Characteristic Impedance of Microstrip on Silicon

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***Abstract*- We compare power-voltage, power-current, and causal definitions of the characteristic impedance of microstrip transmission lines on silicon substrates.**

INTRODUCTION

We compute the traditional power-voltage and power-current definitions of the characteristic impedance [1], [2] of microstrip transmission lines on silicon substrates with the full-wave method of [3] and compare them to the causal definition proposed in [4].

The causal waveguide circuit theory of [4] marries the power normalization of [1] and [2] with additional constraints that enforce simultaneity of the theory's voltages and currents and the actual fields in the circuit. These additional constraints not only guarantee that the network parameters of passive devices in this theory are causal, but they determine the characteristic impedance Z_0 of a single-mode waveguide within a positive constant multiplier.

Reference [5] examines some of the implications of [4], determining the characteristic impedance required by that theory in a lossless coaxial waveguide, a lossless rectangular waveguide, and an infinitely wide metal-insulator-semiconductor transmission line. In this paper we expand that investigation to microstrip lines of finite width on silicon substrates. We use full-wave calculations to compute the power-voltage and power-current definitions of the characteristic impedance of the microstrip lines and a Hilbert-transform relationship to determine the causal characteristic impedance. A comparison shows that the causal characteristic impedance agrees well with some, but not all, of the conventional definitions in microstrip lines.

CHARACTERISTIC IMPEDANCE

Following [1] we define the power-voltage characteristic impedance Z_{PV} from

$$Z_{\mathrm{PV}}(\omega) \equiv \frac{|v_0(\omega)|^2}{p_0(\omega)^*} \qquad \textbf{(1)}$$

and the power-current characteristic impedance Z_{PI} from

$$Z_{\mathrm{PI}}(\omega) \;\equiv\; \frac{p_0(\omega)}{|i_0(\omega)|^2}, \tag{2}$$

where the complex power p_0 of the forward mode is

$$p_0(\omega) \;\equiv\; \int \boldsymbol{e}_{\mathrm{t}}(\omega,\boldsymbol{r}) \times \boldsymbol{h}_{\mathrm{t}}^*(\omega,\boldsymbol{r}) \cdot \boldsymbol{z}\, \mathrm{d}\boldsymbol{r}, \tag{3}$$

ω is the angular frequency, $\boldsymbol{z}$ is the unit vector in the direction of propagation, $\boldsymbol{r} = (x,y)$ is the transverse coordinate, $\boldsymbol{e}_{\mathrm{t}}$ and $\boldsymbol{h}_{\mathrm{t}}$ are the transverse electric and magnetic fields of the forward mode, and the integral of Poynting's vector in **(3)** is performed over the entire cross section of the guide. The voltage v_0 of the forward mode is found by integrating the electric field over a path with

$$v_0(\omega) \;\equiv\; -\int_{\mathrm{path}} \boldsymbol{e}_{\mathrm{t}}(\omega,\boldsymbol{r}) \cdot \mathrm{d}\boldsymbol{l} \tag{4}$$

and current i_0 from

$$i_0(\omega) \;=\; \oint_{\substack{\text{closed}\\ \text{path}}} \boldsymbol{h}_{\mathrm{t}}(\omega,\boldsymbol{r}) \cdot \mathrm{d}\boldsymbol{l}, \tag{5}$$

where $\boldsymbol{l}$ is the unit vector tangential to the path.

The phase angles of the characteristic impedances Z_{PV} and Z_{PI} are equal to the phase angle of p_0, which is a fixed property of the guide. This condition on the phase of the characteristic impedance is a consequence of the power-normalization of the circuit theory; it is required to ensure that the time averaged power in the guide is equal to the product of the voltage and the conjugate of the current [1].

The magnitude of the characteristic impedance is determined by the choice of voltage or current path, and is therefore not defined uniquely by the traditional circuit theories of [1] and [2].

The causal circuit theory of [4] also imposes the power normalization of [1], so the phase angle of the causal characteristic impedance Z_{C} is equal to the phase angle of p_0. However, the causal theory requires in addition that $Z_{\mathrm{C}}(\omega)$ be minimum phase, which implies that

$$\mathcal{H}(\ln|Z_{\mathrm{C}}(\omega)|) \;=\; \arg[p_0(\omega)], \tag{6}$$

where $\mathcal{H}$ is the Hilbert transform. As a result we can determine $\ln|\lambda Z_{\mathrm{C}}|$, where λ is a positive constant that determines the overall impedance normalization, from $\arg(p_0)$ [4]. We can then fix Z_{C} by matching $|Z_{\mathrm{C}}|$ and $|Z_{\mathrm{PV}}|$ or $|Z_{\mathrm{PI}}|$ at a single frequency.

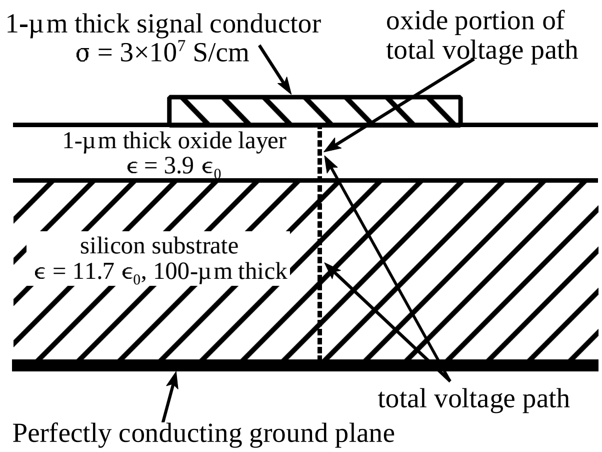

Fig. 1. Microstrip line geometry. The 1-µm thick signal conductor is 5 µm wide.

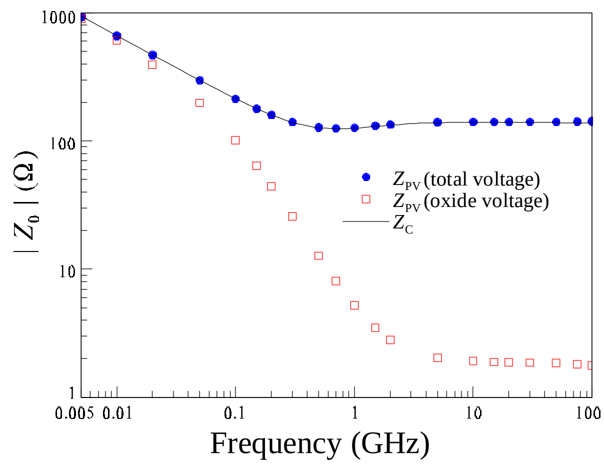

Fig. 2. Comparison of definitions of characteristic impedance of a microstrip line on a 100 Ω·cm substrate. We matched $|Z_C|$ with $|Z_{PV}|$ at 5 MHz.

COMPARISON OF DEFINITIONS

We used the full-wave method of [3] to calculate the characteristic impedance of the 5-µm wide microstrip line of Fig. 1. Figure 2 compares this microstrip's power-voltage and causal definitions of characteristic impedance.

The curve in Fig. 2 labeled "Z_C" is the magnitude of the characteristic impedance determined from the phase of p_0, which we calculated with the full-wave method of [3], and the minimum phase condition **(6)**, as required by the causal circuit theory of [4].

The dots labeled "Z_{PV} (total voltage)" are the magnitudes of the characteristic impedance we calculated with the full-wave method [3] defined with a power-voltage definition. Here the voltage integration path begins in the center of the microstrip line at the ground plane on the back of the silicon substrate and terminates on the signal conductor on top of the oxide. This path corresponds to the vertical dashed line in Fig. 1. The figure shows good agreement between the causal and the power/total-voltage definitions of characteristic impedance.

The traditional circuit theories of [1] and [2] do not uniquely specify the integration path used to define the voltage, and an integration path beginning at the silicon surface and going through the oxide to the signal conductor is equally consistent with those theories. However, Fig. 2 shows that the power-voltage characteristic impedance, which is labeled "Z_{PV} (oxide voltage)" and is defined with a voltage integration path corresponding only to that part of the vertical dashed line in Fig. 1 in the oxide, differs significantly from the characteristic impedance required by the causal theory.

The power/oxide-voltage impedance predicts that the voltage at the input to this microstrip line will respond to a current excitation before the current is turned on, and causes instabilities when this and other network parameters of associated circuit theory are used in conventional temporal simulations [4]. These results are consistent with [5], which also found significant differences between the power/oxide-voltage definition in the infinitely wide metal-insulator-semiconductor transmission line and the causal characteristic impedance Z_C: it appears that not all voltage paths were created equal!

Figure 3 compares the causal, power/total-voltage, and power/current definitions for the microstrip line of Fig. 1, where the integration path used to determine the current exactly encloses the microstrip signal conductor, over a broad range of substrate conductivity. The figure shows that these three definitions of characteristic impedance are in approximate agreement over a broad frequency range.

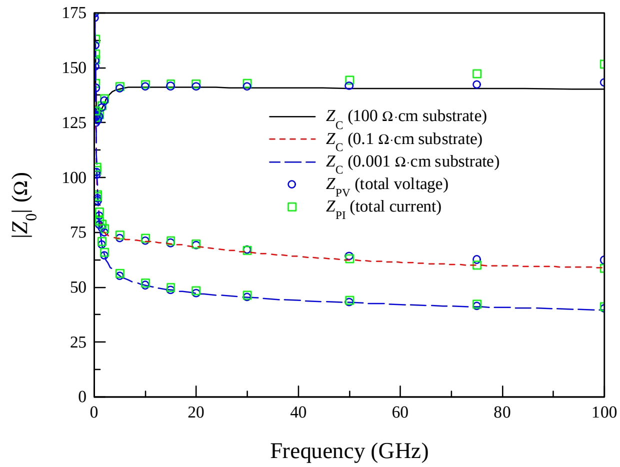

Fig. 3. Comparison of definitions of characteristic impedance of a 5-μm wide microstrip line on substrates of three different conductivities. We matched $|Z_C|$ with $|Z_{PV}|$ at 1 GHz to better illustrate the agreement between the definitions; the dimensions are shown in Fig. 1.

CONCLUSION

We have shown that the characteristic impedance required by the causal power-normalized waveguide circuit theory of [4] agrees well with some, but not all, of the conventional power/voltage and power/current definitions in the microstrip lines we studied.

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