Lumped-Element Models for On-Wafer Calibration*

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***Abstract*- We examine electrical models for lumped-element impedance standards used in on-wafer network-analyzer calibrations. We illustrate the advantages of using models that are complicated enough to replicate the actual electrical behavior of the lumped standards, but do not have more degrees of freedom than absolutely necessary.**

INTRODUCTION

We investigate several models for lumped resistors used in short-open-load-reciprocal/thru (SOLR) and open-short-load-thru (OSLT) on-wafer calibrations. We show that if the model used for the load is not general enough, the resulting lumped-element calibration will not accurately reproduce a multiline thru-reflect-line (TRL) calibration [1], which we treat as our reference calibration. We also show that if the model has too many degrees of freedom, it reproduces noise and other inaccuracies in the measurements on which the model is based, and is not suitable for extrapolation to higher frequencies.

The impedance of lumped-element standards used in on-wafer SOLR and OSLT calibrations must be well modeled if the calibrations are to be valid. Accurate on-wafer SOLR and OSLT calibrations at very high frequencies are based on models for the lumped standards that are derived from measurements performed with the accurate multiline TRL calibration. If the models used to define the lumped-element impedance standards do not correctly define the electrical behavior of the lumped-element standards, large measurement errors can result.

On the other hand, if there are too many degrees of freedom in the models for the lumped-element standards, the models will simply replicate noise and systematic errors in the TRL measurements on which they are based. Therefore, we need to determine an optimal model.

For simplicity, we will restrict our attention to models of lumped resistors, since these are more difficult to model than opens or shorts. In what follows, we will show that resistor models with insufficient degrees of freedom result in inaccurate calibrations. However, models with too many degrees of freedom unnecessarily reproduce measurement errors in the TRL calibrations on which they are based, and are often not well suited for extrapolation. For example, we will see that non-physical polynomial terms in polynomial fits are not statistically significant and do not improve the calibrations. Rather, they reduce our ability to extrapolate the model accurately to higher frequencies. Finally, we will show that an equivalent-circuit model and a low-order polynomial model yield statistical improvements over high-order polynomial models, take best advantage of available measurements, and are well suited for extrapolation.

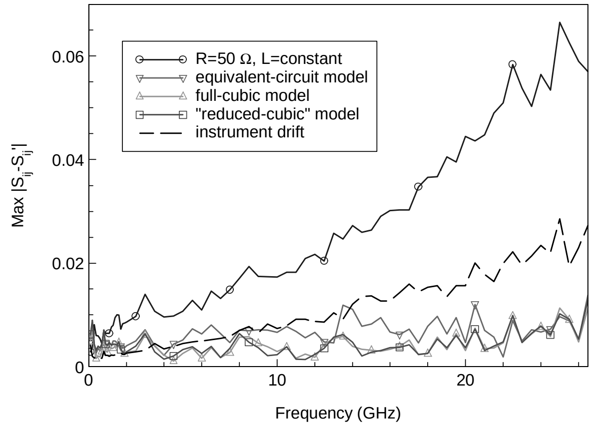

Figure 1. Error bounds for SOLR calibrations with different load standard definitions; all other standards are TRL-corrected.

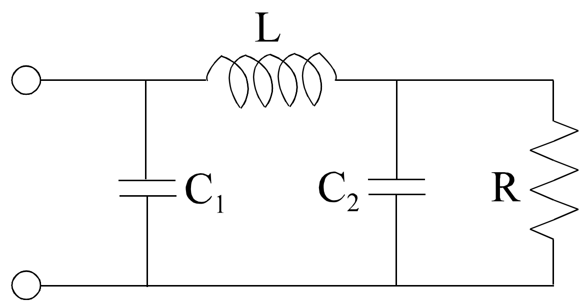

Figure 2. Equivalent-circuit model for lumped-element standards.

CALIBRATION COMPARISONS

We first performed a 50 Ω probe-tip multiline TRL calibration in coplanar waveguide (CPW). We then measured lumped-element standards on a commercial calibration substrate. We fitted our models to the TRL-corrected measurements of the lumped-element standards. Model fits were then used as inputs in a commercial software package to perform our comparison SOLR and OSLT calibrations [2]. The TRL-corrected open and short measurements were also used in the SOLR and OSLT calibrations following the work of Imparato et al. [3]. We performed a final TRL calibration to assess test-set drift and contact repeatability errors. All calibration comparisons were performed using the method of [4].

Figure 1 compares SOLR calibrations based on several models for the load standard to the TRL reference calibration. The most notable observation is that the typical model of constant resistance and inductance is the least accurate of any of the models tested, as shown in the curve marked with open circles. This was true despite our choice of an inductance value that minimized the difference between the SOLR and TRL calibrations. This occurred because this simple model does not have enough degrees of freedom to reproduce the frequency-dependent electrical behavior of the lumped-element resistor.

Figure 1 also compares calibrations based on three other models: a full-cubic polynomial fit, a “reduced-cubic” fit, and an equivalent-circuit model. In the full-cubic model the resistance is of the form $R_0+fR_1+f^2R_2+f^3R_3$ and the reactance is of the form $\omega L_1+\omega^2L_2+\omega^3L_3$. The reduced-cubic model fits the resistance to a simpler polynomial, $R_0+f^2R_2$, and the reactance to $\omega L_1+\omega^3L_3$. The equivalent-circuit model, shown in Figure 2, is an extension of the model developed in [5].

Figure 1 shows that all of these models yield similar results, despite their differing degrees of complexity. The full-cubic, reduced-cubic, and equivalent-circuit models result in error bounds comparable to that of the instrument drift. Thus, our results show that no improvement results from adding the extra, non-physical polynomial terms present in the full-cubic model.

LOAD MODEL ROBUSTNESS

In practice, the measurements used to develop models for lumped-element standards may be degraded by noise, systematic error, or other calibration and measurement problems that are

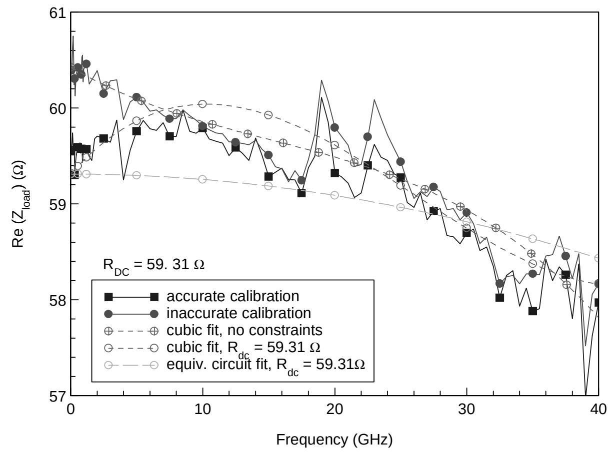

Figure 3. Effect of fitting cubic polynomials and our equivalent circuit to load data corrected with an inaccurate TRL calibration.

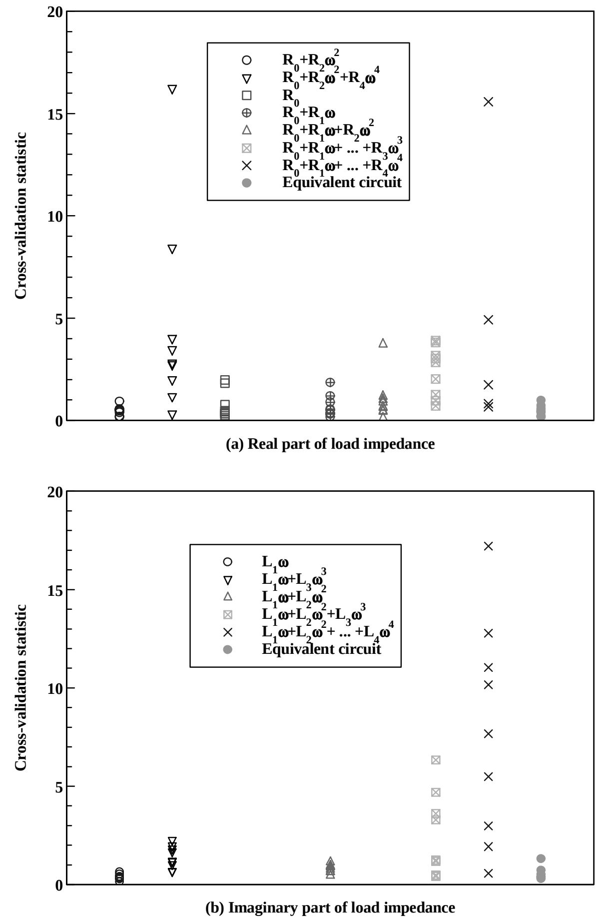

Figure 4.Cross-validation statistic for model predictions versus TRL-corrected measurements for different load models.

difficult to recognize or diagnose. Thus, model selection needs to be based on model robustness; i.e., how insensitive the model is to systematic or random measurement error. We simulated this degradation in our load measurements by correcting our data with two calibrations, one using all five delay lines in our CPW standard set, and a second using only the shortest two delay lines. We then fitted each corrected load measurement to a full-cubic polynomial and to our equivalent-circuit model while constraining the dc resistance. We also fitted an unconstrained full-cubic polynomial for comparison.

Figure 3 shows the results of these measurements for a thin-film resistor that terminates a CPW transmission line printed on a gallium arsenide calibration substrate. The curve marked with solid squares represents the load measurement corrected with respect to our reference TRL calibration, using five delay lines. The curve marked with solid circles is the equivalent data corrected with respect to a less-accurate TRL calibration using only the two shortest delay lines.

We fitted the models to the inaccurately-calibrated load data; these results are shown as dashed lines marked with open circles. We see that the full-cubic and equivalent-circuit models fit the accurate TRL measurement reasonably well, and we note that the root-mean-square estimation errors for the two fits are comparable.

However, at low frequencies both cubic fits display a non-zero slope in the resistance, whether or not we took advantage of the easily-measured dc load resistance to constrain the fits. If we took the Fourier transform of these frequency responses, the corresponding time response would be imaginary, which is not physically realizable. Qualitatively, we observe that the equivalent-circuit model is less sensitive to noise in the data. In the next section we examine this issue quantitatively.

STATISTICAL ANALYSIS

To ascertain the statistical validity of different potential lumped-element models for the load standard, we used the standard technique of cross-validation. In cross-validation, one studies how well the fit of a particular model to an estimation data set can predict the values of another set of validation data. The "best" model is the one that yields the closest agreement between the observed and predicted values of the validation data set. The independence between estimation and validation data sets can highlight such problems as over-fitting. We employed a variation of the k-fold cross-validation method [6].

To analyze our data, we fitted each of the models over the lower three quarters of the total frequency range. Based on this fit, we predicted the values in the upper quarter of the frequency range. The root-mean-square value of the differences between these observed and predicted values represents the calculated cross-validation statistic.

Figure 4 shows our cross-validation statistic for the real and imaginary parts of the load impedance, for each of nine loads, using different fitting strategies. The cross-validation experiment shows that the best fits to the real part of the impedance are achieved with the "ax^2+b" fit and the equivalent- circuit model. The best fits to the imaginary part of the impedance are achieved with a constant inductance (linear reactance) model or the equivalent-circuit model.

CONCLUSIONS

We found that a constant resistance in series with a constant inductance did not have enough degrees of freedom to model lumped-element loads. We also found that high-order polynomials fits cannot be extrapolated accurately, and often result in non-physical fits. However, we found that the low-order polynomial fits and equivalent-circuit models were both sufficiently robust and well suited for extrapolation to higher frequencies.

ACKNOWLEDGMENT

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