Comments on "Conversions Between *S*, *Z*, *Y*, *h*, *ABCD*, and *T* Parameters which are Valid for Complex Source and Load Impedances"

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In his recent article [1], Frickey presents formulas for converting between *Z*, *Y*, *h*, and *ABCD* parameters and a peculiar form of the scattering parameters *S* and transmission parameters *T* which are not in common use among microwave theorists or engineers. The article, along with its reference, defines the scattering and transmission matrices to relate a set of parameters known in the microwave literature as *power* waves [2].

The scattering and transmission matrices conventional in microwave circuit theory relate the *traveling* waves in a transmission line. These matrices are referred to as scattering and transmission matrices by, among many others, [3],[4], [5], and [6]. The traveling waves are counter propagating wave solutions of Maxwell's equations in a transmission line with a dependence in the direction of propagation z of $e^{\pm i\gamma z}$ where γ is the propagation constant of the mode. The traveling wave amplitudes have the property that if a source is placed in an infinite line they decay exponentially away from the source. The ratios of the traveling wave amplitudes are measured by the slotted line method and by the thru-reflect-line vector network analyzer calibration [6].

The differences between the traveling and power waves are great. For example, the traveling wave reflection coefficient of a short is always -1 regardless of the characteristic impedance Z_o of the mode. The Smith chart provides a convenient graphical method of transforming between the traveling wave reflection coefficients and impedances. It is easily seen from equations (12) and (13) of [1] that the power wave reflection coefficient of a short is $-Z_n^*/Z_n$ where Z_n is the power wave normalizing impedance. Thus, except for the case where Z_o is real, the power waves never correspond to the traveling waves. In addition, the Smith chart can not be used to graphically transform between the power waves and impedances except when Z_n is real.

No microwave instrumentation or calibration known to these authors measures power waves. Thus if the equations of [1] were used to determine impedance parameters from the measured scattering parameters, which relate traveling waves [6], significant errors would result. To illustrate this we calculated the impedance of a of a small lumped resistor embedded in a printed coplanar

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waveguide from the measured scattering parameters in the transmission line using the power wave transformation of [1] under the assumption that $Z_n = Z_o$. The scattering parameters of the resistor were measured using the multiline TRL calibration [7] and Z_o was determined with the technique of [8] and [9]. The resistor impedance determined from the power wave transformation is shown in dashed lines in Fig. 1. The resulting impedance is not even remotely related to the anticipated electrical behavior of the small resistor except at very high frequencies where the characteristic impedance of the coplanar lines becomes nearly real. Using other assumptions, such as Z_n constant and real, does not improve the situation.

The appropriate transformation for this measurement problem relates the traveling waves to the voltages and currents. This transformation for Z_o complex is given in [6]. The resistor impedance determined from this transformation is shown in solid lines in Fig. 1 and closely tracks the resistor's anticipated behavior. As expected, the real part of the resistor impedance is roughly equal to its measured dc resistance $R_{dc} = 59.3\ \Omega$ marked on the left of the graph. The imaginary part of the resistor impedance is small and approaches zero linearly at low frequencies.

It is certainly possible to design circuits with power waves. In [6] we provide transformations between the measured traveling wave scattering parameters and either impedance parameters or, equivalently, scattering parameters with respect to a real reference impedance, both useful in the conventional design process. From these parameters it is possible to generate the power wave scattering parameters. In [1] Frickey checks his formulas against a commercial simulator which supports just this type of analysis. The designer should be careful, however, to follow the design rules peculiar to the power wave scattering parameters [2] and to keep clearly in mind the distinction between these and the conventional scattering parameters most commonly used by microwave engineers and theorists.

References

1. Dean A. Frickey, "Conversions Between *S*, *Z*, *Y*, *h*, *ABCD*,and *T* Parameters which are Valid for Complex Source and Load Impedances",*IEEE Trans. Microwave Theory and Tech.*, pp. 205-211, February 1994.

2. K. Kurokawa, "Power waves and the scattering matrix," *IEEE Trans. Microwave Theory and Tech.*, pp. 194-202, March 1965.

3. R. E. Collin, *Foundations for Microwave Engineering*. New York: McGraw-Hill, 1966.

4. N. Marcuvitz, *Waveguide Handbook*. New York: McGraw-Hill, 1951.

5. D. M. Kerns and R. W. Beatty, *Basic Theory of Waveguide Junctions and Introductory Microwave network Analysis*. Oxford: Pergamon Press, 1967.

6. R. B. Marks and D. F. Williams, "A general waveguide circuit theory," *J. Res. Natl. Inst. Stand. Technol.*, vol. 97, pp. 533-561, Sept.-Oct. 1992.

7. R. B. Marks, “A multiline method of network analyzer calibration,” *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 1205-1215, July 1991.

8. R. B. Marks and D. F. Williams, “A general waveguide circuit theory,” *J. Res. Natl. Inst. Stand. Technol.*, vol. 97, pp. 533-561, Sept.-Oct. 1992.

9. D. F. Williams and R. B. Marks, “Transmission line capacitance measurement,” *IEEE Microwave Guided Wave Lett.*, vol. 1, pp. 243-245, Sept. 1991.

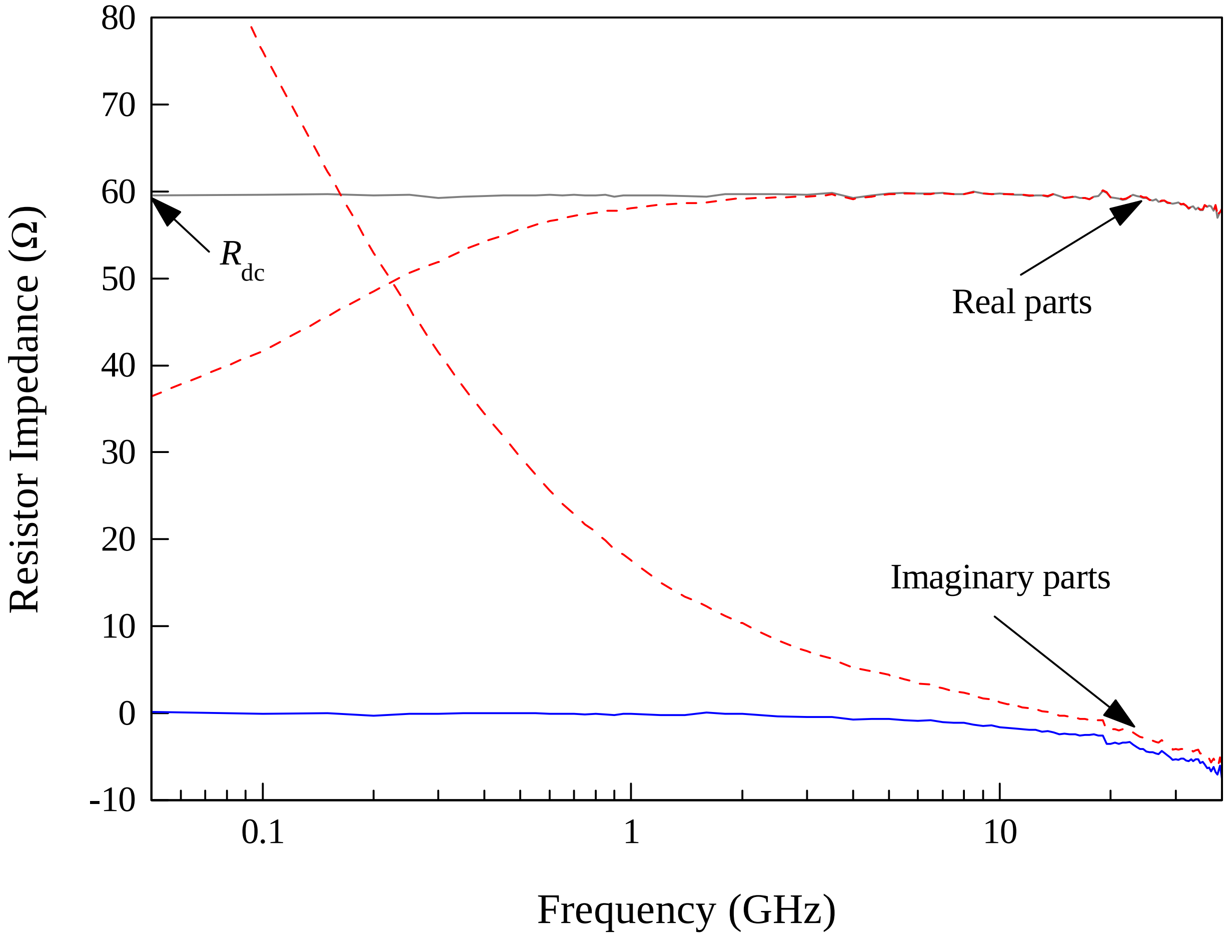

Figure 1. The impedance of a small lumped resistor calculated from scattering parameters measured by the thru-reflect-line calibration. The impedance calculated from the power wave transformation of [1] is labeled with dashed lines. The impedance calculated from the traveling wave transformation of [6] is labeled with solid lines.

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