



# “How Good Are the Internal Conversion Coefficients Now?”

T. Kibédi *Dept. of Nuclear Physics, Australian National University, Canberra, Australia*

T.W. Burrows\* *NNDC, Brookhaven National Laboratory, Upton, NY, U.S.A.*

M.B. Trzhaskovskaya *Petersburg Nuclear Physics Institute, Gatchina, Russia*

C.W. Nestor, Jr. *University of Tennessee, Knoxville, TN, U.S.A.*

P.M. Davidson *Dept. of Nuclear Physics, Australian National University, Canberra, Australia*

\*Email: [burrows@bnl.gov](mailto:burrows@bnl.gov)

# “How Good Are the ICC’s Now?”

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- Summary of Analysis Methods
- Evolution of ICC Theory and Experiment
- Results
- Conclusions

# “How Good Are the ICC’s Now?”

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- Principal Tools:
  - BrIccRAINE: Exact calculations of ICC’s as a function of energy
  - AveTools:
    - Three weighted average methods
      - Limitation of Relative Statistical Weights (LWM)
      - Normalized Residual Method (NRM)
      - Rajeval Technique (RT)
    - After discrepant data identified and removed ( $N > 3$ ):
      - Adopt arithmetic mean of NRM and RT and the larger of the two uncertainties from NRM and RT for experimental
      - Adopt LWM result for comparison to theory

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- Experimental Data:
  - No dipole or M2. Total, K, and L conversion coefficients and K/L ratios considered.
  - Multipolarity must be assigned independent of  $\alpha$ 's and (almost) pure.
  - Individual datum must have  $\Delta\alpha/\alpha \leq 15\%$ . Final average must have  $\Delta\alpha/\alpha \leq 5\%$ .
  - About 99% of the original papers were obtained and checked.
  - 213 data points satisfied these criteria.

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- Comparison of Experiment to theory:
  - $ICC(\text{exp:theory}) = 100 \times (\alpha(\text{exp}) - \alpha(\text{theory})) / \alpha(\text{theory})$
  - Three relativistic Dirac-Fock models:
    - BTNTR – “No hole” approximation
    - RNIT(1) – “Self consistent” approximation
    - RNIT(2) – “Frozen orbital” approximation

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– 17 “slices”

| Mult. | Shell | Mult. | Shell | Mult.                            | Shell |
|-------|-------|-------|-------|----------------------------------|-------|
| All   | All   | E2    | All   | M3                               | All   |
| All   | T+K   | E2    | K     | M4                               | All   |
| All   | Total | E2    | Total | M4                               | K     |
| All   | K     | E3    | All   | M4                               | Total |
| All   | K/L   | E3    | K     | $\Delta\alpha/\alpha \leq 1.5\%$ |       |
| All   | L     | E3    | Total |                                  |       |

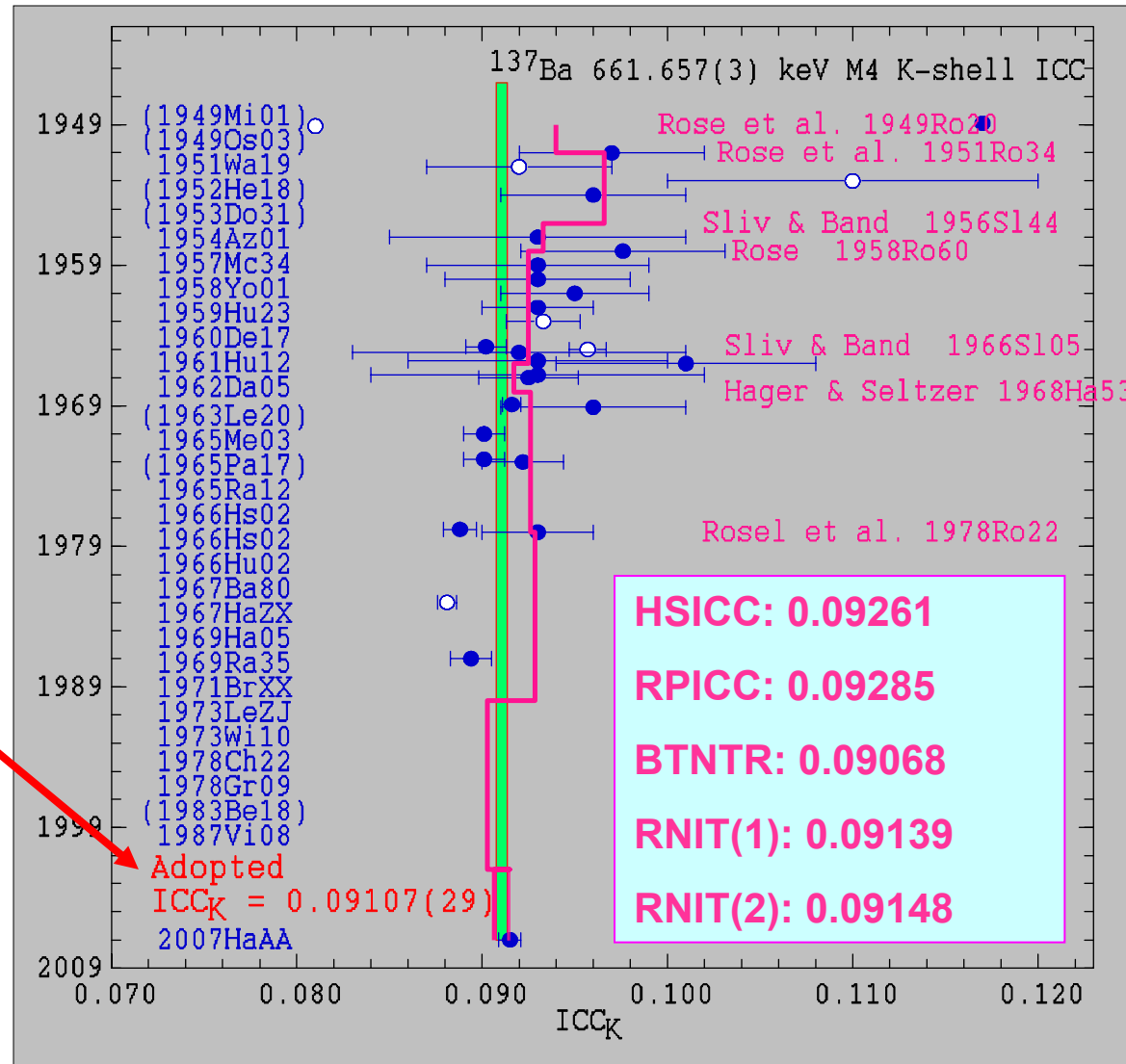
– 188 data points left after exclusion of discrepant data (Compared to 100 in Raman’s original study)

# “How Good Are the ICC’s Now?”

*Adopted*

LWM: 0.09103(28)  
 NRM: 0.09103(28)  
 RT: 0.09110(29)

ENSDF  
 $ICC_K = 0.0904(5)$



# “How Good Are the ICC’s Now?”

| ML  | Shell | N   | BTNTR    |                | RNIT(1)  |                | RNIT(2)  |                |
|-----|-------|-----|----------|----------------|----------|----------------|----------|----------------|
|     |       |     |          | $\chi^2/(N-1)$ |          | $\chi^2/(N-1)$ |          | $\chi^2/(N-1)$ |
| All | All   | 188 | +0.70 40 | 1.82           | -0.61 14 | 1.01           | -0.93 14 | 0.87           |

$\chi^2(\text{critical})=1.25$

Both negative;  
RNIT(1) out by  $4.5\sigma$   
RNIT(2) out by  $6.9\sigma$



# “How Good Are the ICC’s Now?”

| ML  | Shell | N   | BTNTR    |                | RNIT(1)  |                | RNIT(2)  |                |
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| All | All   | 188 | +0.70 40 | 1.82           | -0.61 14 | 1.01           | -0.93 14 | 0.87           |
| All | Tot   | 54  | +0.32 25 | 0.79           | -0.55 24 | 0.76           | -0.71 24 | 0.73           |

$\chi^2(\text{critical})=1.51$

1.3 $\sigma$

2.3 $\sigma$

3.0 $\sigma$

Marginal differences

# “How Good Are the ICC’s Now?”

| ML  | Shell | N   | BTNTR    |                | RNIT(1)  |                | RNIT(2)  |                |
|-----|-------|-----|----------|----------------|----------|----------------|----------|----------------|
|     |       |     |          | $\chi^2/(N-1)$ |          | $\chi^2/(N-1)$ |          | $\chi^2/(N-1)$ |
| All | All   | 188 | +0.70 40 | 1.82           | -0.61 14 | 1.01           | -0.93 14 | 0.87           |
| All | Tot   | 54  | +0.32 25 | 0.79           | -0.55 24 | 0.76           | -0.71 24 | 0.73           |
| All | K     | 72  | +1.5 12  | 3.14           | -0.18 21 | 1.09           | -0.72 21 | 0.80           |

$\chi^2(\text{critical})=1.43$   
 Not favored

# “How Good Are the ICC’s Now?”

| ML  | Shell | N   | BTNTR    |                | RNIT(1)  |                | RNIT(2)  |                |
|-----|-------|-----|----------|----------------|----------|----------------|----------|----------------|
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| All | K     | 72  | +1.5 12  | 3.14           | -0.18 21 | 1.09           | -0.72 21 | 0.80           |
| All | K/L   | 46  | +0.00 31 | 0.83           | -1.64 31 | 0.96           | -1.94 30 | 1.02           |

RNIT(1) and RNIT(2) out by  $>5\sigma$   
 Unexpected

# “How Good Are the ICC’s Now?”

| ML  | Shell | N   | BTNTR    |                | RNIT(1)  |                | RNIT(2)  |                |
|-----|-------|-----|----------|----------------|----------|----------------|----------|----------------|
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| All | K/L   | 46  | +0.00 31 | 0.83           | -1.64 31 | 0.96           | -1.94 30 | 1.02           |
| E2  | All   | 103 | +0.21 23 | 1.01           | -0.77 23 | 0.89           | -0.93 23 | 0.90           |

BTNTR consistent

RNIT(1) and RNIT(2) out by  $>3\sigma$   
 RNIT(2) “follows the trend” being around -0.9

# “How Good Are the ICC’s Now?”

| ML  | Shell | N   | BTNTR    |                | RNIT(1)  |                | RNIT(2)  |                |
|-----|-------|-----|----------|----------------|----------|----------------|----------|----------------|
|     |       |     |          | $\chi^2/(N-1)$ |          | $\chi^2/(N-1)$ |          | $\chi^2/(N-1)$ |
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| E2  | All   | 103 | +0.21 23 | 1.01           | -0.77 23 | 0.89           | -0.93 23 | 0.90           |
| M4  | All   | 50  | +0.98 68 | 3.87           | -0.51 20 | 1.29           | -0.93 20 | 0.72           |

$\chi^2(\text{critical})=1.53$   
Not favored

RT adjusted  $^{193}\text{Ir}$  K  
 $\Delta$  from 3.4% 8 to 3.4% 17  
Problems with  $^{207}\text{Pb}$  K/L

Problems with  
 $^{207}\text{Pb}$  K/L

# “How Good Are the ICC’s Now?”

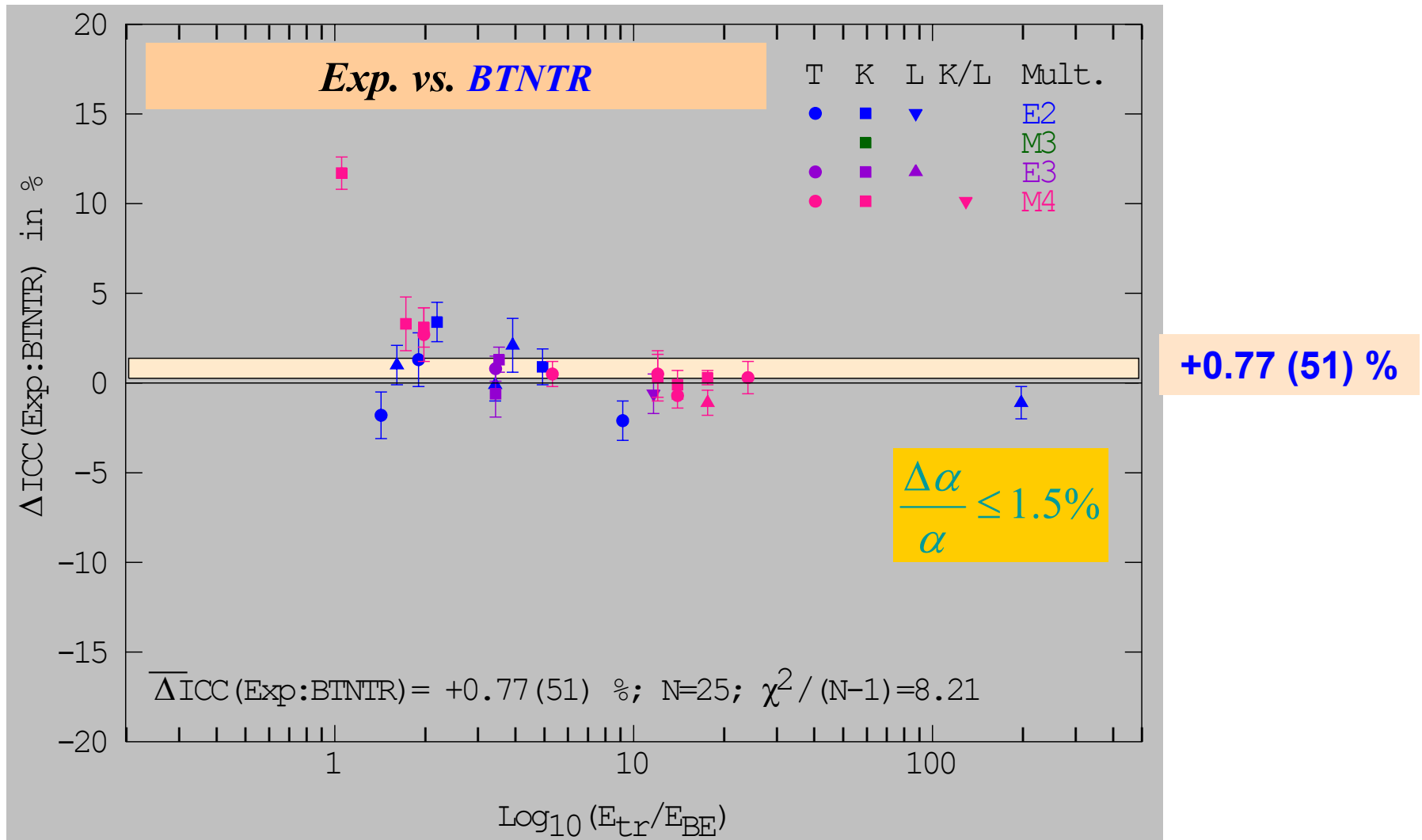
| ML   | Shell | N   | BTNTR    |                | RNIT(1)  |                | RNIT(2)  |                |
|--|-------|-----|----------|----------------|----------|----------------|----------|----------------|
|  |       |     |          | $\chi^2/(N-1)$ |          | $\chi^2/(N-1)$ |          | $\chi^2/(N-1)$ |
| All  | All   | 188 | +0.70 40 | 1.82           | -0.61 14 | 1.01           | -0.93 14 | 0.87           |
| Tot  | All   | 54  | +0.32 25 | 0.79           | -0.55 24 | 0.76           | -0.71 24 | 0.73           |
| K  | All   | 72  | +1.5 120 | 3.14           | -0.18 21 | 1.09           | -0.72 21 | 0.80           |
| K/L  | All   | 46  | +0.00 31 | 0.83           | -1.64 31 | 0.96           | -1.94 30 | 1.02           |
| E2   | All   | 103 | +0.21 23 | 1.01           | -0.77 23 | 0.89           | -0.93 23 | 0.90           |
| M4   | All   | 50  | +0.98 68 | 3.87           | -0.51 20 | 1.29           | -0.93 20 | 0.72           |
| ICC’s known to better than 1.5% relative uncertainty |       |     |          |                |          |                |          |                |
| All  | All   | 25  | +0.77 51 | 8.21           | -0.56 26 | 2.12           | -0.95 17 | 1.06           |

$\chi^2(\text{critical})=1.79$   
Not favored

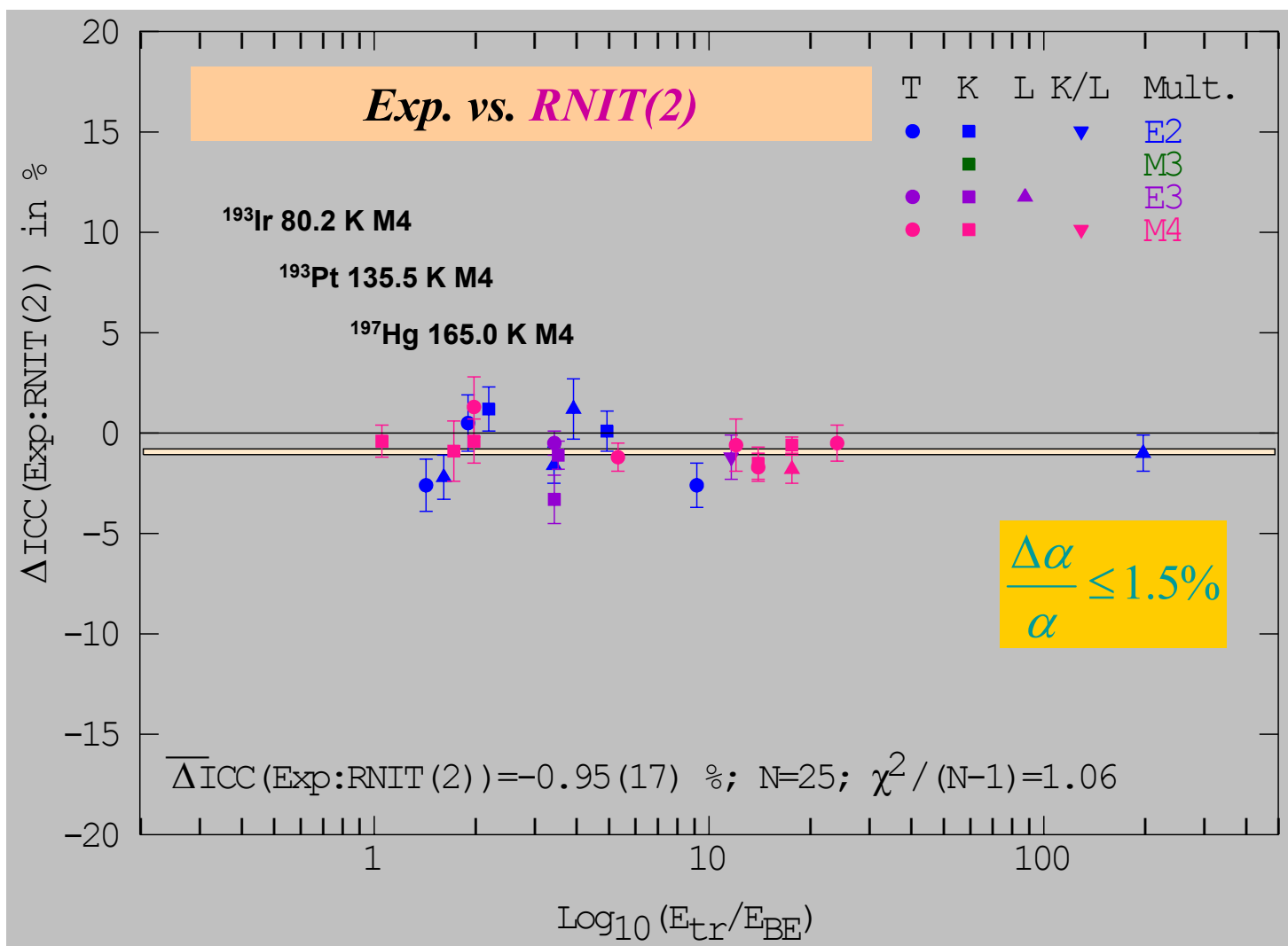
Marginally larger  
than  $\chi^2(\text{critical})$

Favored

# “How Good Are the ICC’s Now?”



# “How Good Are the ICC’s Now?”



-0.95 (17) %



# “How Good Are the ICC’s Now?”

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- BTNTR is definitely not favored
  - $\chi^2/(N-1) > \chi^2(\text{critical})$  for six cases (All/All, All/Tot+K, **All/K**, **M4/All**, **M4/K**, and  **$\Delta\alpha/\alpha \leq 1.5\%$** )
  - Exceptions: BTNTR seems to be favored for All/K/L and E2/Tot.

# “How Good Are the ICC’s Now?”

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- RNIT(2) seems to be somewhat favored over RNIT(1)
  - $\chi^2/(N-1) > \chi^2(\text{critical})$  for  $\Delta\alpha/\alpha \leq 1.5\%$ . Also,  $^{193}\text{Ir}$  M4 K marked as outlier in LWM and adjusted in NRM and RT.
  - Rajeval Technique adjusts the RNIT(1)  $^{193}\text{Ir}$  M4 K uncertainty, from 0.8% to between 1.4% and 1.8%, in four cases

# “How Good Are the ICC’s Now?”

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- As a function of  $R$  where  $R = \log_{10}(E_{tr}/BE_K)$ 
  - $\langle \Delta I_{cc}(\text{Exp:RNIT}(2)) \rangle$  is relatively independent of  $R$
  - $\langle \Delta I_{cc}(\text{Exp:BTNTR}) \rangle$  shows a very strong systematic trend downward until converging with RNIT(1) and RNIT(2) results at  $R \approx 2$
  - $\langle \Delta I_{cc}(\text{Exp:RNIT}(1)) \rangle$  shows a weaker but still definite downward trend until converging with BTNTR and RNIT(2) results at  $R \approx 2$

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