A Causal Microwave Circuit Theory and Its Implications

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***Abstract*- We will describe a new causal power-normalized waveguide equivalent-circuit theory and explore its implications. The new theory marries a power normalization with additional constraints that enforce simultaneity of the theory's voltages and currents and the actual fields in the circuit.**

The causal waveguide circuit theory of [1] requires that $Z_0(\omega)$, the characteristic impedance of the guide, be minimum phase. That is, the theory requires that $\hat{Z}_0(t)=0$ and $\hat{Y}_0(t)=0$ for $t < 0$, where $\hat{Z}_0(t)$ and $\hat{Y}_0(t)$ are the inverse Fourier transforms of $Z_0(\omega)$ and $Y_0(\omega) \equiv 1/Z_0(\omega)$. This condition is required to ensure that the waveguide responds to inputs after, not before, it is excited.

The voltage v and current i in [1] are defined by

$$\boldsymbol{E}_\mathrm{t}(\boldsymbol{r},z) = \frac{v(z)}{v_0}\boldsymbol{e}_\mathrm{t}(\boldsymbol{r})\,;\;\; \boldsymbol{H}_\mathrm{t}(\boldsymbol{r},z) = \frac{i(z)}{i_0}\boldsymbol{h}_\mathrm{t}(\boldsymbol{r}), \qquad \textbf{(1)}$$

where $\boldsymbol{r} = (x,y)$ is the transverse coordinate, $\boldsymbol{E}_\mathrm{t}$ and $\boldsymbol{H}_\mathrm{t}$ are the total electric and magnetic fields in the guide, $\boldsymbol{e}_\mathrm{t}$ and $\boldsymbol{h}_\mathrm{t}$ are the modal electric and magnetic fields of the single propagating mode, and v_0 and i_0 are normalization factors.

The power normalization is achieved by imposing the constraint [2]

$$v_0\, i_0^* = p_0 \equiv \int_S \boldsymbol{e}_\mathrm{t} \times \boldsymbol{h}_\mathrm{t}^* \cdot \boldsymbol{z}\, \mathrm{d}S, \qquad \textbf{(2)}$$

which ensures that the time-average power is $p = \frac{1}{2} v i^*$. The characteristic impedance Z_0 of the waveguide is defined by

$$Z_0 \equiv \left.\frac{v}{i}\right|_{c_-=0} = \frac{v_0}{i_0} = \frac{|v_0|^2}{p_0^*} = \frac{p_0}{|i_0|^2}, \qquad \textbf{(3)}$$

which shows that the power normalization sets the phase of Z_0 equal to the phase of p_0, a fixed property of the guide.

However, the minimum phase constraint imposed by causality is a strong one [3], [4]. It requires that the Hilbert transform of $\ln|\lambda Z_0|$, where λ is a constant, be equal to $\arg(Z_0) = \arg(p_0)$, a fixed property of the guide. As a result, the characteristic impedance of the guide is determined within a constant.

The implications of the new causal circuit theory of [1] are significant. The power flow p_0 is real in a lossless coaxial transmission line, so the phase of Z_0 is 0, and in the causal circuit theory $|Z_0|$ must be constant. We also show that the characteristic impedance of a rectangular waveguide must be proportional to the wave impedance of the guide: the choice $|Z_0| = 1$, allowed in conventional waveguide circuit theories, is not admissible in the causal theory.

REFERENCES

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