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A Metric for Assessing the Degree of Device Nonlinearity and Improving Experimental Design[1]

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Abstract — **We propose a metric that quantifies the degree of nonlinearity of state-space trajectories for purpose of improving the selection of collections of measurement data in the development of behavioral models. We illustrate the method with an off-the-shelf amplifier.**

Index Terms — **Behavioral model, large-signal measurements, metric, nonlinearity, state-space trajectory.**

I. INTRODUCTION

In this paper, we investigate ways of determining whether or not a device is linear from its state-space trajectories, and quantify the degree of nonlinearity with a time-domain-based metric. We then show how this metric leads to an improved method of sampling measured data for the development of behavioral models. We demonstrate the new sampling scheme with a measurement-based model, and demonstrate that it quantitatively improves model accuracy.

The state-space of a device-under-test (DUT) is usually multi-dimensional [1]. Since the dependency on the terminal voltages V_1 and V_2 often dominates, we limit the proposed metric in this work to the (V_1,V_2) plane. A common way of plotting the voltage trajectories of an amplifier is shown in the left plot of Fig. 1. The figure plots the simulated output voltage V_2 of an amplifier on the vertical axis as a function of the input voltage V_1 of the amplifier, under large-signal excitation.

Each trajectory in Fig. 1 corresponds to a single RF cycle of a three-tone multisine excitation with a different input-voltage level. The multisine excitation had an RF carrier frequency of 800 MHz, and we sampled the IF period equidistantly in time to collect the set of RF trajectories. Ref. [2] explains this procedure in detail.

As the input voltage level increases, the trajectories grow larger and become distorted, indicating significant nonlinearity. We see clearly from inspection that the device of Fig. 1 is not linear at the higher input voltage levels. However, determining exactly when the nonlinearity sets in, or how to quantify the degree of nonlinearity is not easy. In this paper, we suggest a way of normalizing these trajectories in order to detect and quantify the degree of nonlinear behavior, and to demonstrate the usefulness of this normalization with a modeling example.

II. NORMALIZED STATE-SPACE TRAJECTORIES

To illustrate our method, we plotted in the left plot of Fig. 2 the (V_1,V_2) trajectories of a 50 Ω transmission line with an electrical length of 110 degrees. Due to the distributed nature of the transmission line, the trajectories are ellipses. Since the curves do not lie on top of each other, we cannot tell by inspection whether the device is truly linear.

We can better tell whether the device is nonlinear by using the scaling properties of linear devices. If we scale the input signal of a linear DUT, its response at the output will be scaled by a similar amount. Utilizing this property, we normalized the (V_1,V_2) trajectories with respect to the peak voltage a_{1p} of the incident traveling voltage wave a_1 in the right side plots of Figs. 1 and 2.

For the 50 Ω transmission line, the result of this normalization is shown on the right plot of Fig. 2. We see that the normalized trajectories are identical, allowing us to identify it easily as linear. In fact, the normalized trajectories of any linear device will always be identical, giving us an unambiguous way of identifying linear devices from their trajectories. However, in the right plot of Fig. 1, we see that the normalized trajectories of our amplifier are not identical, from which we conclude that the amplifier is nonlinear.

III. METRIC FOR STATE-SPACE TRAJECTORIES

We can use the normalized trajectory plots to quantify the degree of nonlinearity of a device. We begin by choosing a reference trajectory corresponding to an input amplitude small enough that the device behaves linearly, but large enough to measure the input amplitude accurately (i.e., well above the noise floor).

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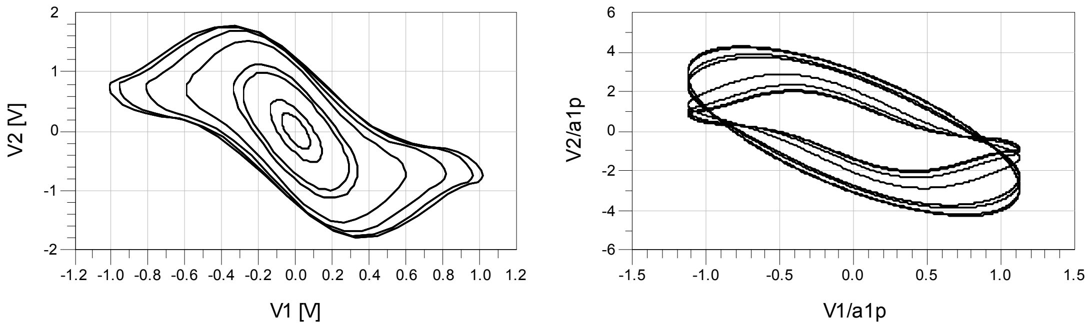

Fig. 1. Unnormalized (left) and normalized (right) trajectories for a simulated amplifier.

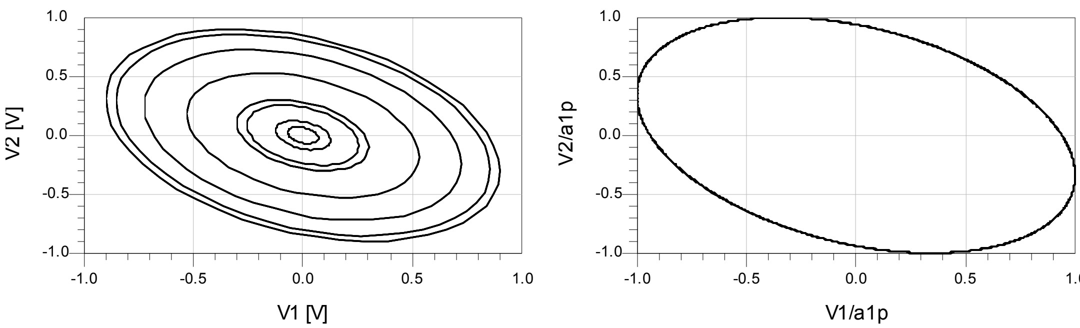

Fig. 2. Unnormalized (left) and normalized (right) trajectories for a short 50 Ohm transmission line.

Next, we calculate the root-mean-square (RMS) value of the orthogonal distance between each normalized trajectory (corresponding to one RF period) and the normalized reference trajectory. To facilitate the calculation, we first phase align each cycle.

If the device is linear, the normalized trajectories will coincide, and the RMS values of these orthogonal distances are zero. On the other hand, if the device is not acting linearly, the normalized trajectories will not coincide, and this will be reflected by nonzero RMS distances between the normalized trajectories and the normalized reference trajectory. A higher RMS value indicates a higher degree of nonlinearity.

Our metric yields a value of zero for the transmission line of Fig. 2. Figure 3 illustrates this metric for the amplifier of Fig. 1. As the peak incident voltage a_{1p} increases, so does the value of sigma.

This metric is defined in the time-domain to have the logical connection to the state-space concept. Whereas a frequency-domain metric could also be defined to express the degree of nonlinearity, the interpretation in terms of state-space (coverage) is no longer straightforward.

IV. Application to Behavioral Modeling

The behavioral modeling scheme of [2] is based on matching a state-space model to measured trajectories. To limit computation time, the number of trajectories used must be restricted. We applied our time-domain-based metric to optimize the selection of these trajectories. The goal was to minimize the number of trajectories while still capturing the nonlinear behavior of the device. Intuitively, we expect that fewer trajectories should be used over the operating ranges where the device is acting linearly, and that a denser grid of trajectories is required where the device is acting nonlinearly.

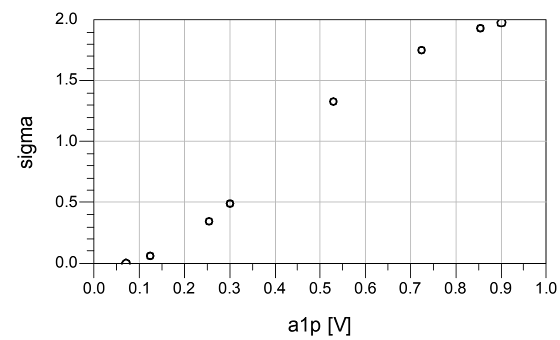

Fig. 3. RMS metric as a function of peak incident voltage for the simulated amplifier of Fig. 1.

As an experimental test case, we used an off-the-shelf coaxial amplifier (different from the simulated amplifier of Fig. 1), which we excited with a nine-tone multisine excitation at 800 MHz. Each tone of the multisine excitation had constant magnitude and phase (chosen to be

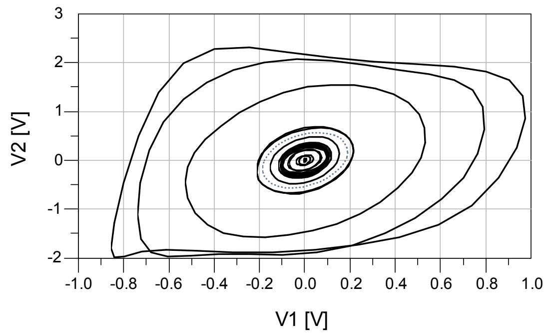

Fig. 4. Twenty temporally equidistant unnormalized trajectories for the measured amplifier. The reference trajectory is gray dotted.

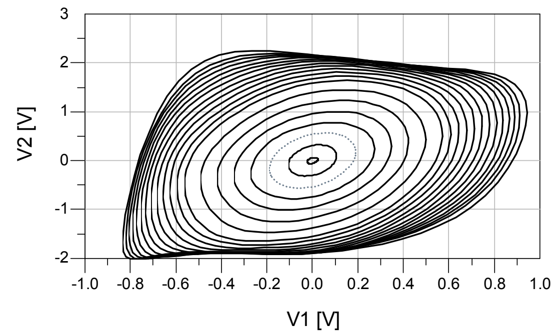

Fig. 5. Twenty unnormalized trajectories for the measured amplifier chosen according to the uniform-valued selection scheme. The reference trajectory is gray dotted.

zero degrees at the calibration reference plane). Twenty RF trajectories based on the equidistant sampling of the IF period (called 'temporally equidistant sampling' in the following) are shown in Fig. 4 and the corresponding RMS values are plotted in Fig. 6. Due to the limited dynamic range of our measurements, we selected the gray dotted trajectory in Fig. 4 as the reference trajectory, rather than the trajectory with the smallest power.

Figure 5 shows another twenty trajectories chosen to more uniformly spread the RMS values of our metric over its total range. We call this the "uniform-valued" selection scheme. We added several trajectories at low RMS values, since the fitting function used to represent the device's behavior would be forced to interpolate strongly if no trajectories at low amplitudes had been included. In other words, the final criterion we used to select suitable trajectories for the uniform-valued scheme was based on a combination of the variation in RMS values, as well as distance between trajectories. This scheme yields better coverage than the temporally equidistant scheme. As we will see later, our proposed method can improve the accuracy of models based on measured data.

Figure 6 compares the RMS values of our metric for the two sampling schemes, with values associated with the temporally equidistant scheme of Fig. 4 represented by triangles, and values associated with the uniform-valued scheme of Fig. 5 represented by circles. The high and rather noisy RMS values from lower a_{1p} are due to the limited dynamic range of the measurement. In Fig. 6, we also notice large jumps between values of our metric using the temporally equidistant scheme of Fig. 4 at the higher input voltages.

We constructed two models to evaluate the influence of the sampling scheme, and compared them in Tables I and II. The model labeled "temporal" corresponds to the temporally equidistant sampling scheme of Fig. 4, while the model labeled "uniform" corresponds to the uniform-valued scheme illustrated in Fig. 5. The number of chosen

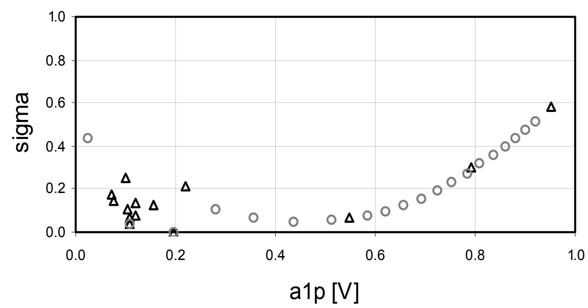

Fig. 6. Values of RMS metric for the temporally equidistant sampling scheme (black triangles) and the scheme yielding approximately uniform distribution of RMS values (gray circles).

trajectories was ten, which means that we used every other trajectory shown in Figs. 4 and 5. The models were trained using an artificial neural network (ANN) with one hidden layer and six hidden neurons.

The two models were subsequently subjected to several test excitations, all of which differ from those used to construct the models. Tables I and II compare measurements and model predictions for two of these test excitations. The tables list values for b_2 corresponding to two of the fundamental tones, and to one spectral component of an intermodulation product. The test excitations are a 17-tone multisine with a constant-amplitude and Schroeder phase spectrum (Table I), and a nine-tone multisine with a constant-amplitude and Schroeder phase spectrum (Table II). In a Schroeder phase spectrum [3], the tones have a particular phase relationship that ensures a low crest factor.

A smaller vector difference indicates better agreement, so that the tables show a clear improvement in the accuracy of the models when the trajectories are selected with the uniform-valued scheme, based on our metric. As the two examples show, the level of improvement depends on the actual experimental conditions.

We also constructed models based on 20 trajectories, but the difference between the two sampling approaches became minor. The reason is that the level of interpolation becomes smaller with increasing number of trajectories. However, note that, as stated in the beginning, we prefer to use fewer trajectories in order to maintain the modelling efficiency.

V. Conclusion

We developed a normalization technique for state-space trajectory plots that allows immediate detection of nonlinear device behavior. We then formulated a temporal metric that expresses the degree of nonlinearity of state-space trajectories. By sampling the state-space trajectories to give uniform distribution in metric values, we obtained higher model accuracy for a very small number of trajectories, compared to equidistant sampling in time. However, we also found that this advantage diminishes with an increasing number of trajectories, as the fitting functions yield higher-quality interpolations. These techniques may be useful for identifying and quantifying nonlinear device behavior, and for obtaining the greatest information possible about the device behavior with the smallest number of measurements (or simulations).

Since this work focused on the (V_1,V_2) voltage plane, a next step would be to generalize this formalism for a state-space of higher dimensions.

Acknowledgement

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	MAGNITUDE			PHASE (degrees)		
	measurement (dBm)	vector difference (dB)		measurement value	phase difference	
		temporal	uniform		temporal	uniform
$b_{2,800.0MHz}$	3.25	-31.24	-41.75	-129.99	1.31	-0.36
$b_{2,800.8MHz}$	3.00	-35.15	-44.58	86.61	1.51	0.72
$b_{2,799.0991MHz}$	-25.45	-38.91	-42.04	138.40	41.23	1.47

Table I. Measured results and corresponding differences between predictions and measurements for the two models. The first two entries correspond to values of b_2 at two of the excitation frequencies, while the third entry corresponds to the value of b_2 at an intermodulation product frequency. The test excitation is a 17-tone multisine test signal having a constant-amplitude and a Schroeder phase spectrum, and a frequency spacing of 100 kHz.

	MAGNITUDE			PHASE (degrees)		
	measurement	vector difference (dB)		measurement	phase difference	
	(dBm)	temporal	uniform	value	temporal	uniform
$b_{2,800.0MHz}$	4.73	-30.57	-44.10	35.91	2.36	0.46
$b_{2,800.8002MHz}$	4.77	-28.69	-35.83	124.76	2.18	1.31
$b_{2,798.9998MHz}$	-31.16	-46.83	-52.19	-141.22	31.37	-8.98

Table II. Measured results and corresponding differences between predictions and measurements for the two models. The first two entries correspond to values of b_2 at two of the excitation frequencies, while the third entry corresponds to the value of b_2 at an intermodulation product frequency. The test excitation is a 9-tone multisine test signal having a constant-amplitude and a Schroeder phase spectrum, and a frequency spacing of 200 kHz.