Accurate Experimental Characterization of Interconnects: A Discussion of "Experimental Electrical Characterization of Interconnects and Discontinuities in High-Speed Digital Systems"[1]

Roger B. Marks and Dylan F. Williams

Abstract—**This paper discusses two issues concerning the accuracy of electrical characterizations of interconnect transmission lines, particularly in regard to a recently published paper. The error in the characteristic impedance may be reduced through an alternative approximation to the capacitance of the transmission line. Furthermore, measurements of both the propagation constant and characteristic impedance, which are the two primary parameters characterizing the line, may be improved by the use of a well-conditioned algorithm.**

A recent paper[1] discusses a procedure for determining the characteristic impedance of lossy transmission lines using measurements of the propagation constant. However, a poor assumption and an ill-conditioned algorithm unnecessarily limit the accuracy of the method.

In order to determine the characteristic impedance Z_o, the authors make use of its relationship with the propagation constant γ, suggesting that their proposal is simply a modification of the relationship proposed in [1]. However, although the above paper[1] suggests that Z_o is *proportional* to γ, [1] presumes an *inverse* relationship between the two. It is more appropriate to consider the above paper[1] to be a modification of [2], since [2] makes use of the *exact* relationship:

$$Z_o = \frac{\gamma}{j\omega[C + G/j\omega]} \tag{1}$$

where ω is the angular frequency and C and G are the capacitance and conductance per unit length, as defined in [2]. The above paper modifies (1) by approximating the bracketed expression as $\varepsilon_{\text{eff}}(\infty)C_o$, where $\varepsilon_{\text{eff}}(\infty)$ is the relative effective permittivity "calculated at high frequencies"[1] and C_o is the capacitance per unit length of the line in "free space," that is, in the absence of the dielectric substrate.

The above paper[1] argues against C directly since "in a homogeneous region of unknown constituency (it) cannot be calculated." While this is undoubtedly true, it *is* possible to calculate C (and G as well) in a region of *known* constituency. Furthermore, as discussed in [3], C may be *measured* by various means; this eliminates the need for calculation. Of course, it is impossible to measure C_o without constructing a dielectric free line.

In order to determine the degree to which the approximation of the above paper is valid, we need to determine the relative effective permittivity $\varepsilon_{\text{eff}}(\omega)$ at "high frequencies." For microstrip lines, as used in the above paper, a high frequency limity of $\varepsilon_{\text{eff}}(\omega)$ exists and is equal to the relative permittivity ε of the substrate [4]. This may be what the authors mean by the "calculated" $\varepsilon_{\text{eff}}(\infty)$. However, since this limit is not approached except at frequencies much higher than those of the quasi-TEM region in which the microstrip is normally

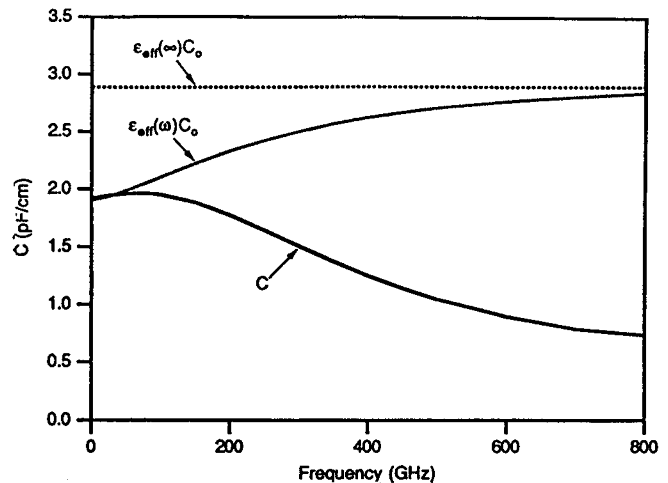

Fig. 1. Results of spectral domain calculations for microstrip (73 μm wide) on a GaAs ($\varepsilon = 12.9$) substrate (100 μm thick). The substrate sits on the bottom of a large conducting box, 4 mm wide and 1.1 mm high. C is the computed capacitance per unit length and $\varepsilon_{\text{eff}}(\omega)C_o$ is the estimate of C used by the above paper at high frequencies. The asymptotè $\varepsilon_{\text{eff}}(\infty)C_o$ is computed from the known limit $\varepsilon_{\text{eff}}(\infty) = \varepsilon = 12.9$.

operated, the product $\varepsilon_{\text{eff}}(\infty)C_o$ is typically a very poor estimate of the quasi-TEM value of C. This is illustrated in Fig. 1 by using spectral domain calculations for a thin, lossless microstrip.

In spite of the use of the term "calculated," it may be that the authors of the above paper intend $\varepsilon_{\text{eff}}(\infty)$ to mean a *measured* value of $\varepsilon_{\text{eff}}(\omega)$ at the high end of some frequency band. This definition, although it improves the approximation (see Fig. 1), remains problematic, for the estimate of C then depends strongly on the frequency ω at which $\varepsilon_{\text{eff}}(\omega)$ is evaluated. Fig. 1 shows the approximation improving as ω falls. However, conductor losses present in any practical line will modify the plot of Fig. 1, which ignores losses, by forcing $\varepsilon_{\text{eff}}(\omega)$ to grow dramatically as ω nears 0. This restricts the possibility of using the low frequency $\varepsilon_{\text{eff}}(\omega)$ instead of $\varepsilon_{\text{eff}}(\infty)$.

In contrast to the large discrepancies between these estimates and the actual values of C, Fig. 1 illustrates that the dc limit of C is a good estimate of C in the low frequency regime. This is the approximation used in [2].

An additional problem with the method of the above paper is the accuracy of the measurement of γ, which is essential to the determination of Z_o. It appears that γ was determined by the technique described in [1], which uses two transmission lines and was originally proposed in [5]. An analysis of that technique confirms the original authors' conclusion [5] that the algorithm is ill-conditioned when the difference $\Delta\varphi$ betweeen the electrical lengths of the two lines is approximately an integral multiple of π. This numerical phenomenon is apparently exhibited in Fig. 3 of the above paper, which demonstrates striking periodic irregularities in Z_o at frequencies roughly consistent with the reported line length.

Although the *algorithm* of the above paper,[1] [1], as well as [5] is ill-conditioned at certain frequencies, the actual problem of determining γ from the measurement of two transmission lines is

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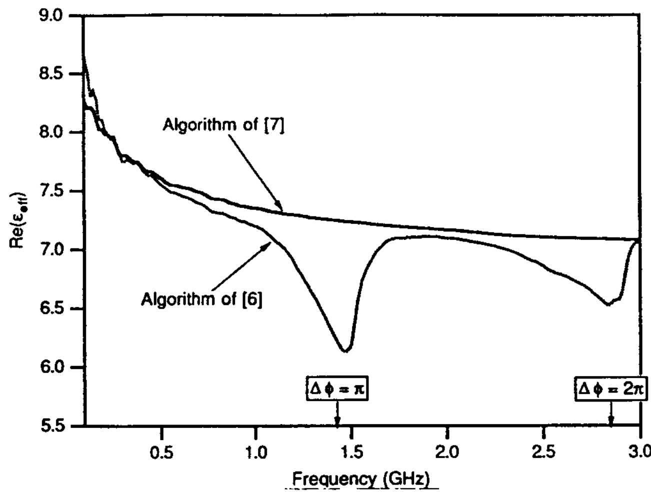

Fig. 2. Real part of effective permittivity determined from network analyzer measurements of two coplanar lines of lengths 40 and 0.54 mm, using the algorithms of [5] and [6]. The arrows indicate the frequencies at which the method of [5] is ill conditioned.

not. Well-conditioned approaches to this problem are well known; for example, [6] describes such an algorithm along with an error analysis demonstrating that it does not suffer from periodic ill-conditioning. Fig. 2 compares $\varepsilon_{\rm eff}$ of a coplanar waveguide transmission line as determined by the algorithms of [5] and [6] operating on identical measurement data. The arrows indicate the frequencies at which the algorithm of [5] is most poorly conditioned. Large discrepancies are apparent; these result in large errors in Z_o when using the method of the above paper. In contrast, [2] obtained a more accurate measurement of Z_o by using the well-conditioned algorithm of [6]. The ability of [6] to further reduce random error through the use of additional transmission lines was not used in generating the results of Fig. 2.

An additional reported drawback of [5] is that, as analysis demonstrates, it assumes that the fixtures are reciprocal. The method fails entirely if reciprocity is violated. Since the cascade matrix which represents the effects of network analyzer imperfections is generally nonreciprocal [7], the method of [5], unlike that of [6], is strictly limited to two-tier calibrations. This drawback does not affect the above paper, which uses two-tier calibrations exclusively.

Author's Reply[2] by Michael B. Steer[3]

I. Introduction

We thank Marks and Williams for their comments on our paper,[1] which presents experimental techniques for characterizing two- and three-terminal interconnects and discontinuities. The comments concern the development of (6) and (7) of our paper.[1] These equations are the mathematical implementation of a technique which C_o calculation and S-parameter measurements of two lengths (or a through and a single length) of a line to determine the complex effective permittivity of the line in the established skin-effect regime.

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Subsequently, the frequency dependent characteristic impedance of the line is determined. Due to space restrictions, it was not possible to present the development of these equations in the paper. The full development is presented in [8] and aspects are referred to here.

II. Theory

The approach taken is to first neglect R and internal inductance, $L_{\rm int}$, of the conductors so that an "approximate" effective relative permittivity, $\hat{\varepsilon}_{r,\rm eff}$, can be determined from the measured propagation constant, γ, of the line. The high frequency asymptote of $\hat{\varepsilon}_{r,\rm eff}$ is then taken as the actual effective permittivity, $\varepsilon_{r,\rm eff}$, at all frequencies. In terms of the per unit length parameters:

$$\gamma = \sqrt{(R + j\omega(L + L_{\rm int}))(G + j\omega C)} = \frac{j\omega}{c}\sqrt{\mu_{r,\rm eff}\varepsilon_{r,\rm eff}} \tag{1}$$

and

$$Z_c = \sqrt{\frac{R + j\omega(L + L_{\rm int})}{G + j\omega C}} = Z_0\sqrt{\frac{\mu_{r,\rm eff}}{\varepsilon_{r,\rm eff}}}s \tag{2}$$

where $\omega = 2\pi f$, f is frequency, $\mu_{r,\rm eff}$ and $\varepsilon_{r,\rm eff}$ are the effective relative permeability and permittivity, respectively, and the free space impedance of the line with ideal conductors is

$$Z_0 = \frac{1}{C_0 c}. \tag{3}$$

$L_{\rm int}$ is due to current internal to the conductors and is asymptotically zero at high frequencies as the skin effect is fully established. Consequently, if the line is embedded in a nonmagnetic media:

$$\mu_{r,\rm eff} = \frac{R + j\omega(L + L_{\rm int})}{j\omega L} \tag{4}$$

and

$$\varepsilon_{r,\rm eff} = \frac{G + j\omega C}{j\omega C_0}. \tag{5}$$

L, C, and $G/\omega C$ are relatively independent of frequency and $R << \omega L$ at high frequencies since R has a $\sqrt{f}$ dependence of the skin effect. Thus $\lim_{f\to\infty}\mu_{r,\rm eff}(f) = 1$ and $\varepsilon_{r,\rm eff}$ is the high frequency asymptote of $\hat{\varepsilon}_{r,\rm eff}(f)$ where the intermediate dielectric constant

$$\hat{\varepsilon}_{r,\rm eff}(f) = -\gamma^2(f)c^2/\omega^2 \tag{6}$$

is obtained by substituting $\mu_{r,\rm eff} = 1$ in (1). Note that $\hat{\varepsilon}_{r,\rm eff}(f)$ approximates the dielectric constant only at high frequencies, since at low frequencies it includes significant contributions from $\mu_{r,\rm eff}$. In (6), γ is the measured propagation constant and is a by-product of the conventional TRL calibration procedure [5]. (An improved technique for determining γ was recently introduced in [6].) At lower frequencies, $L_{\rm int}$ and R become important so that $\mu_{r,\rm eff} \neq 1$. In particular, rearranging (1):

$$\mu_{r,\rm eff}(f) = -\frac{\gamma^2(f)c^2}{\varepsilon_{r,\rm eff}\omega^2}. \tag{7}$$

Combining (2), (3), and (7) yields

$$Z_c(f) = -j\frac{\gamma(f)}{\varepsilon_{r,\rm eff}\omega C_0}. \tag{8}$$

Equations (7) and (8) are just [footnote 1, eqs. (6), (7)].

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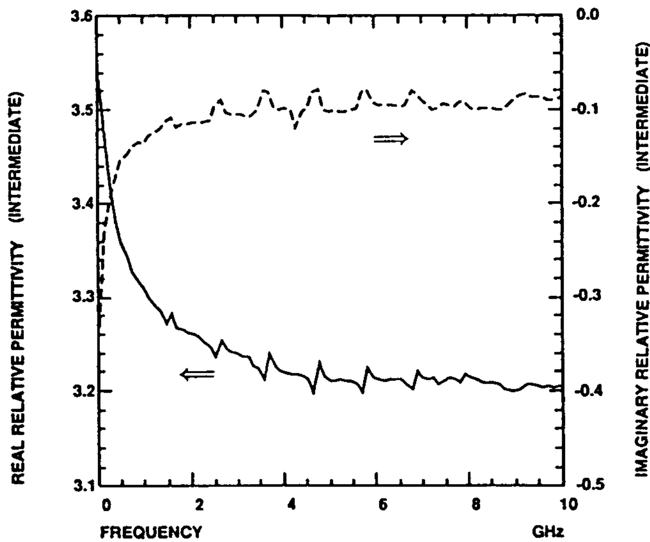

Fig. 3. Approximate relative dielectric constant $\hat{\varepsilon}_{r,\text{eff}}$ versus frequency.

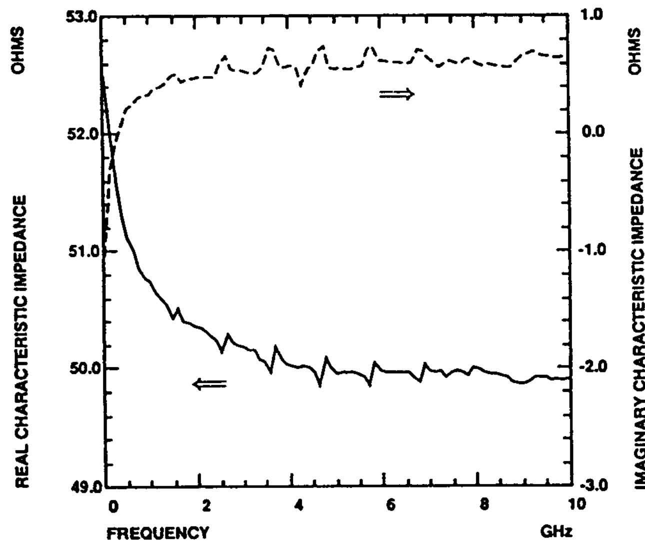

Fig. 4. Characteristic impedance versus frequency.

III. Measurements

The previous method was used in conjunction with a conventional TRL set of measurements, to determine Z_c of a 3-in long embedded microstrip line on an FR-4 printed circuit board. The width and thickness of the copper microstrip were 8 and 3.6 mils, respectively with a measured dc resistance of 1.37 Ω/m. It was 5.6 mils above the ground plane and C_0 was found to be 31.6 pF/m by using boundary element analysis. Note that the skin depth is half the microstrip thickness at 1.2 MHz.

From measurements of a through and the 3-in section of line, γ was determined, and subsequently from (6), $\hat{\varepsilon}_{r,\text{eff}}$ evaluated. In Fig. 3, $\hat{\varepsilon}_{r,\text{eff}}$ is plotted and it is seen to asymptotically approach a limit at high frequencies (as previously assumed) indicating that $\varepsilon_{r,\text{eff}} = 3.21$ and the effective dielectric loss tangent is 0.03. Substituting this and C_0 in (8), Z_c, shown in Fig. 4, was calculated. Z_c and $\hat{\varepsilon}_{r,\text{eff}}$ have similar frequency characteristics, since in both cases the frequency dependence is largely determined by $\mu_{r,\text{eff}}(f)$.

IV. Discussion and Conclusion

Marks and Williams discuss two alternative methods, described in [5], for determining the capacitance, C, of the line by assuming that the conductance, G, of the line is negligible. The first of these uses extrapolated low frequency S-parameter measurements and dc resistance measurement to determinge the quasi-static line capacitance. Uncertainties in extrapolating to dc are reflected in the capacitance estimate. The second approach is erroneous as shown in the Appendix.

The method presented here for determining the characteristic impedance of a line uses the measured propagation constant and high frequency estimation to determine the effective permittivity. The inherent assumption is that the capacitance of the line is independent of frequency. However, at very high microwave frequencies, the microstrip capacitance will increase as the field distribution changes and more field lines are concentrated in the substrate provided that the field distribution remains quasi-TEM. Nevertheless, we believe that this technique yields the best estimate of the characteristic impedance of the line at high frequencies.

Marks and Williams also present spectral domain simulation to 1000 GHz of a microstrip line. At 100's of gigahertz, quasi-TEM propagation on the line is unlikely and the field becomes nonconservative. Thus it is important that the method by which C is evaluated be specified. One could equally well define $C = \varepsilon_{r,\text{eff}} C_0$. The behavior of C at millimeter wave frequencies as reported in Fig. 1 has little meaning.

Appendix

The second technique presented in [3] for determining the capacitance of a line neglects the difference between the characteristic impedance of the line, Z_c, and the reference characteristic, Z_{ref}, of the measurement system. Following the development in [3], for a small lumped resistor, $R_{\text{load,dc}}$ as low frequencies:

$$Z_{\text{ref}}\left(\frac{1+\Gamma_{\text{load}}}{1-\Gamma_{\text{load}}}\right) \equiv Z_{\text{load}} \approx R_{\text{load,dc}} \tag{9}$$

where Γ_{load} is the measured reflection coefficient of the load referred to as Z_{ref}. Substituting (9) in

$$\gamma/Z_c = G + j\omega C \tag{10}$$

results in

$$C[1 - j(G/\omega C)] \approx \frac{\gamma}{j\omega\alpha R_{\text{load,dc}}}\left(\frac{1+\Gamma_{\text{load}}}{1-\Gamma_{\text{load}}}\right) \tag{11}$$

where $\alpha = Z_c/Z_{\text{ref}}$ and in general is complex. The important point to note is that Z_c of the line must be known or assumed before C can be calculated in this technique. Thus the technique cannot be used to determine Z_c. In contrast, the corresponding equation in [3], their (6), does not include α. However, the line examined had a Z_c of approximately 50 Ω so that $\alpha \approx 1$ since $Z_{\text{ref}} = 50\Omega$ and so the error is not evident.

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Comment on "Accurate Experimental Characterization of Interconnects: A Discussion of 'Experimental Electrical Characterization of Interconnects and Discontinuities in High-Speed Digital Systems' "[1]

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In my reply to a letter by Marks and Williams[1] commenting on recent work by myself and others [1] statements, now seen to be in error, were made as to one of the capacitance measurement techniques in [2] being erroneous. Both of the capacitance measurements presented by Williams and Marks in [2] are correct and I apologize to the authors. I was too hasty in rationalizing the discrepancy between the capacitances of lines calculated using their technique and those directly measured by a capacitance meter. The important point to note in [2] is that Γ_{load} [2, (5)] is not the actual reflection coefficient of the load but rather that determined using the TRL calibration technique. Γ_{load} is then with respect to the reference impedance of the TRL procedure which happens to be equal to the actual characteristic impedance of the line [3].

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