2 warming is small enough that the effect of the warming on the statistics of the time series can be neglected, so that, aside from the annual cycle, the statistics of the data for $t \leq t_*$ may be assumed to be stationary. Temperatures $T_s(t)$ are expressed as deviations from the climatic mean established from this earlier portion of the data set, so that

$$\langle T_s(t) \rangle = 0 \quad t \leq t_* \tag{1}$$

In principle, of course, some interannual variability may be attributable to external factors such as volcanic or solar activity, and one would want to correct the data for these effects, but until convincing quantitative estimates of the size of these effects become available, such variability will probably have to be included in the 'natural variability' of the climate.

The later portion of the data set $T_s(t)$, $t > t_*$, where significant warming due to the CO_2 increase is predicted, is expressed as deviations from the climatic mean established from the earlier portion. Climate models attempt to predict the warming $\Delta T_s(t)$ expected in the temperatures at the various stations:

$$\langle T_s(t) \rangle = \Delta T_s(t) \quad t > t_* \tag{2}$$

The warming is a function of time because of the gradual increase in CO_2 with time and because of the response characteristics of the atmosphere-ocean system.

We cannot, of course, test the prediction (2) directly, since we have access only to the data set $T_s(t)$ from 'our own' planet. The warming trend at any one station may be indistinguishable from natural variations in temperature until the trend has become very large. But we should be able to find early evidence of the warming by constructing a weighted average of the data, averaging over stations and months to reduce the natural variability. Denote this weighted average by

$$A = \sum_{s=1}^{S} \sum_{t=0}^{T_W} w_s(t) T_s(t_+ + t) \tag{3}$$

The average A starts with the (year, month) t_+ and ends T_W months later. For example, the averaging period might be chosen to be the year 1975, with t_+ = (January 1975) and T_W = 11 months. The weight $w_s(t)$ assigned to each station s may depend on the month t and so can vary with the seasons. The weighting may be normalized to

$$\sum_s \sum_t w_s(t) = 1 \tag{4}$$

We shall omit writing the limits on the sums from now on since they do not change.

If the surface temperatures are behaving as predicted in (2) by the climate models, then the expected value of the weighted average (3) is

$$\langle A \rangle = \sum_s \sum_t w_s(t)\, \Delta T_s(t_+ + t) \tag{5}$$

However, the weighted average A obtained from our data set will probably differ from zero whether or not the CO_2 warming has occurred, simply because of the climatic noise in averages such as those described by *Leith* [1973] arising from the natural, unpredictable variability of the atmosphere. We therefore need some measure of the level of climatic noise. Since A is a sum over many independent data, its statistics are likely to be nearly Gaussian (normal). A good measure of its natural variability is therefore its expected variance, which can be computed from periods identical in length to the one used in (3) but displaced many years earlier to start at t_-, when there is no climatic warming. For example, we might choose t_- to be (January 1925).

Denote the weighted average over the earlier period starting at t_- by A_-,

$$A_- \equiv \sum_s \sum_t w_s(t)\, T_s(t_- + t) \tag{6}$$

Its variance is

$$\langle A_-^2 \rangle = \left\langle \left[\sum_{s_1} \sum_{t_1} w_{s_1}(t_1)\, T_{s_1}(t_- + t_1) \right] \left[\sum_{s_2} \sum_{t_2} w_{s_2}(t_2)\, T_{s_2}(t_- + t_2) \right] \right\rangle$$

$$= \sum_{s_1} \sum_{s_2} \sum_{t_1} \sum_{t_2} w_{s_1}(t_1)\, w_{s_2}(t_2) \langle T_{s_1}(t_- + t_1)\, T_{s_2}(t_- + t_2) \rangle \tag{7}$$

The quantity in angular brackets is just the lagged covariance between temperatures at pairs of stations s_1 and s_2:

$$C_{s_1 s_2}(t_1, t_2) \equiv \langle T_{s_1}(t_- + t_1) T_{s_2}(t_- + t_2) \rangle \quad (8)$$

In the period before the climatic warming, the covariance should not depend on the particular year t_- for which it is evaluated, though it will still depend on the season. A discussion of variables with seasonally varying statistics has recently been given by *Hasselmann and Barnett* [1981]. By using all of the data before t_* and assuming that the climatic statistics are stationary from year to year, we may estimate (8) using standard statistical procedures. Let us therefore assume that the covariance (8) has been obtained from our data set. We are then able to estimate the variance expected for A_- for any weighting as

$$\langle A_-^2 \rangle = \sum_{s_1} \sum_{s_2} \sum_{t_1} \sum_{t_2} w_{s_1}(t_1)\, w_{s_2}(t_2) C_{s_1 s_2}(t_1, t_2) \quad (9)$$

The confidence with which we can assert that a climatic warming has occurred depends on how large the ratio $A/\langle A_-^2 \rangle^{1/2}$ is. The larger the ratio is, the less likely is it that the value for A we have obtained can be due to natural variability. Since the statistics of A are nearly Gaussian and $\langle A_-^2 \rangle$ is the expected variance of A, confidence limits can be established for A. The expected value of the ratio $A/\langle A_-^2 \rangle^{1/2}$, if the climate models are correct, is $\langle A \rangle / \langle A_-^2 \rangle^{1/2}$, where $\langle A \rangle$ is given by (5). By choosing the weights $w_s(t)$ to maximize this ratio we obtain a weighting scheme that is optimal for detecting climatic change if it is occurring along the lines suggested by the models. If the models are wrong and the climate does not warm as much as they suggest, then the weighting scheme will fail to show a climatic change, as measured by $A/\langle A_-^2 \rangle^{1/2}$: in the absence of climatic change, no matter what the weighting selected, this ratio will exceed 2 with less than 2.3% probability.

The weighting $w_s(t)$ that maximizes $\langle A \rangle / \langle A_-^2 \rangle^{1/2}$ can be found in the usual fashion, by setting the derivatives of the ratio with respect to each weight equal to zero and solving the resulting equations for the weights. The set of equations obtained by this procedure is

$$\sum_{s'} \sum_{t'} C_{ss'}(t, t') w_{s'}(t') = W \Delta T_s(t_+ + t) \quad (10)$$

where W is a constant which must be adjusted so that the weights $w_s(t)$ are properly normalized as in (4). Note that (10) is a simple linear equation for $w_s(t)$: the covariance $C_{ss'}(t, t')$ is determined from the data before the warming, and $\Delta T_s(t_+ + t)$ is the predicted warming. Provided that there are not too many stations S and not too many months $(T_W + 1)$, (10) can be solved by straightforward numerical procedures. However, an approximate solution to the equations can be found that will prove useful.

Useful Approximation for Weights

To obtain some insight into the behavior of the weighting, let us solve (10) approximately by assuming either that the covariance $C_{ss'}(t, t')$ decreases rapidly as the separation between stations s and s' increases and the number of months between t and t' increases or that $w_s(t)$ and $\Delta T_s(t_+ + t)$ vary slowly with station position and month t if C does not decrease so rapidly. This may be a good approximation for surface temperature in middle and low latitudes where the spatial variability in the statistics is comparable to the spatial variability in the amount of warming expected. It may be poorer in polar regions where correlations may extend over distances large in comparison with the spatial variability in the amount of warming there. It may also prove inadequate if long-range correlations are found in monthly averaged surface temperature as strong as those found by *Wallace and Gutzler* [1981] in the 500-mbar geopotential height fields.

Having made these assumptions, define the quantity

$$V_s(t) = \sum_{s'} \sum_{t'} C_{ss'}(t, t') \quad (11)$$

Roughly speaking, this quantity may be interpreted as the product

$$V_s(t) \sim 2C_{ss}(t, t) N_s \tau_s \quad (12)$$

where $C_{ss}(t, t)$ is the interannual variance of monthly averaged temperature at station s for month t, N_s is the number of stations significantly correlated with station s, and τ_s is the typical correlation time, in months, of temperatures in the neighborhood of s. All of these quantities could depend on season. Given the assumptions stated above, the solution to (10) is approximately given by

$$w_s(t)/W \approx \Delta T_s(t_+ + t)/V_s(t) \quad (13)$$

Equation (13) shows that $w_s(t)$ behaves as we might have guessed intuitively: stations and months where the predicted warming is large are weighted more; stations and months where temperature variability is large are weighted less; stations and months that are correlated with many others are weighted less (since the information contributed by the others is relatively less).

It is interesting to calculate the ratio $\langle A \rangle / \langle A_-^2 \rangle^{1/2}$ to see how large it is expected to be if the climate models are correct. By substituting (10) into (9) we obtain the exact result

$$\langle A \rangle^2 / \langle A_-^2 \rangle = (1/W) \sum_{s} \sum_{t} w_s(t) \Delta T_s(t_+ + t) \quad (14)$$

which is approximately given by

$$\frac{\langle A \rangle^2}{\langle A_-^2 \rangle} \approx \sum_{s} \sum_{t} \frac{[\Delta T_s(t_+ + t)]^2}{V_s(t)} \quad (15)$$

It is independent of the normalization factor W, of course. Equation (15) may be interpreted as the signal-to-noise ratio of the climatic change, or as the amount of 'information' about the climatic change that can be extracted from the data. By choosing the weights according to (13) we have extracted the maximum amount of evidence of climatic change possible with a linear average of the data. If the ratio in (15) is larger than, say, 4, we begin to have a reasonable chance of discovering the climatic change in the data at the 2.3% significance level.

Since the signal-to-noise ratio is maximized when the weightings are chosen to satisfy (10), the first derivatives of the ratio with respect to the weights vanish. The power of the optimal weighting method is therefore not sensitive to small errors made in determining the weights.

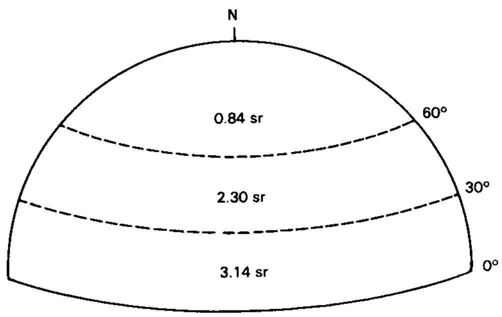

Fig. 1. The geographical zones into which the northern hemisphere is divided. The area in steradians of each zone is shown within each zone.

Special Remarks

Determination of the optimal weighting requires an estimate of the covariance matrix $C_{ss'}(t, t')$, since the weights $w_s(t)$ are solutions of the linear equations (10) in which the covariance matrix elements appear as coefficients. The covariance matrix is not known exactly and must be estimated from a data set of finite length. Inevitable sampling error in the covariance estimates must be kept in mind in implementing the scheme described here.

Suppose, for example, that monthly averaged data from 100 stations are available for a period 50 years prior to the time when significant warming is believed to have occurred and that the optimal weighting for each station for each month of the year is to be determined. Given this situation, even assuming that there were no correlation from one month to the next, one would be trying to estimate covariances of 100 variables with 50 independent samples. An eigenvalue analysis of the sample covariance matrix would find at most 49 nonzero eigenvalues; i.e., the covariance matrix would be highly singular.

Some care must therefore be taken that the number of independent weights does not exceed the number of independent samples available in the data set. One way to reduce the number of independent weights is to divide the earth up into regions within which the statistics are not too inhomogeneous, so that the station data are replaced by regional averages. The weighting can then be determined region by region instead of station by station. The analysis of the next section provides an example of this approach. A slightly more abstract but in principle quite useful approach is to use a subset of empirical orthogonal functions to represent the spatial variability of the data, as suggested by *Hasselmann* [1979]. Allowance for seasonal variability in the weighting can lead to sampling problems similar to those met in trying to represent spatial variability of the weighting in too much detail. Reduction of the number of temporally independent weights can be achieved by parameterizing seasonal variability of the weights with a few sinusoidal modes to represent the annual cycle. It may be useful to model the correlations in the data from one month to the next using low-order autoregressive models such as those described, for example, by *Jenkins and Watts* [1968], generalized to allow for seasonal variations in the statistics along the lines of *Hasselmann and Barnett* [1981]. Monte Carlo studies may be needed to verify that the treatment of the data set is consistent with the number of data available.

Adjustment of the weighting at each station and month independently to optimize the chances of detecting climatic change may be prevented by the size of the data set, but appropriately reducing the degrees of freedom in the weighting and introducing statistical models for the covariance matrix will allow one to make the best possible use of the data set consistent with its size. The effort involved in modeling the covariance matrix, if required, can be substantial, but the matrix is interesting in its own right as a statistical characterization of the atmosphere. Fortunately, the effort need not be repeated to determine new optimal weights $w_s(t)$ if later theoretical developments should lead to different predictions of the climatic shift $\Delta T_s(t)$, since this changes only the right-hand side of (10).

Once the covariance matrix is obtained and the optimal weighting is determined, a theoretical estimate of the level of natural variability in the weighted average A of (3) follows from (9) and (10). If the data set is long enough, a more satisfactory estimate may be obtained by simply applying the weighting to successive portions of the data set before the signal is believed to be significant and estimating the variability of the weighted average directly from the data. A substantial disagreement between the two estimates would indicate that the statistical model chosen for the covariance matrix is a very poor representation of its actual nature, including the possibility that a substantial amount of long–time scale 'noise' is present in the data that cannot be accounted for by using short-term statistics. In the latter case, the investigator must return to first principles, identify the source of the long–time scale variability, and either remove it from the data or include it in the estimate of the intrinsic climatic noise. Only when an acceptable estimate of the noise intrinsic to the average A is obtained can one establish confidence limits on whether or not climatic change has occurred.

The intrinsic variability of A can be used to gauge how well the theoretical prediction for climatic change as measured by A agrees with the value of A calculated from the data. There is less than 5% chance of their differing by more than $2\langle A_-^2\rangle^{1/2}$ (two standard deviations).

3. APPLICATION TO THE CO_2 WARMING

We shall attempt here to estimate how much improvement might be possible, using the weighting scheme described in the previous section, in our ability to detect the climatic warming produced by the CO_2 increase. The estimate is necessarily a rough one, since the detailed statistical studies of the temperature records needed to obtain a more accurate estimate have not yet been done.

The weighting scheme is used to best advantage when the weights are allowed to vary from station to station and from day to day. Little is lost, however, by using monthly averaged data, since the warming is believed to be occurring slowly and since the variation of statistics with season occurs mostly on this time scale. More is lost by not treating each geographical region individually, but the covariance statistics needed for such a treatment are not readily available. We shall therefore restrict ourselves to the northern hemisphere, divided into three zones separated by latitudes 30° and 60°, as illustrated in Figure 1. This division is suggested by the study of the statistics of zonally, monthly averaged surface temperature by *Weare* [1979], who found no significant correlation between temperatures averaged over 15° latitude bands on either side of latitudes 30° and 60°N, but did find significant (0.4–0.5) correlation within each of the three zones of Figure 1. Zonal averages taken at latitudes 15°, 45°, and 70° will be used to represent the behavior of temperatures in the tropical, mid-latitude, and polar zones respectively. We shall allow the weighting factors in (13) to vary with season and from one zone to another.

The weighting factors are given in (13) in terms of the warming expected and the covariance statistics of the temperature in each zone. We shall use values for the warming taken from the climate model of *Manabe and Stouffer* [1980]. Values shown in Figure 2 for the warming are zonal averages over continents and oceans. Manabe and Stouffer's results for a 1200-ppm concentration of CO_2 have been scaled down by a factor [*Ramanathan et al.*, 1979; *Madden and Ramanathan*, 1980] of log 1.1/log 4 = 0.07 to obtain the warming expected from a 10% (30 ppm) increase in CO_2 concentration. This corresponds to the increase in concentration believed to have occurred since the early 1900's to about 1972. The model results do not include the time-dependent effects of the ocean heat capacity, which may delay the warming by a decade or more [*Offenborn and Grassl*, 1981] and alter its geographical character [*Schneider and Thompson*, 1981]. For the covariance statistics of zonally averaged temperature we use standard deviations of monthly average 1000-mbar temperature, averaged over seasons, computed by *Oort* [1982] from 10 years of data beginning in 1963. Standard deviations are shown in Figure 3. The spring and autumn seasons are adjusted so that they are equal. Our estimates of the correlations in time and space of zonally averaged temperature are based on the study of *Weare* [1979]. Correlations between zones are neglected, and 1-month-lagged correlations of the temperature in each zone are estimated to have the values given in Table 1. Correlations are assumed to be constant throughout the year.

Before investigating the use of the optimal weighting of the data to detect the warming, let us evaluate the possibility of detecting the warming in an individual zone. Let us compute the 'signal-to-noise' ratio for the seasonal warming $\Delta T_j(t)$ given in Table 1 for seasonally averaged (i.e., 3-month

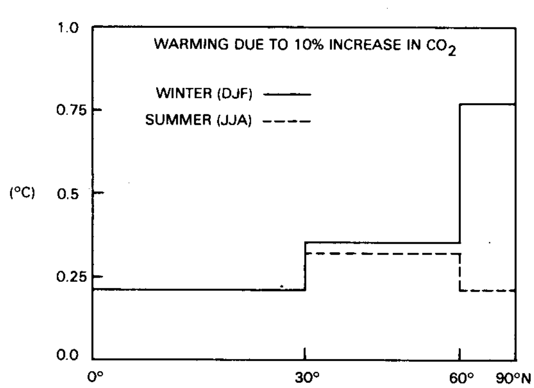

Fig. 2. Surface warming in degrees Celsius due to 10% increase in CO_2 as a function of latitude and season, obtained by scaling the results of *Manabe and Stouffer*'s [1980] calculation by a factor of log 1.1/log 4. Warming values expected for the high-, middle-, and low-latitude zones are 0.49°, 0.35°, and 0.21° and 0.56°, 0.35°, and 0.24°C for the spring and autumn respectively.

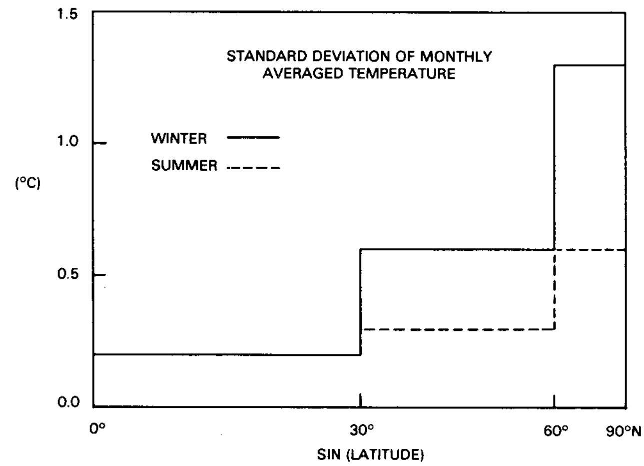

Fig. 3. Standard deviations of monthly, zonally averaged temperature for each season and zone. Values for spring and autumn are forced to be equal and are 1.25°, 0.35°, and 0.2°C for the high-, middle-, and low-latitude zones respectively.

averaged) temperatures in each zone. The natural variability of seasonally averaged temperature may be estimated using *Jones*' [1975] result for time averages,

$$\overline{T} = \frac{1}{N} \sum_{i=1}^{N} T(i) \tag{16}$$

over N months, where i labels the month:

$$\langle \overline{T}^2 \rangle = \frac{M_N}{N} \langle T^2 \rangle \tag{17}$$

with

$$M_N = 1 + 2\left[c\left(1 - \frac{1}{N}\right) + c^2\left(1 - \frac{2}{N}\right) + \cdots + c^{N-1}\left(\frac{1}{N}\right)\right] \tag{18}$$

where $\langle T^2 \rangle$ is the variance of monthly averaged temperature and c is the 1-month-lag correlation of temperature. We have assumed that the temperature's statistical behavior in time can be adequately represented by a first-order Markov process. The quantity M_N is the effective time, in months, between independent samples in an N-month average.

The signal-to-noise ratio $[\Delta T_j(t)]^2/\langle \overline{T}^2 \rangle$ for unweighted seasonally averaged temperature in each zone is shown in Figure 4. We show the signal-to-noise ratio rather than its square root because the ratio is linear in the number of data used to detect the signal; that is, an average over two seasons should have twice as large a signal-to-noise ratio as an average over one season (neglecting correlations between the seasons). None of the numbers in Figure 4 exceeds 4, and so at the 2.3% significance level we would not expect to

TABLE 1. Lagged Correlations of Monthly, Zonally Averaged Temperature, Assumed Independent of Season

	1-Month-Lag Correlation
90°–60°	0.2
60°–30°	0.2
30°–0°	0.5

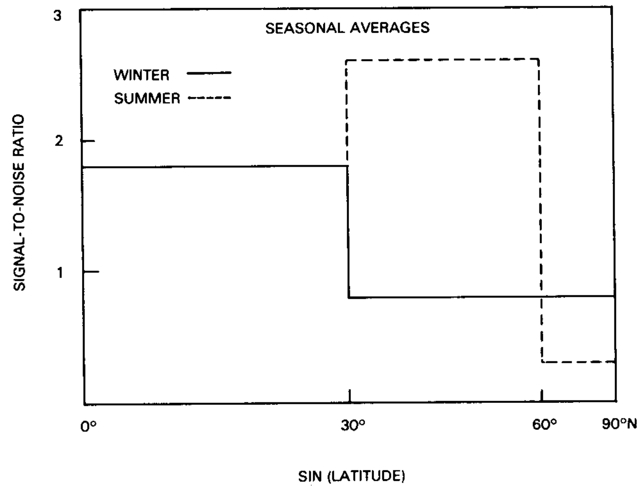

Fig. 4. The signal-to-noise ratio $[\Delta T_j(t)]^2/\langle \bar{T}^2 \rangle$ for seasonally, zonally averaged temperature for the warmings indicated in Figure 2. Values for high, middle, and low latitudes are 0.4, 2.4, and 1.8 and 0.5, 2.4, and 2.5 for spring and autumn respectively. Ratios greater than 4 represent a two-standard deviation warming.

detect the warming in the temperature averaged over a single season. However, it is clear from Table 4 that if our estimate of the level of variability of seasonal averages is correct and if the warming is indeed occurring at the levels assumed here, then by averaging over enough seasons we should be able to see some evidence of a climatic warming. The implications of the magnitudes of these ratios are discussed further in section 4. Note that when the values in Figure 4 are divided by the area in steradians of each zone to obtain a measure of the amount of detectable signal per unit area of the earth, one finds a maximum value in mid-latitudes in the summer, in agreement with the results of *Wigley and Jones* [1981], although the values for spring and autumn in mid-latitudes are close enough that on the basis of our calculations alone one could not rule these seasons out as being more favorable.

We next investigate the improvement in our ability to detect a climatic warming using the optimal weighting scheme of section 2. Consider first the case of trying to detect climatic warming for a single season. The weighting for each zone is given approximately by (13) and (17):

$$w_j(t) \propto \Delta T_j(t)/M_3\langle T^2 \rangle \tag{19}$$

where M_3 is defined in (18) and $\langle T^2 \rangle$ is the variance of monthly averaged temperature for the season of interest. These weights are shown in Table 2. Note that the polar regions should generally be weighted much less than the lower latitudes, except in the winter.

TABLE 2. Weighting Factors for Each Zone From Equation (19) Optimally Designed for Detecting a Shift in Seasonal Mean Temperature

	Winter	Spring	Summer	Autumn
90°–60°	1	1	1	1
60°–30°	0.8	3.4	2.2	3.0
30°–0°	2.2	3.2	1.7	3.2

The factors have been normalized by dividing by the area of each zone to obtain the weighting appropriate for each station for stations distributed uniformly on the surface of the earth. The weighting for the polar zone has been arbitrarily set equal to 1 for each season.

TABLE 3. Change in Hemispherically Averaged Temperature ΔT_0, Defined in Equation (20), and Climatic Noise of Hemispherically Averaged Temperature, $\langle T_0^2 \rangle^{1/2}$, From Equation (21)

	Winter	Spring	Summer	Autumn
ΔT_0, °C	0.34	0.30	0.25	0.33
$\langle T_0^2 \rangle^{1/2}$, °C	0.20	0.16	0.12	0.16

The hemispheric average warming ΔT_0 expected from a 10% increase in CO_2 is obtained from an area-weighted average of each zone,

$$\Delta T_0 = \sum_{j=1}^{3} \Omega_j \, \Delta T_j/2\pi \tag{20}$$

where Ω_j is the area in steradians of zone j given in Figure 1 and ΔT_j is the warming of that zone from Figure 2. The climatic noise in the hemispherically averaged temperature is estimated from

$$\langle T_0^2 \rangle = \sum_{j=1}^{3} \Omega_j^2 \langle \bar{T}_j^2 \rangle/(2\pi)^2 \tag{21}$$

where we use *Jones*' [1975] expression (17) for the variance $\langle \bar{T}_j^2 \rangle$ in the seasonally averaged temperature of zone j. We have neglected correlations among temperatures in the different zones, which is suggested by *Weare*'s [1979] study as mentioned above. In Table 3 we show for each season the quantities ΔT_0 and $\langle T_0^2 \rangle^{1/2}$. In Figure 5 we plot versus season the signal-to-noise ratio for the hemispherically averaged temperature and, for comparison, the signal-to-noise ratio expected using the weights of Table 2, computed using (15). Note that more is gained by using the weighting scheme in the nonsummer months than in the summer.

Let us turn now to annual averages of the temperature. Since the correlation time is short in high latitudes (Table 1) and the variance changes little with season in low latitudes

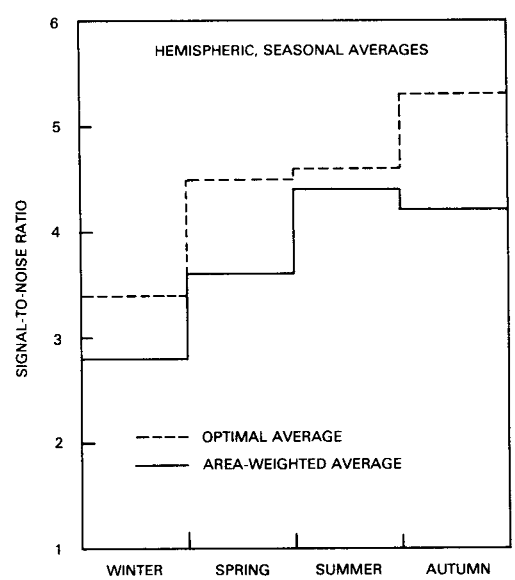

Fig. 5. Signal-to-noise ratio as a function of season for seasonally, hemispherically averaged temperature and for seasonally averaged temperature optimally weighted geographically using weights from Table 2.

TABLE 4. Weighting Factors for Annually Averaged Temperatures

	Winter	Spring	Summer	Autumn
90°–60°	1	0.7	1.3	0.8
60°–30°	0.8	2.3	2.8	2.3
30°–0°	1.7	1.7	1.7	2.0

Weights are normalized as in Table 2, with the polar winter zone weighting arbitrarily set equal to 1.

where the correlation times are long, we may use (17) to compute the variances of annual averages, with $N = 12$ and $\langle T^2 \rangle$ the annually averaged variance, with values for M_{12} obtained from (18) using the correlations listed in Table 1.

The optimal weightings for each season and latitude are shown in Table 4. The normalization used is such that all entries in Table 4 would equal 1 if the conventional area average was optimal. The signal-to-noise ratio $\langle A \rangle^2 / \langle A_-{}^2 \rangle$ expected using the weights in Table 4 for a weighted average over the northern hemisphere and over one year is found from (15) to be

$$\langle A \rangle^2 / \langle A_-{}^2 \rangle = 14 \tag{22}$$

For comparison, the hemispherically (area-weighted), annually averaged surface warming $\langle \Delta \bar{T}_0 \rangle$ (overbars here indicate annual averages) calculated using (20) is 0.30°C, and the standard deviation of hemispherically, annually averaged temperature $\langle \bar{T}_0{}^2 \rangle^{1/2}$ calculated using (21) is 0.09°C, so that the signal-to-noise ratio $\langle \Delta \bar{T}_0 \rangle^2 / \langle \bar{T}_0{}^2 \rangle$ expected for hemispheric annual temperature is

$$\frac{\langle \Delta \bar{T}_0 \rangle^2}{\langle \bar{T}_0{}^2 \rangle} = \left(\frac{0.3}{0.09} \right)^2 = 11 \tag{23}$$

Thus, by using optimal weighting of the data, our ability to detect a climatic warming is improved by about 25% relative to using the more conventional hemispheric, annual temperature as a measure of global warming, in the sense that the weighted average produces the same signal-to-noise ratio as would be obtained using the conventional average with 25% more data.

4. DISCUSSION

In the previous section we estimated the signal-to-noise ratio we should expect to see if the surface warming due to a 30-ppm increase in the CO_2 concentration in the atmosphere were to occur as predicted. For area-weighted averages over a single year of northern hemisphere data we obtained a value for the ratio $\langle \Delta \bar{T}_0{}^2 \rangle / \langle \bar{T}_0{}^2 \rangle \approx 11$, or a 3.3 standard deviation shift in the mean. The optimal weighting scheme would give an even larger ratio. Such values would suggest that the temperature rise since the beginning of the century ought to be easily observable in the data. Recent analyses of surface temperature data by *Borzenkova et al.* [1976] and *Hansen et al.* [1981] do show a substantial temperature rise up until the 1940's, but this warming was followed by a marked cooling, noted earlier by *Angell and Korshover* [1977], during the period when the CO_2-related warming ought to be most evident. Although our main purpose here is to investigate the use of the optimal weighting scheme for detecting climatic change rather than to evaluate the chances of detecting the climatic warming due to the CO_2 increase, the discrepancy between our estimates of the observability of the warming and what the data actually show merits some discussion. In any case, some of the considerations raised here are likely to be issues in any search for evidence of climatic change.

Let us try, then, to obtain a slightly more realistic estimate of the detectability of the warming, including the effect of having only a limited number of data to work with. Our estimate of the signal-to-noise ratio depends on, among other things, the accuracy of the predictions of *Manabe and Stouffer*'s [1980] climate model. Most models of other workers seem to predict similar amounts of warming, though seasonal and geographical details vary. A few models have predicted substantially smaller temperature increases, but *Ramanathan* [1981] has identified physical processes left out of these models that appear to explain the models' lower sensitivities to a CO_2 increase. The temperature increase predicted by *Manabe and Stouffer*'s [1980] model is, however, for the eventual warming expected after the climate system has equilibrated with the new atmospheric concentration of CO_2. The values we have used therefore almost certainly overestimate the amount of warming that has actually occurred because of the delay in the warming caused by the large heat capacity of the oceans. Suppose we use as a rough approximation to the atmospheric concentration of CO_2 an exponentially increasing function of the form

$$[CO_2] \text{ (ppm)} = 300 + 30e^{(t-1972)/25} \tag{24}$$

which would assign to the year 1972 a 30-ppm increase of CO_2 over the levels typical of the early part of the century. The rise in the hemispherically averaged temperature, calculated as in section 3, assuming instantaneous equilibration of the climate to the CO_2 concentration, would be given by

$$\Delta T \text{ (°C)} = 0.3e^{(t-1972)/25} \tag{25}$$

The results of *Bryan et al.* [1982] suggest that the effect of the oceans is to slow the approach to equilibrium, with temperatures reaching 60–80% of their equilibrium values within a few years but requiring many decades thereafter to climb appreciably further toward equilibrium. If their results are taken as representative, the effect of the oceans is to reduce the warming at any time to about 70% of the equilibrium value, so that the temperature rise would in effect be delayed about 10 years:

$$\Delta T \text{ (°C)} = 0.3e^{(t-1982)/25} \tag{26}$$

We turn next to our estimate of the natural variability of temperature in the northern hemisphere. We have almost certainly underestimated it, since we have assumed that data are uniformly available over the hemisphere and that all interannual variability can be explained by variability of the atmosphere on time scales of a few months. The latter assumption entered in our use of a first-order Markov process with the 1-month-lag correlations given in Table 1. A factor of 2 or 3 underestimate in the variance of the hemispherically averaged temperature is not impossible. The analysis of *Yamamoto and Hoshiai* [1979] shows a variance of $0.26(°C)^2$ for annually averaged northern hemisphere temperature for the period 1951–1977. Combined with the probable overestimation of the strength of the warming just discussed, it is not unlikely that we have overestimated the signal-to-noise ratio for the year 1972 by as much as a factor of 4 to 6.

TABLE 5. Probable Signal-to-Noise Ratios Using Data From Three Periods All Beginning in 1946 and Ending in 1975, 1980, and 1985 Respectively

	Hemispheric, Annual Averages		Optimal Averages	
Period Covered by Data	Signal-to-Noise Ratio	Probability $<2\sigma$ Warming	Signal-to-Noise Ratio	Probability $<2\sigma$ Warming
1946–1975	2.3 (1946–1957 vs. 1968–1975)	68%	3.3	57%
1946–1980	4.6 (1946–1959 vs. 1971–1980)	44%	6.6	28%
1946–1985	8.7 (1946–1962 vs. 1975–1985)	17%	12.6	6%

The second column shows the signal-to-noise ratios that can be achieved by subtracting means over the two spans given in parentheses below the ratios. The fourth column shows probable signal-to-noise ratios resulting from the optimal seasonal and geographical weighting scheme described in section 3 and also using an optimal weighting of each year based on the signal (26) believed to be present. The third and fifth columns show the probability of seeing less than a two-standard deviation change in the climate during the period.

One should also keep in mind that the signal-to-noise ratios given are only the most probable values. Even if the predicted (most probable) warming is 3.3 standard deviations, for example, there is still 1 chance in 10 that natural variability would lead to a mean temperature more than 1.3 standard deviations lower than the most probable mean, i.e., that we would reject the hypothesis of climatic change at the 5% significance level since the warming for that year would be less than 2 standard deviations.

The chances of detecting a warming are obviously improved by comparing averages over many years instead of the average temperature for just a single year, since the signal-to-noise ratio increases roughly linearly with the number of years averaged over. On the other hand, the strength of the predicted warming decreases earlier in the century, and the early portion of the data set must be kept in reserve to establish the climatic norm with which averages over the later data are compared. To see what values the signal-to-noise ratios might reach by averaging over many years within these constraints, let us examine the possibility of detecting a warming using data from 1946 to recent times. Assume that the signal grows exponentially as in (25), with the 10-year delay due to the oceans included. Assume also a more conservative estimate of the variance of hemispherically, annually averaged temperature of $3 \times 0.008(°C)^2 = 0.024(°C)^2$ (i.e., 3 times as large as the estimate obtained in section 3), and assume that there is negligible correlation in the temperature from one year to the next.

As a measure of the warming, let us compare the mean temperature over a span of N_y years ending near the present to an earlier span of N_x years starting from 1946. The difference between the two means has an intrinsic variance $\langle \Delta_-^2 \rangle$, in the absence of any warming, given by

$$\langle \Delta_-^2 \rangle = \left(\frac{1}{N_x} + \frac{1}{N_y} \right) \langle T^2 \rangle \qquad (27)$$

where $\langle T^2 \rangle = 0.024(°C)^2$ is our estimate of the variance of the annually averaged temperature. The expected difference between the two means is

$$\langle \Delta \rangle = (\overline{\Delta T})_y - (\overline{\Delta T})_x \qquad (28)$$

where $(\overline{\Delta T})_x$ is the average of (25) over the N_x earlier years, and $(\overline{\Delta T})_y$ is the corresponding average for the later N_y years. By adjusting N_x and N_y, we can find the choice that maximizes the signal-to-noise ratio $\langle \Delta \rangle^2 / \langle \Delta_-^2 \rangle$ and gives us the best chance of finding a significant difference in temperature between the two periods.

This has been done for three periods ending in 1975, 1980, and 1985 respectively, and the resulting maximum signal-to-noise ratios are shown in the second column of Table 5. For comparison we show in the fourth column of Table 5 the improvement possible using the optimal geographical and seasonal weighting scheme described in section 3, which gives a 25% improvement in signal-to-noise ratio, and in addition weighting each year of data in the record so as to take maximum advantage of the exponential trend expected in the data, which gives another 13% improvement over simply comparing means from the end and beginning of the record, as was done in the second column of Table 5.

In the third column of Table 5 is shown the probability of finding less than a 2σ warming in hemispherically averaged temperature (and so concluding that there has been no climatic change at the 5% significance level). It is remarkable that even with 30 years of data (1946–1975) there is still almost a 70% chance of seeing less than a two-standard deviation warming over that period. It must be remembered, of course, that our assumptions for the noise levels and signal strengths are rather conservative: we have decreased the signal-to-noise ratio by a factor of 3 from what our original estimate would have been by using $0.024(°C)^2$ for the variance of the annually averaged temperature; we have lost another factor of 0.49 by including the delaying effect of the ocean and a factor of 0.6 because the warming during the 30-year period is only 75% as large as what occurred since the beginning of the century; and with only 30 years of data some additional noise is introduced in not being able to establish the climatic mean before the warming (i.e., the first term on the right-hand side of (27)).

On the other hand, given our assumptions, there is less than a 7% chance of seeing hemispheric cooling during this period, which is what actual surface temperature data seem to show [e.g., *Borzenkova et al.*, 1976; *Yamamoto and*

Hoshiai, 1979; *Hansen et al.*, 1981]. This suggests that one or more of the assumptions on which Table 5 is based are wrong. The warming may be less than has been predicted and delayed by the oceans more than we have assumed. Equally likely is that there is more long-term variability of the climate than our estimates have allowed for. Many sources for such variability have been suggested. Some of the variability may eventually be accounted for and removed from the data, such as that due to material deposited in the atmosphere by volcanic activity [*Mitchell*, 1961; *Robock*, 1981; *Hansen et al.*, 1981]. Some may be due to the integrating effects of the 'slow' components of the climate system, such as ice sheets, the deep oceans, vegetation, etc. [*Hasselmann*, 1976]. Much work needs to be done in establishing limits on the likely magnitude of variability from these sources to make unambiguous detection of the CO_2-related warming possible soon. In several more decades, with continued increase in the CO_2 concentration, the effects of the warming should be clear enough that subtle statistical considerations may not be so necessary.

5. CONCLUSION

The optimal weighting of data described here is designed to increase the probability of detecting a predicted climatic shift in the data. In an ideal world, the prediction of the climatic change would be made before scrutinizing the available data. In practice one must trust that a prediction based on sound physical principles is objective enough that it can be considered to be determined independently of the knowledge gained from study of a data set.

Implementation of the method requires that a data set be long enough that a satisfactory estimate of the noise level in the data set can be made. As climate model runs become longer and the ability of climate models to represent the natural variability of the climate improves, it may eventually be possible for the models themselves to suggest optimal weightings of data sets to detect climatic change.

As with any statistical method designed to detect long-term trends in data, the method described here assumes that all long-time scale variability, apart from the suspected signal, can be accounted for as a statistical residue of shorter-time scale processes, in the same way that *Leith* [1973] has expressed variability of climatic averages in terms of shorter-term statistics. Estimates of the noise level on time scales comparable with the length of the data set must be based on theoretical and modeling efforts. For detecting the CO_2 warming in surface temperature data one needs a data set at least 30 years long, and preferably longer. It seems likely from the analysis of the previous section that some additional information about longer-term variability is going to be required.

The approach described here generates, in effect, a linear filter that converts a many-variable time series into a single-variable time series best suited to revealing a predicted climatic change. If new variables are admitted to the averaging scheme, the power of the method to detect the climatic change is increased, if the variables added are not highly correlated with the ones already included in the average. In this sense an ideal index of the climatic change would be a weighted average of all atmospheric variables for which sufficient data can be assembled. However, one is inevitably faced with the problem of verifying that the cause of the climatic change revealed by the index is indeed the one postulated. The forces which affect climate are so interconnected that documentation of the climatic change in some detail is required before one begins to feel confident that the true source of the climatic change has been identified. This requires that different atmospheric variables be analyzed separately to see if each is behaving according to model predictions. Unfortunately, the greater the detail with which one looks at the data, the smaller the signal-to-noise ratio becomes, so that one must wait longer to verify climatic change in detail than to see the first evidence of it.

There may be some tendency for global climatic change, at least at the surface of the earth, to occur in much the same way no matter what its external cause, as *Wigley and Jones* [1981] have remarked, so that identification of the cause of the change may be difficult. The atmosphere may 'prefer' to vary on long-time, global scales in only a few ways. To the extent that this is true, the optimal weighting will tend to be uniform in time and space. Considering the inhomogeneity of the statistics on the earth, the tendency for the estimates for the weights obtained in section 3 toward uniformity is already remarkable. This may also be evidence that a kind of 'fluctuation-dissipation' relation may be governing the behavior of the atmosphere, although it is not clear how a result of this nature might be derived from *Leith*'s [1975] formulation of the idea.

Despite this tendency, it appears that observations of other variables might enable one to distinguish the warming due to the CO_2 increase from that due, say, to an increase in the solar output. *Madden and Ramanathan* [1980] remark, for example, that the CO_2 warming should be accompanied by stratospheric cooling.

Although the method described here can in principle be used to construct an optimal test for climatic change that can be applied to an entire data set, a more satisfactory use of it might be to construct a kind of annual index of global temperature from the optimal weighting for a year of data, as was done in the example in section 3, with different seasons and geographical areas weighted differently, and then to construct a time series of these yearly averages, which can then be treated in much the same way as the more familiar time series of global, annually averaged temperatures. *Epstein* [1982], for example, has discussed in some detail tests aimed at detecting climatic change in a time series of globally averaged temperatures. Such methods could be applied to the time series of optimally weighted annual averages.

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REFERENCES

Anderson, T. W., *An Introduction to Multivariate Statistical Analysis*, pp. 126–153, John Wiley, New York, 1958.

Angell, J. K., and J. Korshover, Estimate of the global change in temperature, surface to 100 mb, between 1958 and 1975, *Mon. Weather Rev.*, *105*, 375–385, 1977.

Borzenkova, I. I., K. Ya. Vinnikov, L. P. Spirina, and D. I. Stekhnovsky, Change in air temperature of the northern hemisphere for the period 1881–1975, *Meteorol. Gidrol.*, *7*, 27–35, 1976.

Bryan, K., F. G. Komro, S. Manabe, and M. J. Spelman, Transient climate response to increasing atmospheric carbon dioxide, *Science*, *215*, 56–58, 1982.

Epstein, E. S., Detecting climate change, *J. Appl. Meteorol., 21,* 1172–1182, 1982.

Hansen, J., D. Johnson, A. Lacis, S. Lebedeff, P. Lee, D. Rind, and G. Russell, Climate impact of increasing atmospheric carbon dioxide, *Science, 213,* 957–966, 1981.

Hasselmann, K., Stochastic climate models, 1, Theory, *Tellus, 28,* 473–485, 1976.

Hasselmann, K., On the signal-to-noise problem in atmospheric response studies, in *Meteorology Over the Tropical Oceans,* edited by D. B. Shaw, pp. 251–259, Royal Meteorological Society, Bracknell, Berkshire, England, 1979.

Hasselmann, K., and T. P. Barnett, Techniques of linear prediction for systems with periodic statistics, *J. Atmos. Sci., 38,* 2275–2283, 1981.

Jenkins, G. M., and D. G. Watts, *Spectral Analysis and Its Applications,* Holden-Day, San Francisco, Calif., 1968.

Jones, R. H., Estimating the variance of time-averages, *J. Appl. Meteorol., 14,* 159–163, 1975.

Leith, C. E., The standard error of time-average estimates of climatic means, *J. Appl. Meteorol., 12,* 1066–1069, 1973.

Leith, C. E., Climate response and fluctuation dissipation, *J. Atmos. Sci., 32,* 2022–2026, 1975.

Madden, R. A., and V. Ramanathan, Detecting climate change due to increasing carbon dioxide, *Science, 209,* 763–768, 1980.

Manabe, S., and R. J. Stouffer, Sensitivity of a global climate model to an increase of CO_2 concentration in the atmosphere, *J. Geophys. Res., 85,* 5529–5554, 1980.

Mitchell, J. M., Jr., Recent secular changes of global temperature, *Ann. N. Y. Acad. Sci., 95,* 235–250, 1961.

Offenborn, W., and H. Grassl, Time-dependent ocean surface temperature variations caused by an increase of CO_2 content, *Contrib. Atmos. Phys., 54,* 107–122, 1981.

Oort, A. H., Global atmospheric circulation statistics, 1958–1973, *NOAA Prof. Paper 14,* U.S. Government Printing Office, Washington, D. C., in press, 1982.

Ramanathan, V., The role of ocean-atmosphere interactions in the CO_2 climate problem, *J. Atmos. Sci., 38,* 918–930, 1981.

Ramanathan, V., M. S. Lian, and R. D. Cess, Increased atmospheric CO_2: Zonal and seasonal estimates of the effect on the radiation energy balance and surface temperature, *J. Geophys. Res., 84,* 4949–4958, 1979.

Robock, A., A latitudinally dependent volcanic dust veil index, and its effect on climate simulations, *J. Volcanol. Geotherm. Res., 11,* 67–80, 1981.

Schneider, S. H., and S. L. Thompson, Atmospheric CO_2 and climate: Importance of the transient response, *J. Geophys. Res., 86,* 3135–3147, 1981.

Stefanick, M., Space and time scales of atmospheric variability, *J. Atmos. Sci., 38,* 988–1002, 1981.

Wainstein, L. A., and V. D. Zubakov, *Extraction of Signals From Noise,* translated from Russian by R. A. Silverman, Prentice-Hall, Englewood Cliffs, N. J., 1962.

Wallace, J. M., and D. S. Gutzler, Teleconnections in the geopotential height field during the northern hemisphere winter, *Mon. Weather Rev., 109,* 784–812, 1981.

Weare, B. C., Temperature statistics of short-term climatic change, *Mon. Weather Rev., 107,* 172–180, 1979.

Wigley, T. M. L., and P. D. Jones, Detecting CO_2-induced climatic change, *Nature, 292,* 205–208, 1981.

Yamamoto, R., and M. Hoshiai, Recent change of the northern hemisphere mean surface air temperature estimated by optimal interpolation, *Mon. Weather Rev., 107,* 1239–1244, 1979.

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