Reciprocity Relations for On-Wafer Power Measurement[1]

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Abstract

The implications of expressions relating the forward and reverse transmission coefficients of a waveguide junction derived from the Lorentz reciprocity condition are explored. The two terms in the relation, the phase of the reference impedance in the guide and a new reciprocity factor, lead to an asymmetric scattering parameter matrix when one of the transmission lines connected to the junction is lossy.

Introduction

This work explores the general relationship between the forward and reverse transmission coefficients of a reciprocal waveguide junction, as discussed in detail in [1]. In [1] and [2], the constraint placed by the Lorentz reciprocity condition on a junction connecting two uniform waveguides and consisting only of reciprocal media is derived. The result, which is based upon a new circuit theory applicable to lossy hybrid modes such as those found in coplanar waveguide (CPW) or microstrip lines, is

$$\frac{S_{21}}{S_{12}} = \frac{K_1}{K_2}\,\frac{1-jIm(Z_{r1})/Re(Z_{r1})}{1-jIm(Z_{r2})/Re(Z_{r2})}\;, \tag{1}$$

where S_{ij} are the scattering parameters (S-parameters) of the junction and Z_{r1} and Z_{r2} are the reference impedances at the two ports of the junction. The S-parameters and reference impedances are defined in [2]. In (1) the reciprocity factor K_n is defined as

$$K_n \equiv \frac{\overline{p}_{on}}{{p_{on}}^*}\;, \tag{2}$$

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where

$$p_{on} \equiv \int_{\sigma_n} \boldsymbol{e}_n \times \boldsymbol{h}_n^* \cdot \hat{\boldsymbol{n}} \, dS, \tag{3}$$

$$\overline{p}_{on} \equiv \int_{\sigma_n} \boldsymbol{e}_n \times \boldsymbol{h}_n \cdot \hat{\boldsymbol{n}} \, dS, \tag{4}$$

and $\mathbf{e}_n$ and $\mathbf{h}_n$ are the modal transverse electric and magnetic fields across the guide. The surface of integration σ_n is coincident with the reference plane in the nth guide, and $\hat{\mathbf{n}}$ is the unit vector normal to σ_n and in the direction of propagation in that guide. A time dependence $e^{j\omega t}$ is also assumed.

For the actual S matrix, the reference impedance Z_{rn} equal to the characteristic impedance Z_{on}. The constraint placed by (1) on the actual S matrix is determined by setting $Z_{rn}=Z_{on}$.

Partially Filled Waveguide

If the phase of the electric field is constant across a waveguide, then the magnitude of the reciprocity factor K for that guide must be 1. In some waveguides the phase of the electric field of the propagating mode is not constant. A waveguide partially filled with a lossy dielectric is an example.

The magnitude of the reciprocity factor of the dominant mode of a rectangular waveguide partially filled with a lossy dielectric was calculated following Harrington [3] and is plotted in Figure 1. The continuity of the normal component of the electric displacement across the air-dielectric boundary forces the electric field to change phase across that boundary. This results in a complex reciprocity factor with magnitude less than 1, as illustrated in Figure 1. If the lossy dielectric which partially fills this guide is terminated so as to form a junction connecting to a hollow rectangular waveguide of the same dimensions, application of (1) to that junction shows that, even when all reference impedances are chosen to be real, the forward and reverse transmission coefficients are unequal.

Coaxial Lines

The phases of the electric and magnetic fields are nearly constant across many common guides. Coaxial lines, hollow rectangular and circular waveguide, and, to a lesser extent, quasi-TEM lines are examples. Thus we expect the magnitude of the reciprocity factor to be nearly 1 in these guides.

The reciprocity factor of 2.4 mm coaxial air lines was investigated using the calculation technique of Daywitt [4], which rigorously includes the penetration of fields into lossy metal conductors. The phases of the electric and magnetic fields of this guide are nearly constant and the magnitude of K is nearly 1 at low frequencies. At 50 GHz, the highest frequency at which the 2.4 mm line is useful, the magnitude of K deviated from 1 by less than 3×10^{-10}. Thus in coaxial lines the phase of the characteristic impedance is the only significant factor in (1).

Marks and Williams [5] noted that the characteristic impedance of coaxial air lines varies greatly at low frequencies where, in the limit, the phase angle of Z_0 approaches -45°. Thus the contribution of the phase of the impedance in (1) cannot be ignored at low frequencies. This is illustrated in Figure 2.

The Experimental Determination of $|S_{21}/S_{12}|$

The magnitudes of S_{21} and S_{12} of a waveguide junction may in principle be determined directly from microwave power measurements. The procedure begins with the measurement of the power transferred from a reflectionless source into a reflectionless power meter. Then port 1 of the junction is connected to the source and port 2 to a second reflectionless power meter. The ratio of the two powers is $|S_{21}|^2$. $|S_{12}|^2$ may also be measured by reversing the experiment. The quotient $|S_{21}/S_{12}|$ then tests the reciprocity condition. If the experiment is performed on a reflectionless junction and $|S_{21}| \neq |S_{12}|$, the difference in the measured power ratios is entirely due to the preferential absorption of power traveling in one of the two directions within the junction.

In [6] we reported a similar experiment for a junction connecting a 2.4 mm coaxial line and a coplanar waveguide (CPW) line. The waveguide junction was a microwave probe, and power from a microwave power source was transferred through it to a thermistor bead mounted in a short section of CPW.

In the actual experiment reported in [6], the product $S^{o}_{21}S^{o}_{12}$ was determined by the two-tier TRL de-embedding technique, allowing the ratio $|S^{o}_{21}/S^{o}_{12}|$ to be determined without a reverse power measurement. Furthermore, neither the microwave source nor the thermistor bead was reflectionless. To take that into account, the transducer efficiency η of the microwave probe and the short section of CPW line which it contacted, given by

$$\eta \equiv \frac{P_L}{P_A}, \tag{5}$$

was measured. Here P_A is the power available from the source and P_L is the power delivered to the load. The transducer efficiency η is the equivalent of the transducer power gain described in [7] or the inverse of the transducer loss described in [8]. In the experiment, P_A was determined by first connecting the source to a calibrated coaxial sensor head and measuring the power dissipated in the sensor head. Then the reflection coefficients of the source and sensor head were measured and P_A calculated from the data. P_L was determined by a dc-substitution technique.

The transducer efficiency of the probe (including the short section of CPW line) is related to its pseudo-scattering parameters by

$$\eta = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2-2Im(\Gamma_L)Im(Z_{r2})/Re(Z_{r2}))}{|(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L)-S_{21}S_{12}\Gamma_S\Gamma_L|^2}, \tag{6}$$

where Γ_S and Γ_L are the reflection coefficients of the microwave source and thermistor bead, respectively, and Z_{r2} is the reference impedance at the CPW port at which the thermistor bead is attached. Rearrangement of (6) then allows us to write $|S_{21}/S_{12}|$ strictly in terms of measured quantities:

$$\left|\frac{S_{21}}{S_{12}}\right| = \frac{\eta\ |(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L)-S_{21}S_{12}\Gamma_S\Gamma_L|^2}{|S_{21}S_{12}|(1-|\Gamma_S|^2)(1-|\Gamma_L|^2-2Im(\Gamma_L)Im(Z_{r2})/Re(Z_{r2}))}. \tag{7}$$

In the squares of Figure 3, we have plotted $|S_{21}/S_{12}|$ for the case in which both reference impedances are chosen equal to the corresponding characteristic impedance. In the experiment, S_{11}, S_{22}, and $S_{21}S_{12}$, the scattering parameters of the intervening probe and line, were measured using the two-tier multi-line TRL de-embedding technique of Marks [9]. The characteristic impedance Z_{r2} of the CPW was determined from its propagation constant using the technique of Marks and Williams [5]. The agreement is good, and $|S^o_{21}/S^o_{12}|$ deviates significantly from 1, especially at the low frequencies. At very low frequencies the prediction from (1) approaches $1/\sqrt{2}$ because the phase angle of Z_{o2} approaches -45° [5].

For comparison we have also plotted $|S_{21}/S_{12}|$, represented by circles in the figure, after transformation to a real reference impedance. The measured data plotted in Figure 3 are compared to the predictions of (1) under the assumption that $|K_1|=|K_2|=1$ (see dashed and solid lines in Figure 3). The agreement is again quite good.

Conclusions

We explored a general condition relating the forward and reverse transmission coefficients of a reciprocal junction connected to uniform but otherwise arbitrary waveguides. The condition differs from the usual relation equating the two transmission coefficients in that it involves a reciprocity factor and the phase angle of the reference impedances in each guide connected to the junction.

An example of a rectangular waveguide partially loaded with a lossy dielectric showed that the magnitude of the reciprocity factor may deviate significantly from 1 in some circumstances. In coaxial lines constructed with lossy metals, the magnitude of the reciprocity factor is nearly 1, but the phase of the characteristic impedance must be considered at low frequencies.

We presented experimental evidence for coplanar lines showing that the ratios of the magnitudes of the actual forward and reverse transmission coefficients of a microwave probe deviate significantly from 1 in practical measurement situations. This has many practical implications for on-wafer measurements in which the reciprocity condition is used to determine the forward and reverse transmission coefficients of from their measured product.

References

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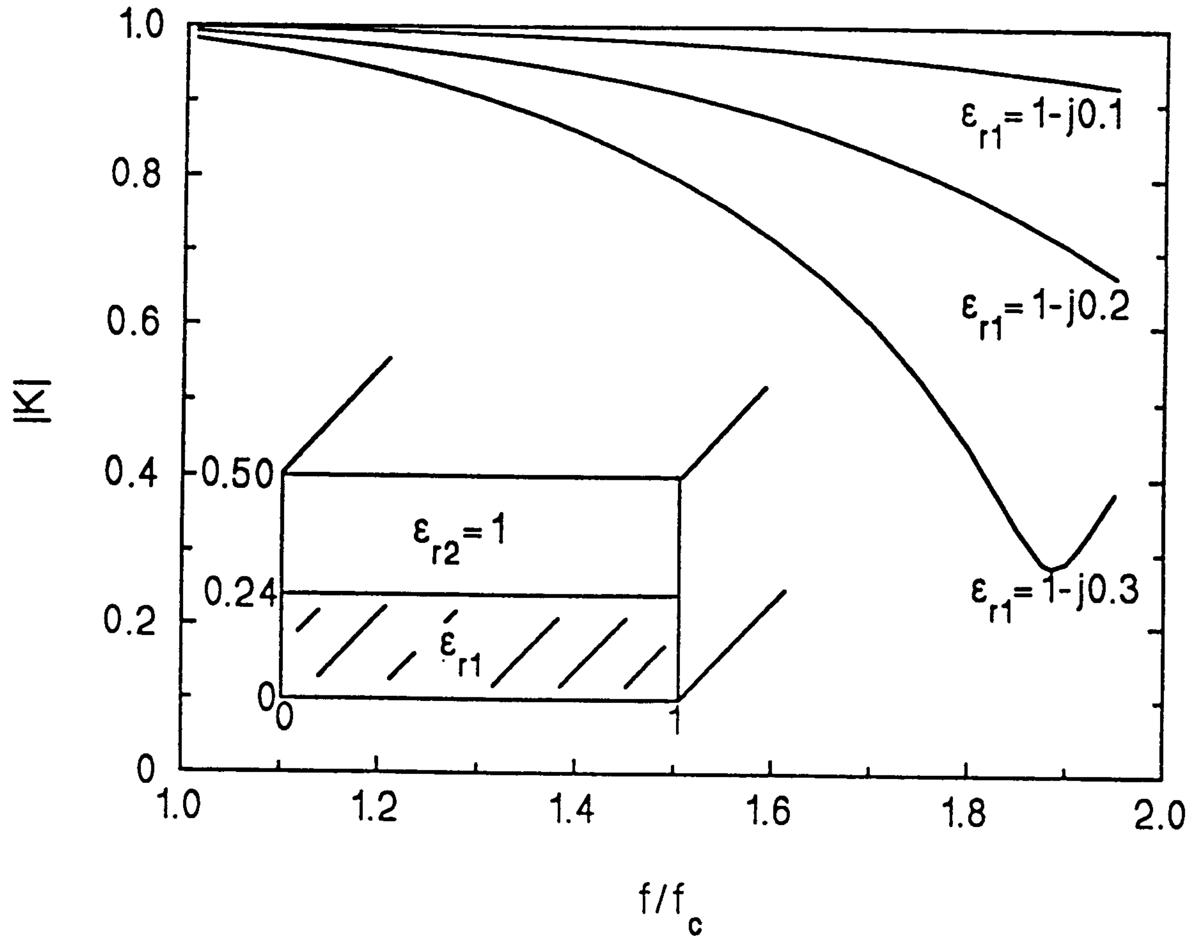

Figure 1. The magnitude of the reciprocity factor for the dominant mode in a waveguide partially filled with a lossy dielectric. The frequency is normalized to the cutoff frequency of the mode in the empty guide.

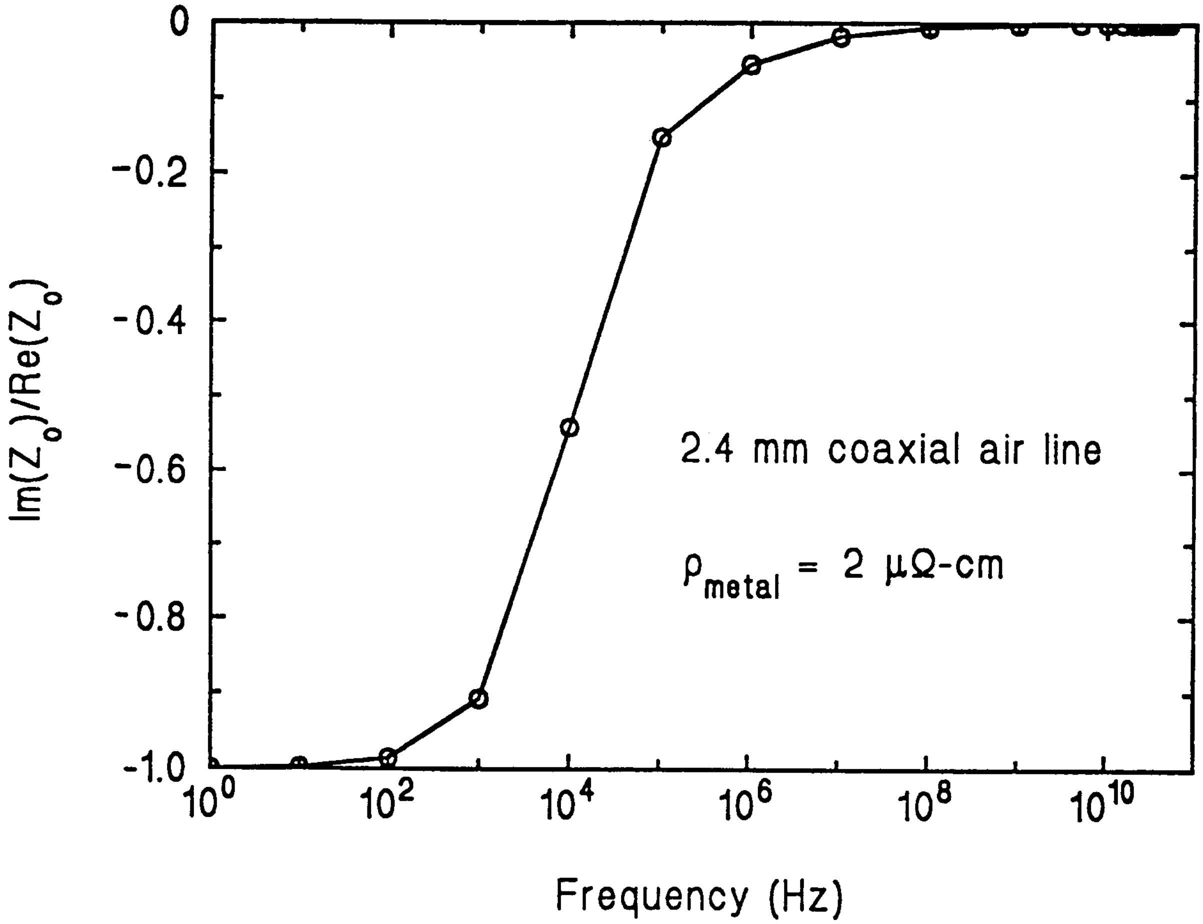

Figure 2. $Im(Z_o)/Re(Z_o)$, equal to the tangent of the phase of Z_o, for a 2.4 mm coaxial line with a center conductor of diameter of 1.042 mm and metal resistivity of 2 μΩ·cm. The plotted values were calculated using the results of Daywitt [4].

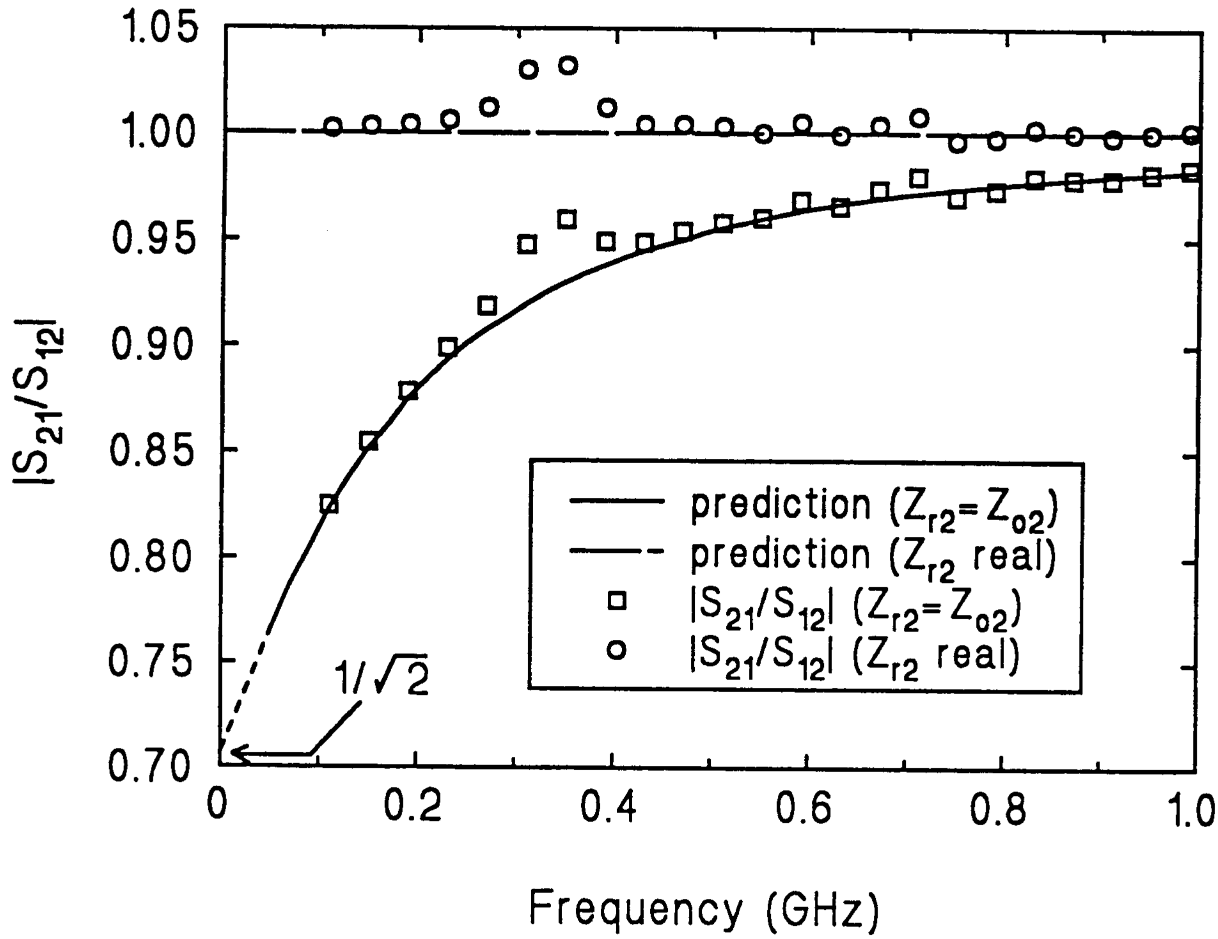

Figure 3. Measurements of $|S_{21}/S_{12}|$ (with $Z_{r2}=Z_{o2}$ and Z_{r2} real) based on (7) compared with the values calculated from (1) under the assumption $|K_1|=|K_2|=1$. The calculated and measured results agree closely, and $|S_{21}/S_{12}|$ deviates significantly from unity, especially for complex Z_{r2}. At very low frequencies the prediction using $Z_{r2}=Z_{o2}$ approaches $1/\sqrt{2}$ because the phase angle of Z_{o2} approaches $-45°$ [5].