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Frequency-Dependent Transmission Line Parameters

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Recent theoretical developments have resulted in the rigorous definition of equivalent circuit parameters for non-TEM transmission lines. This summary provides a brief review of the theory and presents calculated and measured results illustrating the behavior of these parameters. This behavior directly affects the electrical performance of electronic chip interconnects.

In a uniform transmission line, we denote the normalized electric and magnetic fields of a single mode propagating in the $+z$ direction as $\boldsymbol{e}e^{-\gamma z}$ and $\boldsymbol{h}e^{-\gamma z}$, where $\boldsymbol{e}$ and $\boldsymbol{h}$ are independent of z. The integral of the Poynting vector over the cross section S gives the net complex power p_o crossing the transverse plane z=0 as

$$p_o \equiv \int_S \boldsymbol{e}_t \times \boldsymbol{h}_t^* \cdot \boldsymbol{z}\, dS \ . \tag{1}$$

The microwave voltage v_o can be defined by the path integral

$$v_o \equiv -\int_{path} \boldsymbol{e}_t \cdot d\boldsymbol{l} \ , \tag{2}$$

where the arbitrary path lies in S. We define the modal characteristic impedance as

$$Z_o \equiv |v_o|^2 / p_o^* \ . \tag{3}$$

Z_o and p_o are of identical phase, uniquely determined by the structure of the mode [1]. The magnitude of Z_o depends on the path used to define v_o.

In order for the real equivalent circuit parameters C, L, G, and R to be related to γ and Z_o in the conventional way, they must be defined so as to satisfy

$$j\omega C + G \equiv \frac{\gamma}{Z_o} \tag{4}$$

and

$$j\omega L + R \equiv \gamma Z_o \ . \tag{5}$$

Although Eqs. (4) and (5) provide unique definitions of the four circuit parameters, it is possible to cast them into the form [1]

$$C = \frac{1}{|v_o|^2}\left[\int_S \varepsilon' |\boldsymbol{e}_t|^2 dS - \int_S \mu' |h_z|^2 dS\right], \tag{6}$$

$$L = \frac{1}{|i_o|^2}\left[\int_S \mu' |\boldsymbol{h}_t|^2 dS - \int_S \varepsilon' |e_z|^2 dS\right], \tag{7}$$

$$G = \frac{\omega}{|v_o|^2}\left[\int_S \varepsilon'' |\boldsymbol{e}_t|^2 dS + \int_S \mu'' |h_z|^2 dS\right], \tag{8}$$

$$R = \frac{\omega}{|i_o|^2}\left[\int_S \mu'' |\boldsymbol{h}_t|^2 dS + \int_S \varepsilon'' |e_z|^2 dS\right]. \tag{9}$$

Here $\varepsilon \equiv \varepsilon' - j\varepsilon''$ and $\mu \equiv \mu' - j\mu''$. In passive media, the four real components ε', ε'', μ', and μ'' are all non-negative.

Equations (6) through (9) have many computational applications due to the quadratic form in which the fields appear. They also permit apportioning the circuit parameters among the cross-sectional regions. For example, L is the sum of an "external" inductance in the dielectric and an "internal" inductance in the imperfect metal.

For a lossless TEM line, G and R vanish, as do the second integrals in C and L. The remaining integrals in C and L are the conventional expressions for the capacitance and inductance per unit length, as given by Collin [2]. When the dielectric is lossy but μ'' is zero, the mode may remain TEM but a shunt conductance G, given by the first term of Eq. (8) as in [2], is present.

When the metal boundaries are lossy or the dielectric is inhomogeneous, the mode is non-TEM. The second integrals in C and L, absent in [2], are quadratic in the longitudinal fields and may, in some quasi-TEM cases, prove to be negligible compared to the first terms. The expressions for C and G in general include contributions due to fields inside the metal that are often unappreciated. A nonzero series resistance R, given by the second integral in Eq. (9), may also appear whenever e_z and ε'' are nonzero; the integral extends over a lossy dielectric as well as an imperfect conductor. Equation (9) reduces to a line-integral expression for R [2] when the surfaceimpedance approximation is invoked and the dielectric is lossless.

We have investigated the microwave equivalent circuit parameters of a lossy 2.4 mm coaxial airline. The calculations of Fig. 1, which include the effects of field penetration into the metal [3], show that the microwave capacitance C of a 2.4 mm coaxial line deviates only slightly from the electrostatic capacitance. The calculations also show that, in contrast to the conventional assumption, G is nonzero.

Figure 2 shows the contributions of the inner conductor, outer conductor, and air regions of a 2.4 mm airline to its total inductance L. At high frequencies, the contributions of the conductors are small compared to the air region contribution, which is nearly equal to the perfectly-conducting inductance L_0. At low frequencies, the contributions to L from the conductors become significant. As the fields fully penetrate the inner conductor, its contribution to L approaches the magnetostatic value of 50 nH/cm. In contrast, as the fields penetrate the outer conductor, L becomes dominated by the "internal" (metal) inductance of the outer conductor at low frequencies. The resistance of the airline approaches its magnetostatic value at low frequencies, and the resistance of the outer conductor becomes significant at high frequencies (see Fig. 3).

The equivalent circuit parameters of a coplanar waveguide line, calculated from data provided by Heinrich [4], are shown in Figs. 4 and 5. Again, R deviates from the line's measured dc resistance per unit length R_{dc} as ω increases, while L approaches the external inductance L_{ext}. At the low frequencies where the fields fully penetrate the conductors, R dominates over ωL and L exceeds L_{ext}.

In contrast, C and $G/(\omega C)$ are nearly constant with frequency, with C increasing by only 0.8% from its quasi-static value at the highest frequency plotted. At low frequencies, $G/(\omega C)$ is dominated by dielectric loss; it increases slightly at high frequencies due to transverse currents in the metal lines.

We have developed several techniques for determining the low-frequency limit of C from direct measurements [5]. Exploiting the nearly constant nature of C and the smallness of $G/(\omega C)$ leads to a method of determining Z_0 from γ using Eq. (4). γ can be determined from frequency-domain [6] or time-domain [7] measurements. The frequency-dependent L and R are then determined from Eq. (5). This is illustrated in Fig. 4, where these measurements are compared to the calculations. This measurement technique is particularly useful when the equivalent circuit parameters must be accurately determined but the material parameters are not well known.

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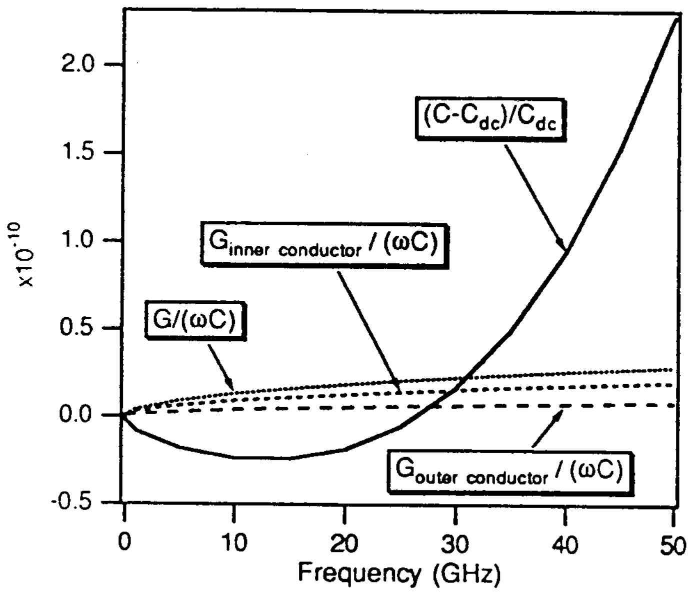

Figure 1: C and G for 2.4 mm coaxial line with metal conductivity 50 M S/m, computed by method of [3].

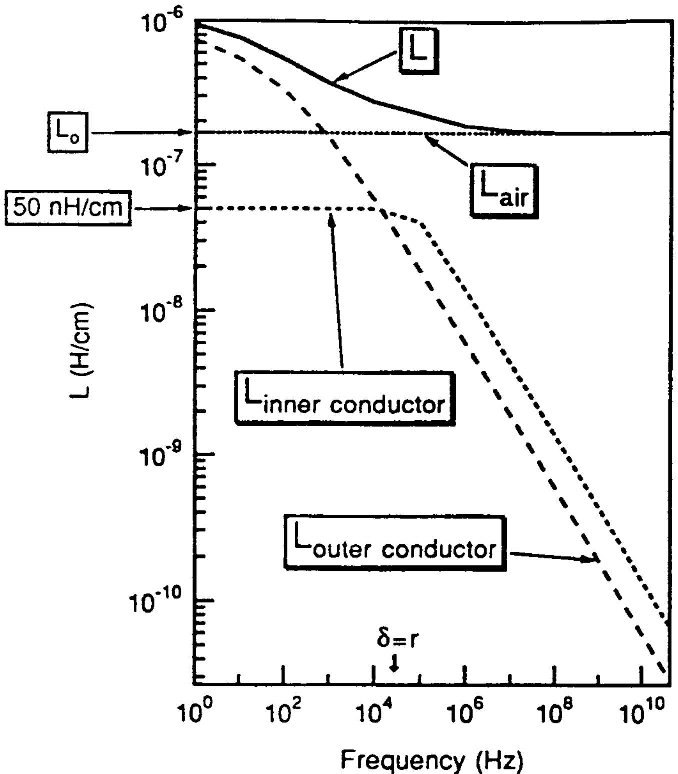

Figure 2: Contributions to L for line of Fig. 1. Arrow marks the frequency at which skin depth δ equals center conductor radius r.

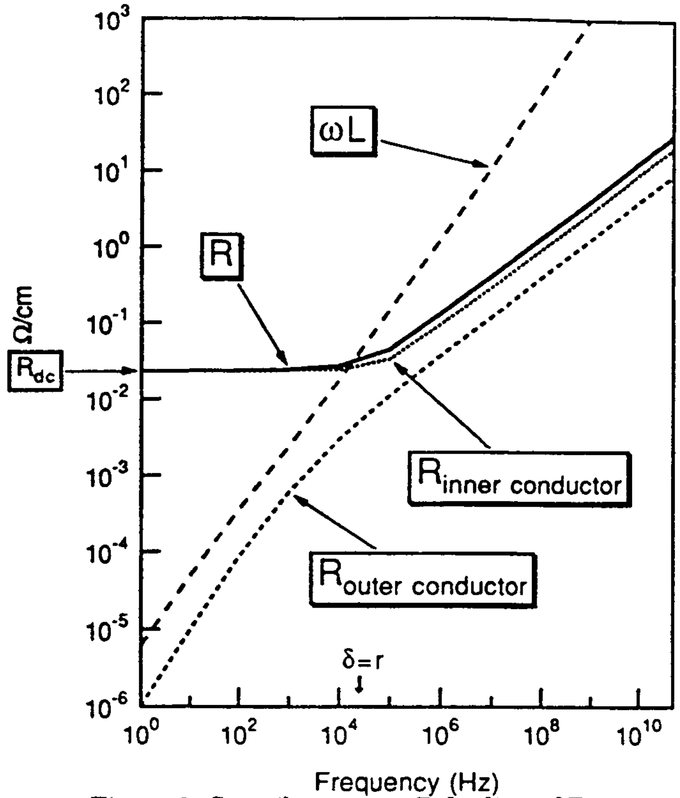

Figure 3: Contributions to R for line of Fig. 1. ωL is shown for comparison.

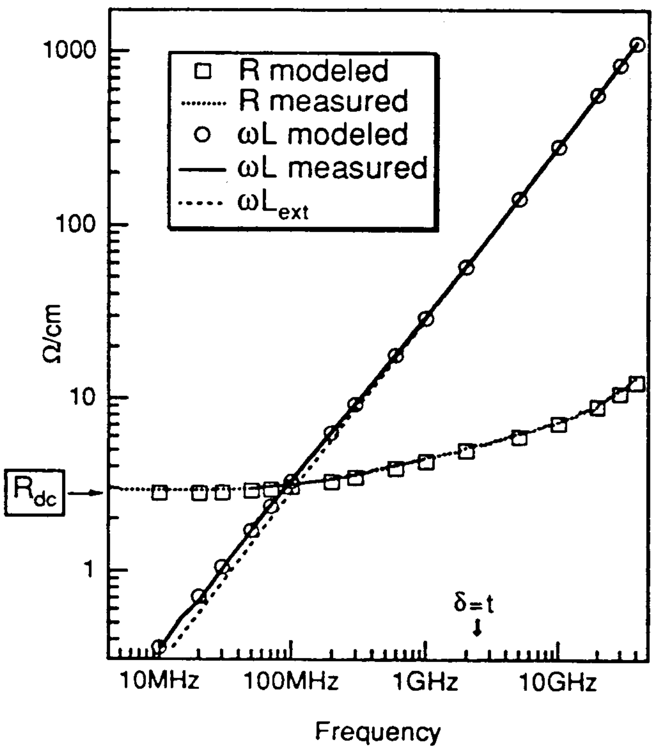

Figure 4: R and ωL for CPW line (see Fig. 5) from calculations of Heinrich [4] and from measurement. The estimate L_{ext} is L measured at 40 GHz.

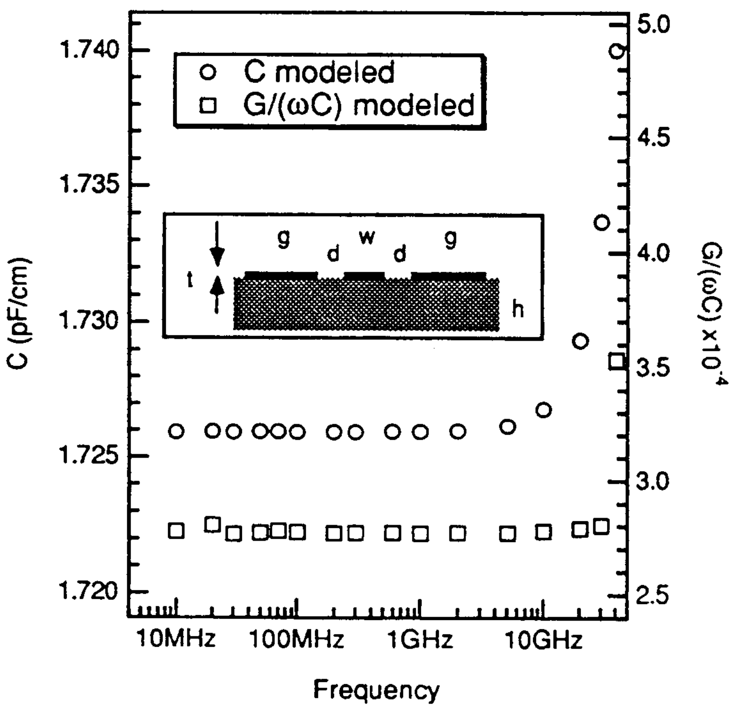

Figure 5: Calculated parameters of CPW line whose dimensions are shown in inset. Here w=71 μm, d=49 μm, g=250 μm, h=500 μm, and t=1.61 μm. The substrate has relative permittivity ε=12.9 and loss tangent 0.0003. The metal conductivity is 36.02 MS/m.