Transmission Line Capacitance Measurement

Dylan F. Williams and Roger B. Marks

Abstract—**The capacitance of coplanar lines is measured with two new techniques, one utilizing the resistance of the line and the other that of a resistor embedded in the line. The results of both measurements agree closely with calculations. A technique for directly comparing the capacitance of two similar transmission lines is also demonstrated. The relevance of these measurements to the determination of characteristic impedance is discussed.**

I. Introduction

This letter proposes two methods for the measurement of C, the capacitance per unit length of a transmission line. A knowledge of C is required in a recently-presented method [1], [2] for the determination of the frequency-dependent characteristic impedance using a measurement of the propagation constant. The methods described here are applicable to quasi-TEM lines but not necessarily to other types of waveguides.

For any transmission line mode, the (real) generalized circuit parameters C, G, R, and L are defined in terms of the characteristic impedance Z and propagation constant γ by [1], [3]:

$$\frac{\gamma}{Z} \equiv j\omega C + G \tag{1}$$

and

$$\gamma Z \equiv j\omega L + R. \tag{2}$$

While the phase of Z is a fixed property of the transmission line, its magnitude is arbitrary. In [1], Z is defined via a power-voltage relationship. The magnitude of Z therefore depends on the choice of path over which the voltage is defined. As can be seen from (1) and (2), the normalization can also be fixed by the specification of either C, G, R, or L.

In one of the methods proposed here, the normalization of the voltage is specified through R, as we demand that the low-frequency limit of R is equal to the measured dc resistance of the transmission line. The other measurement of C is based on the measured dc resistance of a load resistor. Both of these measurements implicitly define the endpoints of the voltage path integral to lie on the conductors.

The essence of the schemes for measurement of Z is the assumption that C is nearly independent of frequency and metal conductivity and that $G/\omega C$ is negligible. These conditions, which are supported in [1], are assumed valid here.

Manuscript received April 30, 1991. This work was supported in part by the Naval Air Systems Command under Contract N00019-89-C-0150 and in part by the NIST Director's Reserve.

The authors are with the National Institute of Standards and Technology, Mail Code 813.01, 325 Broadway, Boulder, CO 80303.

IEEE Log Number 9102355.

While the theory and techniques discussed in this letter are applicable to a number of quasi-TEM guides, we here determine approximate values of C for a set of CPW lines fabricated under the MIMIC Phase 3 program, which we shall call the MIMIC lines. These lines had an average center conductor width of 74.3 μm, gap between the center conductor and the ground planes of 48.5 μm, and gold metalization thickness of approximately 1.5 μm. The lines were fabricated on a semi-insulating GaAs substrate with an approximate thickness of 500 μm. The assumptions made concerning the behavior of C and G may actually break down at extremely low or high frequencies in the MIMIC lines, although we think they are good between 5 MHz and 40 GHz, the frequency range considered in this investigation.

II. Determination of a Line's Capacitance from Its dc Resistance

The imaginary part of the product of (1) and (2) is

$$RC + LG = Re\left(\frac{\gamma^2}{j\omega}\right). \tag{3}$$

G is small for many CPW lines at microwave frequencies [1], [2], and calculations demonstrate that typically $LG \ll RC$. If R is approximately equal to the dc resistance per unit length of the line R_{dc}, an easily measurable quantity, then (3) becomes

$$C \approx \frac{1}{R_{dc}} Re\left(\frac{\gamma^2}{j\omega}\right). \tag{4}$$

The propagation constant of the MIMIC lines was measured from 5 MHz to 1 GHz by performing a thru-reflect-line (TRL) calibration using the algorithm of Marks [4]. The approximate values of C calculated from (4) are plotted in the curve of Fig. 1 labeled "$R \rightarrow R_{dc}$."

These approximate values are expected to deviate significantly from the actual value except at low frequencies, where the current in the conductors is highly uniform and the approximation $R \approx R_{dc}$ is valid. For this reason, a least squares fit of a quadratic to the approximation of C was used to extrapolate to dc. This extrapolated value of C is listed in the first row of Table I.

At extremely low frequencies R should become independent of frequency; that is, its first derivative should be zero there. The measured values of C in Fig. 1 indicate that R has a nonzero first derivative even at the lowest frequencies at which we made measurements. This suggests that the extrapolation is in error by a small amount, no more than a few percent in this case.

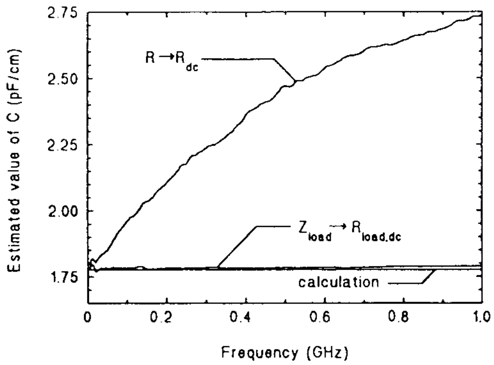

Fig. 1. Approximate values of C from the techniques described in this work are plotted. Assumptions made in deriving the approximations in the measured curves improve at low frequencies.

TABLE I
LOW-FREQUENCY CAPACITANCE

Assumption	C	$(G/\omega C)$	C/C_{50}
$R \to R_{dc}$	1.787 pF/cm		1.016
$Z_{load} \to R_{load,dc}$	1.783 pF/cm	< 0.004	1.014
Calculated	1.776 pF/cm	0.0015	1.010

The measured and calculated values of capacitance are compared. The measurements are compared to $C_{50} = 1.759$ pF/cm, the nominal capacitance we calculated for a 50 Ω CPW line on a GaAs substrate.

III. Determination of Line Capacitance from a Lumped Load

For a small lumped resistor at low frequencies we expect that,

$$Z\frac{1+\Gamma_{load}}{1-\Gamma_{load}} \equiv Z_{load} \approx R_{load,dc}, \tag{5}$$

where $R_{load,dc}$ is the dc resistance of the lumped load and Γ_{load} is its complex measured reflection coefficient. Substitution of (5) into (1) gives

$$C[1 - j(G/\omega C)] \approx \frac{\gamma}{j\omega R_{load,dc}} \frac{1+\Gamma_{load}}{1-\Gamma_{load}}. \tag{6}$$

The TRL calibration algorithm of Marks [4] was used to determine γ and the reflection coefficient of a small lumped resistor embedded in the MIMIC lines from 5 MHz to 1 GHz. The approximate values of C calculated from (6) are plotted in the curve of Fig. 1 labeled "$Z_{load} \to R_{load,dc}$." A least-squares fit of a quadratic to the measured values of C was used to extrapolate the approximate values of C to dc where Z_{load} is expected to most nearly equal $R_{load,dc}$. This extrapolated value of C is listed in the second row of Table I.

Approximate values of $G/\omega C$ are also obtained with this technique. It was difficult to extrapolate $G/\omega C$ to dc because of noise in the data, but it appeared that $G/\omega C$ was less than 0.004.

IV. Calculation from the Dimensions of the Line

The measured dimensions of the lines were used in conjunction with a spectral domain technique to calculate the capacitance of the related lossless line with zero metal thickness. The dielectric constant was assumed to be 12.9 [5] and an approximate correction [6] was applied to account for the metal thickness. The calculated capacitance of the lines, plotted in the curve in Fig. 1 labeled "calculation," was approximately constant, as anticipated. This calculated capacitance should be approximately equal to the capacitance of the actual line, since the theory of [1] and [2] indicates that the capacitance is an extremely weak function of metal conductivity. $(G/\omega C)$ for the lines was estimated to be 0.0015 using [7] and the GaAs loss tangent of 1.6×10^{-3} [8]. These values are listed in the third row of Table I.

V. Direct Comparison of Lines

We fabricated in our laboratory a set of CPW lines that were nominally identical to the MIMIC lines. Our lines had a measured center conductor width of 74.0 μm and gap between the center conductor and outer ground planes of 48.6 μm.

Denoting quantities related to our lines with the subscript "NIST," we have from (1),

$$\frac{C[1 - j(G/\omega C)]}{C_{NIST}[1 - j(G/\omega C)_{NIST}]} = \left(\frac{Z_{NIST}}{Z}\right)\left(\frac{\gamma}{\gamma_{NIST}}\right). \tag{7}$$

To approximate Z_{NIST}/Z, we assumed that the reflection coefficient Γ_{NIST} of our lines measured with respect to a calibration performed in the MIMIC lines was determined solely by the difference in characteristic impedance. We then computed Z_{NIST}/Z from

$$\Gamma_{NIST} \approx \frac{(Z_{NIST}^2 - Z^2)\sinh(\gamma l_{NIST})}{2Z_{NIST}Z\cosh(\gamma l_{NIST}) + (Z_{NIST}^2 + Z^2)\sinh(\gamma l_{NIST})}, \tag{8}$$

where γl_{NIST} is the product of γ_{NIST} and the length of the NIST line. Equation (7) was then used to determine the ratio C/C_{NIST}.

As long as the lines are similar enough that the assumption implicit in (8) remains valid, this method determines C/C_{NIST} at all frequencies. However, since we expect this ratio to be nearly constant, we have averaged the data, which were taken from 50 MHz to 40 GHz. The average was weighted using frequency-dependent error estimates derived by attributing all of the error in C/C_{NIST} to errors in γ_{NIST} and Γ_{NIST}. The required unknowns were determined by fitting to actual measured variations for lines of widely varying center conductor widths.

The ratios of C/C_{NIST}, as determined by calculation, measurement, and this direct comparison method, are listed in Table II; the table shows close agreement.

VI. Conclusion

Various methods for determining the capacitance per unit length C of quasi-TEM lines were found to approximately agree with each other and with calculation. Once C is known, the magnitude and phase of the characteristic

TABLE II
CAPACITANCE RATIO

Technique	C	C_{NIST}	C/C_{NIST}
Calculation	1.776 pF/cm	1.759 pF/cm	1.010
$R \to R_{dc}$	1.787 pF/cm	1.769 pF/cm	1.010
Direct comparison			1.008

The ratio of the capacitances of the MIMIC and NIST lines determined by different methods are compared.

impedance of the line can be determined from its propagation constant, an easily measured quantity. Applications in circuit design and in the comparison of measurements based on transmission line calibration standards suggest themselves.

REFERENCES

[1] R. B. Marks and D. F. Williams, "Characteristic impedance determination using propagation constant measurement," *IEEE Microwave Guided Wave Lett.*, vol. 1, no. 7, pp. 141-143, June 1991.

[2] D. F. Williams and R. B. Marks, "The interpretation and use of S-parameters in lossy lines," presented at *36th Automatic RF Techniques Group Conf. Proc.*, Monterey, CA, Nov. 1990.

[3] J. R. Brews, "Transmission line models for lossy waveguide interconnections in VLSI," *IEEE Trans. Electron Dev.*, vol. 33, pp. 1356-1365, Sept. 1986.

[4] R. B. Marks, "A multiline method of network analyzer calibration," *IEEE Trans. Microwave Theory Tech.*, vol. 39, no. 7, pp. 1205-1215, July 1991.

[5] M. N. Afsar, J. R. Birch, and R. N. Clarke, "The measurement of the properties of materials," *Proc. IEEE*, vol. 74, no. 1, pp. 183-199, Jan. 1986.

[6] T. Kitazawa, Y. Hayashi, and M. Suzuki, "A coplanar waveguide with thick metal coating," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, no. 9, pp. 604-608, Sept. 1976.

[7] K. C. Gupta, R. Garg, and I. J. Bahl, *Microstrip Lines and Slotlines.* Dedham, MA: Artech House, 1979, pp. 68-69.

[8] C. A. Balanis, *Advanced Engineering Electromagnetics.* New York: Wiley, 1989, p. 62.