Characteristic Impedance Determination Using Propagation Constant Measurement

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Abstract—**A method is demonstrated where by the characteristic impedance of transmission lines may be easily determined from a measurement of the propagation constant. The method is based on a rigorous analysis using realistic approximations to account for the effects of imperfect conductors. Numerical studies indicate that high accuracy is possible, and experiments using coplanar waveguide demonstrate the advantage of the method in the interpretation of *S*-parameters.**

I. Introduction

Microwave automatic network analyzers may be precisely calibrated using the TRL method, in which the "reference impedance" is set by the characteristic impedance Z_o of the section of transmission line used as a standard. In the presence of significant conductor loss, Z_o is difficult to compute since it depends strongly on the frequency and conductivity [1]. *S*-parameter measurements taken with respect to an unknown, frequency-dependent reference impedance can be difficult to interpret. Furthermore, since Z_o is complex in the presence of loss, the measured reflection coefficient of a passive device may exceed unity in magnitude [2]. This further clouds the interpretation.

If Z_o can be determined, a complex impedance transformation can be used to shift to a constant, real reference impedance in which these *S*-parameter anomalies are eliminated. One approach to this problem is presented in [3], which argues that Z_o can be determined from a measurement of the propagation constant γ and knowledge of the "free-space capacitance." This idea is attractive since γ is readily determined during the TRL calibration. However, the relationship between Z_o and γ assumed by [3] is presented without supporting arguments, and the results surprisingly indicate that $Im(Z_o)$ is positive and that $Re(Z_o)$ falls at the lower frequencies instead of rising due to the internal inductance. Our analysis shows that [3] implicitly neglects the conductor loss, which is typically the dominant effect in planar lines.

This letter will demonstrate the feasibility of an alternative indirect measurement of Z_o through measurement of γ. The method arises from a careful analysis and, while approximate, can be quite precise. We demonstrate the validity of the approach with numerical studies of lossy coaxial cable and coplanar waveguide (CPW) as well as with CPW measurements.

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Some of the results discussed here have been presented previously in conference [4].

II. Theory

In any uniform transmission line, the normalized transverse electric and magnetic fields of a single mode propagating in the $+z$ direction will be denoted by $\boldsymbol{e}e^{-\gamma z}$ and $\boldsymbol{h}e^{-\gamma z}$, where $\boldsymbol{e}$ and $\boldsymbol{h}$ are independent of z. Integrating over the transverse across section S, we define the complex power carried by the forward-propagating mode as

$$p_o \equiv \int_S \boldsymbol{e} \times \boldsymbol{h}^* \cdot \boldsymbol{z}\, dS, \qquad (1)$$

where $\boldsymbol{z}$ is the longitudinal unit vector. Define the constant v_o, analogous to a voltage, by the integral

$$v_o = -\int_{\text{path}} \boldsymbol{e} \cdot d\boldsymbol{l} \qquad (2)$$

over some specified path. Finally, define the characteristic impedance by

$$Z_o \equiv |v_o|^2 / p_o^*. \qquad (3)$$

Note that its phase is equal to that of p_o, but p_o is independent of the phase of $\boldsymbol{e}$ and $\boldsymbol{h}$. Hence the phase of Z_o is not arbitrary but unique. The magnitude of Z_o depends on the path used to defined v_o.

The ratio and product of γ and Z_o can be expressed [5]

$$\frac{\gamma}{Z_o} = j\omega C + G \equiv \frac{j\omega}{|v_o|^2}\left[\int_S \epsilon |\boldsymbol{e}|^2\, dS - \int_S \mu^* |h_z|^2\, dS\right] \qquad (4)$$

and

$$\gamma Z_o = j\omega L + R \equiv \frac{j\omega}{|i_o|^2}\left[\int_S \mu |\boldsymbol{h}|^2\, dS - \int_S \epsilon^* |e_z|^2\, dS\right], \qquad (5)$$

where $i_o \equiv v_o / Z_o$. If the real circuit parameters C, G, L, and R can be determined, either (4) or (5) can be used to compute Z_o from a measurement of γ. Unfortunately, they are typically unknown. Our analysis of quasi-TEM lines [6], however, suggests some useful approximations. In particular, G is negligible ($G \ll \omega C$) when the substrate loss is low and the transverse currents in the conductors are weak, as is typically true except at very high frequencies. Thus (4)

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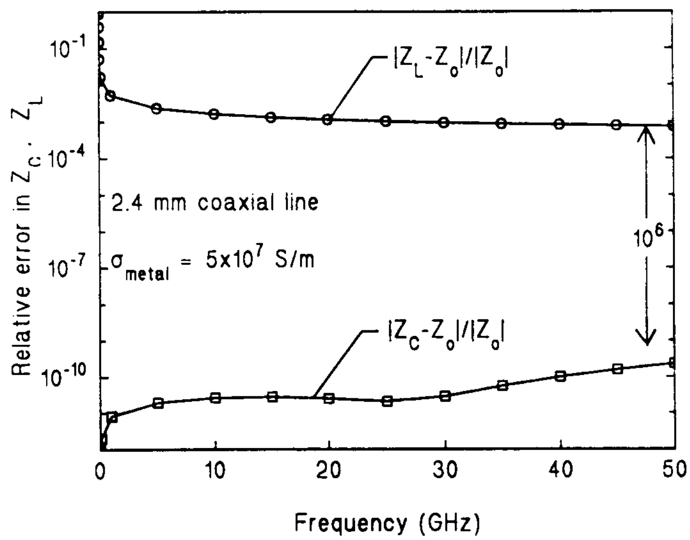

Fig. 1. Relative error in Z_C and Z_L, computed from calculated γ, with respect to calculated Z_o of a 2.4-mm coaxial line with conductivity near that of copper. All calculations were done using the method of Daywitt [7].

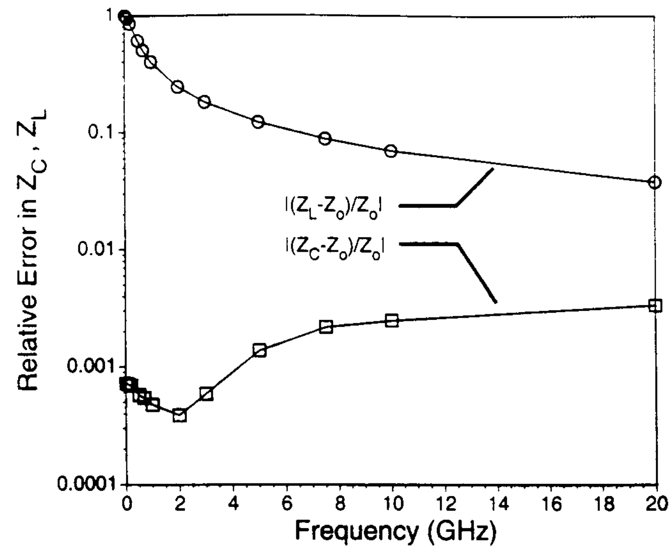

Fig. 2. Relative error in Z_C and Z_L for a CPW line of 0.7-μm gold on 500-μm GaAs. The center conductor is 73-μm wide, the ground plane 350 μm, and the gap between them 49 μm. Computations of Z_o and γ were provided by Heinrich.

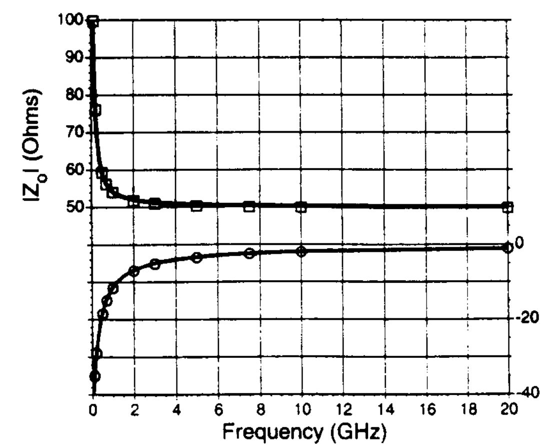

Fig. 3. Characteristic impedance of CPW line described in Fig. 2. Upper and lower curves plot $Re(Z_C)$ and $Im(Z_C)$, respectively, based on a measurement of γ. Plotted points, indicated by the squares and circles, were computed by Heinrich using a model of the same line.

allows the determination of the *phase* of Z_o from a measurement of the phase of γ, which in turn allows for the transformation to a real reference impedance. If, in addition, C is known, then (4) allows the determination of the *magnitude* of Z_o from γ. Since, as can be shown by analysis, the dependence of C on frequency and conductivity is typically quite weak, C may be well approximated by C_o, the dc-capacitance assuming perfect conductors.

In contrast, L and R depend strongly on the conductivity and frequency due to the varying currents inside the metal. Furthermore, the condition of small R ($R \ll \omega L$) may be violated at low frequencies. Therefore, the use of (5) to estimate either the phase or magnitude of Z_o is inherently problematic. Nevertheless, (5) is the basis of the method proposed in [3].

III. Examples

The validity of our new approach was confirmed in several ways. A numerical study of lossy coaxial cable was undertaken using the method developed by Daywitt [7]. Fig. 1 compares the estimate Z_c, as computed from (4) with the assumptions that $G = 0$ and $C = C_o$, to the calculated Z_o. The two values are virtually indistinguishable over the entire band. By contrast, Fig. 1 also illustrates the analogous estimate Z_L, which uses (5) and the assumptions that $R = 0$ and $L = L_o$, in effect the assumptions of [1]. Although the agreement with Z_o is fair at the high end of the band, it is entirely incorrect as the low end and is always worse than Z_C by many orders of magnitude.

A similar comparison for coplanar waveguide (Fig. 2) is based on computations of Z_o and γ performed by Heinrich using the method of [1]. Here the value of C at the lowest computed frequency (10 MHz) was used as an estimate of C_o. Again, the estimate Z_C is excellent, while Z_L is poor. The phase of Z_C is within 0.2 degrees of the phase of Z_o across the band.

In both coaxial and CPW calculations, the phase of Z_L is not even of the same *sign* as that of Z_o, Z_L having always a positive imaginary part. The characteristic impedance reported by [3] also had a positive imaginary part.

Experimental evidence comes from CPW measurements. Fig. 3 displays Z_C computed from a measurement of γ and an estimate of C_o. Also plotted are a number of values computed by Heinrich. The agreement is good, especially considering the drastic shift in both phase and magnitude at the low frequencies.

FIg. 4 shows the measured reflection coefficient of a small resistor terminating a length of CPW. At the high end of the band, the match is fairly good and the reflection coefficient is nearly real, as we expect of a resistor in a lossless line. On the other hand, the curve begins to deviate below about 5 GHz. Assuming that this deviation is caused not by a variation of the load impedance but by a change in Z_o, we used the estimate Z_C to transform to a real reference impedance. The transformed data, also shown in Fig. 4, demonstrate that, with a real, constant reference impedance, the resistor does indeed look like a real, constant-impedance load.

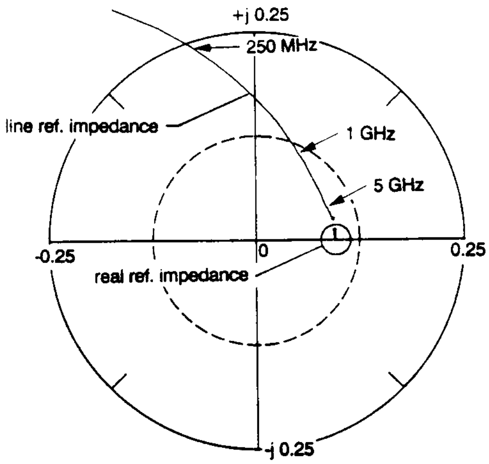

Fig. 4. Measured reflection coefficient, from 150 MHz to 15 GHz, of a small resistor attached to the CPW line described in Fig. 2. One impedance reference is the impedance Z_o of the line; the other is approximately real and constant.

We also performed an experiment in which a high-loss CPW line was terminated with a low-loss short circuit. The reflection coefficient of the offset short with Z_o as the reference impedance exceeded unity in magnitude. However, the reflection coefficient using a real reference impedance lay inside the unit circle.

IV. Conclusion

The characteristic impedance of a lossy transmission line may be readily determined using the propagation constant, which is easily and accurately measured with a vector network analyzer. Although the variation of Z_o is typically severe at low frequencies, an appropriate impedance transformation effectively eliminates the effects of the transmission line. The resulting impedance-transformed S-parameters better reflect the nature of the device under test.

Although Figs. 1 and 2 indicate that the accuracy of the measurement decreases somewhat with frequency, they also suggest that the accuracy will be very high for frequencies well into the millimeter-waveband. This is confirmed by our own measurements to 40 GHz, which are not shown in Figs. 3 and 4 simply for clarity of the display.

References

[1] W. Heinrich, "Full-wave analysis of conductor losses on MMIC transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 1468-1472, Oct. 1990.

[2] N. Marcuvitz, *Waveguide Handbook*. London: Peter Peregrinus, 1986, p. 27.

[3] J. S. Kasten, M. B. Steer, and R. Pomerleau, "Enhanced through-reflect-line characterization of two-port measuring systems using free-space capacitance calculation," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 215-217, Feb. 1990.

[4] D. F. Williams and R. B. Marks, "The interpretation and use of S-parameters in lossy lines," in *36th ARFTG Conf. Dig.*, Automatic RF Techniques Group, Monterey, CA, Nov. 29-30, 1990, pp. 84-90.

[5] J. R. Brews, "Transmission line models for lossy waveguide interconnections in VLSI," *IEEE Trans. Electron Devices*, vol. 33, pp. 1356-1365, Sept. 1986.

[6] R. B. Marks and D. F. Williams, "Applications of a universal microwave circuit theory," to be submitted to *IEEE Trans. Microwave Theory Tech.*

[7] W. C. Daywitt, "Exact principal mode field for a lossy coaxial line," *IEEE Trans. Microwave Theory Tech.*, accepted for publication.