Detection of Tones Masked by Noise: A Comparison of Human Observers with Digital-Computer-Simulated Energy Detectors of Varying Bandwidths

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This paper is dedicated to my parents, who started me on the road to model building with tinker toys and erector sets and patiently encouraged and supported the completion of my formal education.

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Abstract

Prior investigators of auditory signal processing have supposed that when a human observer attempts to detect the presence of a signal tone masked by thermal noise, the observer much like a detector consisting of a band-pass filter, centered at the tone frequency, followed by a power or energy detector. The width of the observer's hypothetical band-pass filter is called the *critical bandwidth*.

There have been two methods of estimating the critical bandwidth. One method involves comparing the masking effectiveness of noises of varying pass-band width. This method has led to critical bandwidth estimates ranging from 65 to 500 Hz for tone signals of 800 or 1000 Hz. The main problem with the first method is that the critical bandwidth of the observer may vary when the width of noise pass-band is varied. The second method is based on the unreasonable assumption that the critical bandwidth is the only unknown factor influencing the observer's performance.

Three recent studies have used an electrical or computer-simulated filter followed by an energy detector to analyze the same noise-masked tone stimuli that were presented to

human observers. The results showed a positive correlation between the responses of the observers and the energy output of the filter.

These experiments suggested the approach of the present experiment: a more direct method for estimating the critical bandwidth. Stimuli were presented to observers; and, also, for the same stimuli, the energy passed by filters of different pass-band width was measured and correlated with the observer's responses. The bandwidth of the filter giving the largest correlation gave an estimate of the critical bandwidth.

The stimuli were 60 0.1-second bursts of band-pass (250-750 Hz) noise, approximately half of which had added to them a 500 Hz pure-tone signal. The stimuli were tape recorded and presented five times to four observers for yes-no judgments as to the presence or absence of the tone. The tape was then digitized at a sampling rate of 4000 samples/second, and the total energy passed by digital single-tuned filters centered at 500 Hz and having bandwidth of 10, 20, 41, 81, 176, 303, 494, and 720 Hz was computed for each stimulus.

For comparison with previous research the performance of the observers was compared with that of the digitally simulated filter model. The totals of the responses of two of the observers separated the tone-plus-noise stimuli from the noise-only stimuli as well as outputs of filters 68 and 135 Hz in width, whereas the other two performed worse than the widest filter. When the observer's response totals were rank correlated with the filter outputs, all four observers' responses correlated best with outputs of the narrowest (10, 20, 41 Hz) digital filters for the stimuli containing tone signals, but the responses to the noise-only stimuli correlated best with the output of wider (81-303 Hz) filters. A filter-bank model is proposed in qualitative explanation of the results.

Introduction

A noise-masked tone detection experiment involves asking an observer to judge whether or not a weak pure tone signal has been added to a background stimulus of Gaussian noise. The source of the tone is the sinusoidal voltage from an oscillator; the source of the noise is the randomly fluctuating voltage from a vacuum tube. The stimuli are mixed electronically and then impressed across the earphone terminals. The observer is asked whether or not he hears the tone or he is asked to judge whether or not the tone was added. If the levels of the tone and noise are set appropriately (with the power per cycle of the noise in the frequency region of the tone about 10 dB below the power of the tone alone), the observer will sometimes report hearing the tone and sometimes not. The proportion of such detections is the basic datum of the experiment.

The experimental situation has been studied for two somewhat separable reasons. One is the attempt to build a general theory of psychophysics: a theory to explain the judgments of observers in difficult, uncertain detection tasks. In these studies, essentially non-stimulus variables have been shown to influence judgments. Examples of such stimuli are the preceding sequence of stimuli and judgments and the reward structure of the experimental situation (Friedman et al., 1966; Swets, 1961). Alternatively, the noise-masked tone detection experiment is used to create and test the adequacy of conceptions of how the auditory system processes the acoustic stimulus and to measure fundamental properties of the system as suggested by the conceptions. This latter reason is the concern of this paper. The conception under consideration is that of a simple linear band-pass filter, whose output is measured by a power or energy detector. The obvious measurement suggested by this model is the width of the filter pass band. The experimental part of this study attempts such a measurement. A diagram of the model appears in Figure 1a. The stimulus x(t) is input to the narrow band filter, whose frequency response appears in the graph below the diagram. The output y(t) of the filter feeds into the energy detector, which outputs the decision statistic ϕ. If the statistic ϕ is larger than the criterion K, the model of the observer judges that the tone was present in the stimulus. If ϕ is less than K, the tone is judged to be absent. The critical bandwidth of the model (Figure 1b) is the difference between the two frequencies for which the pure tone power attenuation is twice that of the signal tone frequency (for which the attenuation is assumed to be minimal).

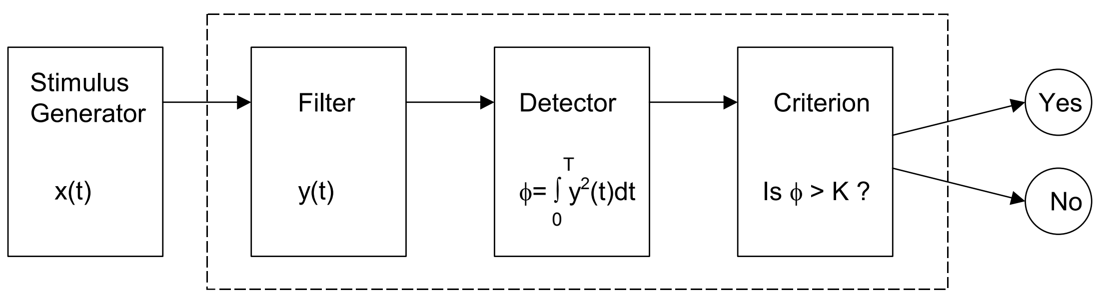

Figure 1a. Diagram of the critical bandwidth detection model (see text for explanation). The dotted lines enclose the hypothetical observer.

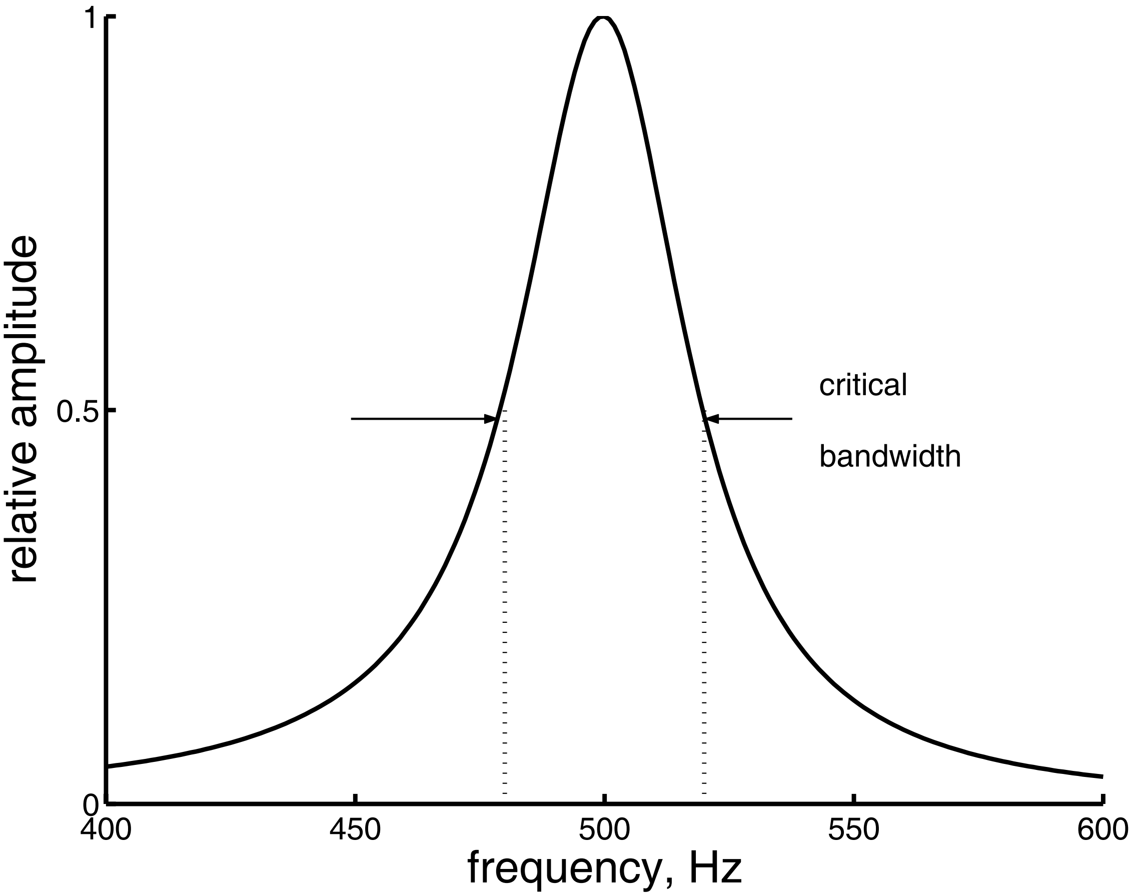

Figure 1b. Filter power output for sine wave inputs of equal amplitude but varying frequency. The distance between the dotted lines is the critical bandwidth.

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Critical Bandwidth Measurements Using Noise of Varying Pass-Band Width

The narrow-band linear filter model for noise-masked tone detection was introduced by Fletcher (1940, 1953); for simplicity he assumed that the filter's amplitude-frequency response was rectangular and centered about the tone frequency. His detection rule was that a continuous tone is at threshold level (reported to be heard 50% of the time) when the tone power is a certain constant times the noise power passed by the filter. If N_0 is the power per cycle of a wide-band masking noise, W_0 is the observer's critical bandwidth, and S is the power of the tone signal at threshold, Fletcher's rule is that

$S/N_0 = c\ W_0$. (Equation 1)

Fletcher reasoned that if the noise were pre-filtered by an external rectangular filter similarly centered but having a pass-band wider than that of the observer, the observer's threshold would be unaffected. However, when the external filter pass-band becomes narrower than that of the observer, the threshold, recorded as the ratio of threshold signal power to noise power per cycle, should be proportional to the width of the filter. Letting W represent the width of the external filter pass-band, we have

$S/N_0 = c\ W,\ W < W_0$. (Equation 2)

Fletcher measured this threshold for external filter pass-bands of 30, 200, 500, 1000, 4000, and 8000 Hz. His results are plotted in Figure 2. The horizontal part of the solid line represents the region in which the observer's filter is assumed to be narrower than the external filter (Equation 1). The 45° part of the curve represents on the log-log plot the region where the observer's filter is assumed to be wider that the external filter (Equation 2). Fletcher set the height of the curve by setting c=1, so that at threshold the noise power was assumed to be equal to the tone power. The elbow where the two straight parts of the curve meet is at the width of the observer's filter. These *critical bandwidths*, as Fletcher named them, appear in Table 1. Fletcher considered their accuracy to be within a factor of two.

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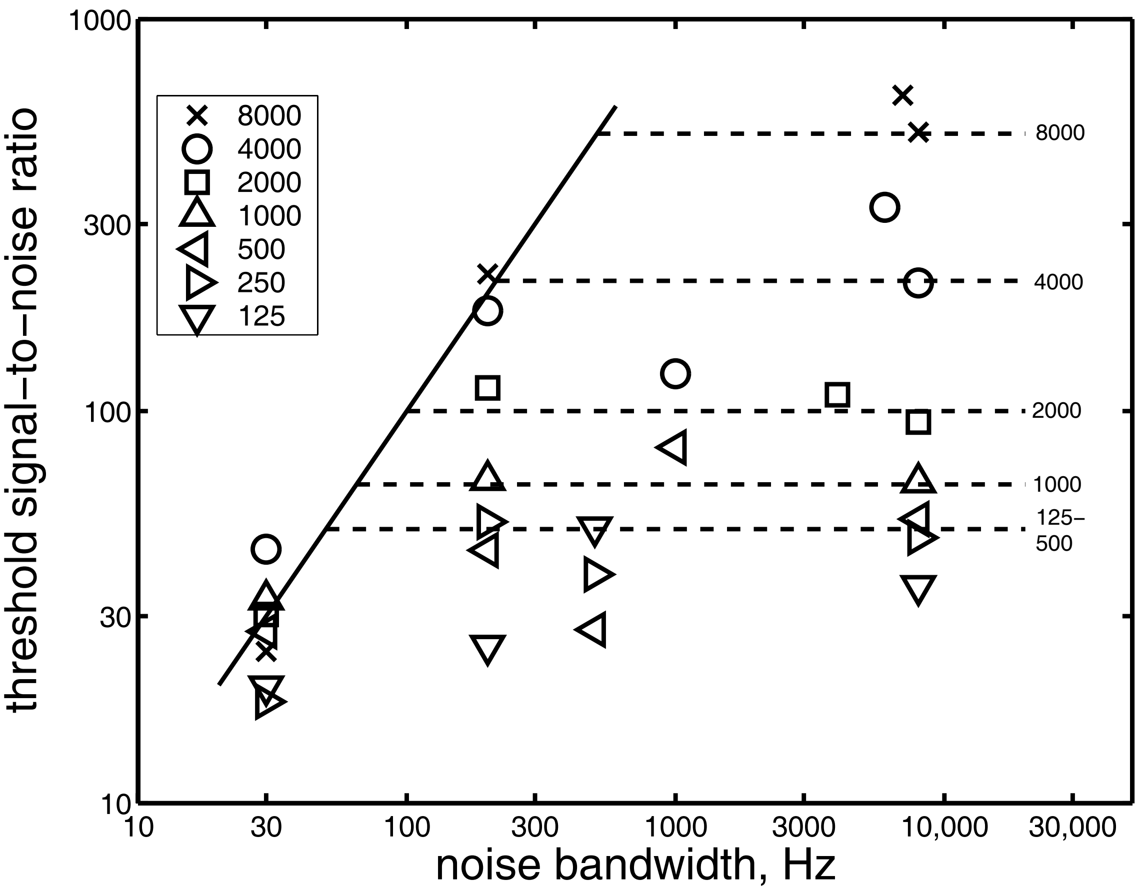

Figure 2. Fletcher's threshold signal-to-noise ratio vs. width of noise band. The legend symbols give the tone frequency in Hz. The solid line is the prediction for noise bandwidths less than the critical bandwidth. The horizontal lines are predictions for noise

bandwidths greater than the critical bandwidth. The estimated critical bandwidths for 500 and 1000 Hz are 48 and 65 Hz, respectively.

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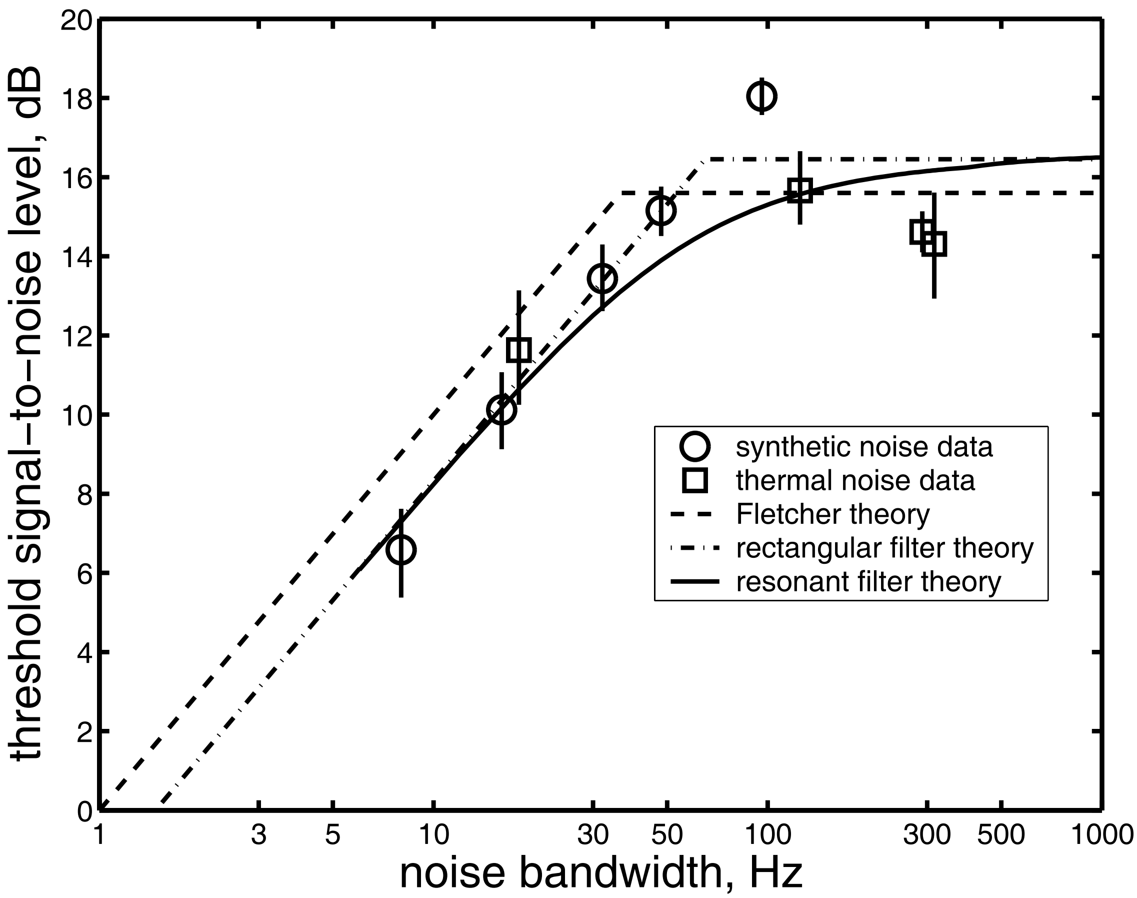
Investigator	**Tone Frequency in Hz**	**Tone Duration in seconds**	**Critical Bandwidth Estimate in Hz**
Fletcher (1953)	500	continuous	48
	1000	continuous	65
Schafer et al. (1957)	800	1.5	65
Hamilton (1957)	800	0.4	150
	800	0.1	150
Swets, Green, and Tanner (1962)	1000	0.1	95
van den Brink (1964)	800	0.5	150
	800	0.1	200
de Boer (1962)	reanalysis of other research		200
Green and Swets (1966)	reanalysis of other research		500

Table 1. Critical bandwidths measured by varying the bandwidth of the masking noise.

Other investigators have repeated Fletcher's basic experiment with more refined experimental techniques and more sophisticated analyses. Shafer, Gales, Shewmaker, and Thompson (1950) collected more data in the narrow-band region. They used noise synthesized from equal amplitude sine waves of random phase whose frequencies were spaced one Hz apart. In this way they obtained noise with bandwidths of 8, 16, 32, 48, and 96 Hz. They also used narrow-band filtered thermal noise with bandwidths of 20, 50, 200, 450, and 500 Hz. Their threshold measurements appear in Figure 3. The data are more suggestive of a gradual transition than of a sharp transition at the critical bandwidth, so these authors suggested the data could be better fitted by assuming the observer's filter had the amplitude-frequency response of a single-tuned filter, which would alter the amplitude of a sine wave with frequency f by the factor
$[1 + (f_0/B)^2 (f/f_0 - f_0/f)^2]^{-0.5}$,
where f_0 is the frequency at which the filter has minimum attenuation and B is the half-power bandwidth of the filter (the difference between the two frequencies at which the power attenuation is 3 dB). Their "eyeball" curve fitting yielded estimates for the

equivalent rectangular power bandwidth of 65 Hz and a sensitivity ratio c = 0.68 for their two low center frequencies, 200 and 800 Hz. The corresponding value of B is 43 Hz.



threshold signal-to-noise level, dB
0
2
4
6
8
10
12
14
16
18
20
1
3
5
10
30
50
100
300
500
1000
noise bandwidth, Hz
synthetic noise data
thermal noise data
Fletcher theory
rectangular filter theory
resonant filter theory

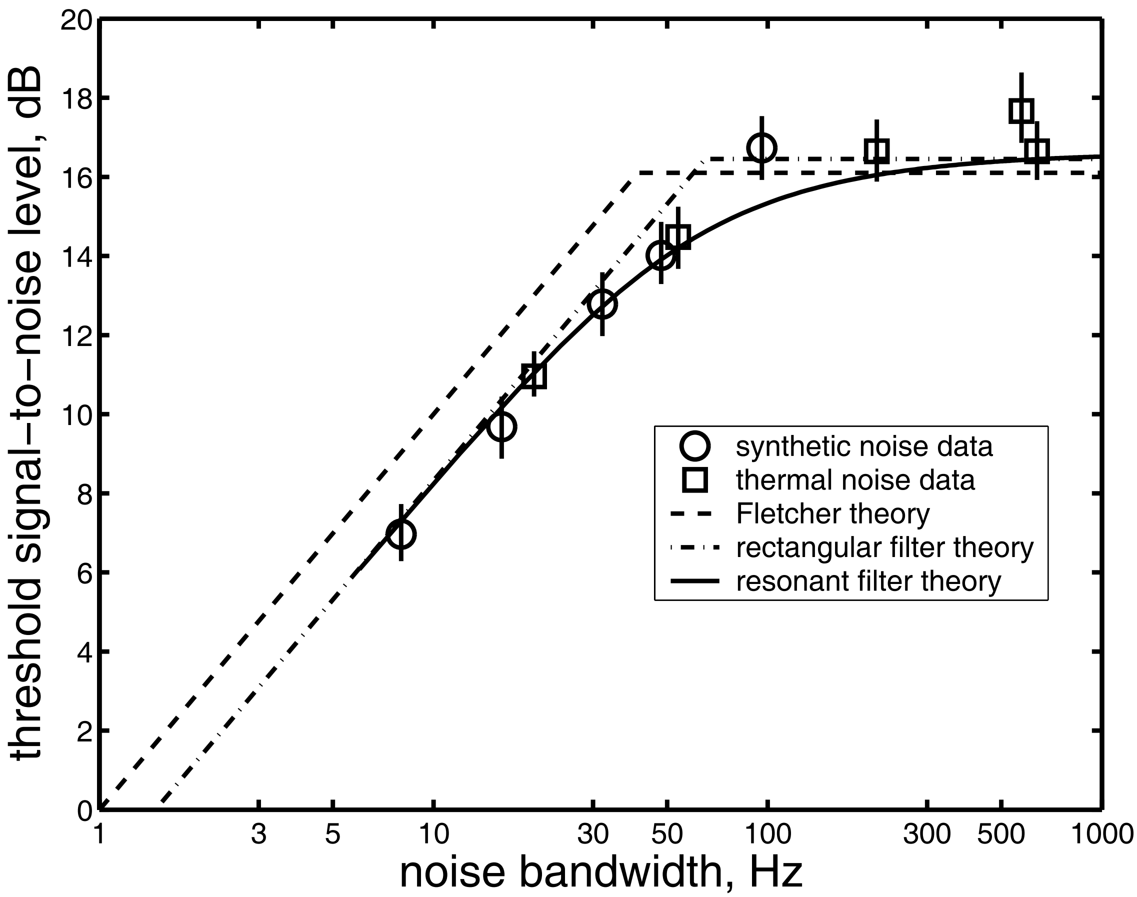


Figure 3. Threshold signal-to-noise ratios as function of masking noise bandwidth from Schafer et al. (1950). (Above) Masking of a 200 Hz pure tone centered in the noise band. (Below) Masking of an 800 Hz pure tone centered in the noise band. Ordinates are threshold signal tone level minus noise spectral density level in dB. Abscissas are noise bandwidth in Hz. Circles indicate noise synthesized from tones, squares indicate thermal noise. Vertical bars cover the 98 percent confidence interval for the mean. The Fletcher theory critical bandwidths are 36 and 41 Hz, the rectangular filter critical bandwidth estimates are both 65 Hz.

A similar analysis was done by Swets, Green, and Tanner (1962). They used a single-tuned external filter to avoid the technical difficulties of trying to approximate a rectangular amplitude-frequency response. First they plotted detection performance as a function of noise level for wide-band noise. Then they filtered the noise and recorded the resulting improvement in detection performance. Under the assumption that the improvement in the case of the band-pass noise is due to a drop in noise power passing through the observer's filter, this drop in noise power was estimated by finding the drop in power for wide-band noise that yielded equivalent improvement in detection performance. The width of the observer's internal filter was estimated as the width of a filter whose output would drop this amount when the internal filter is introduced. Internal filter widths were estimated for external filter widths of 10, 20, 30, 40, 50, 60, and 90 Hz. The results, averaged over the different external filter conditions and the three observers, yield different estimates depending on the assumed shape of the observer's filter characteristic. The assumption of a single-tuned filter gives an average estimate of 41 Hz as the half-power bandwidth, whereas if the shape is assumed to be Gaussian or rectangular the average half-power bandwidths are 79 and 95 Hz respectively. The frequency of the tone in this experiment was 1000 Hz and the duration of the tone was 0.1 second.

The effect of signal tone duration upon critical bandwidth measurements is available from experiments by Hamilton (1957) and by van den Brink (1964). At a tone frequency of 800 Hz they obtained threshold ratios of signal power to noise power density for a range of external filter widths and signal durations. Using the width of an external filter where the threshold begins to fall as the estimate of the critical bandwidth, Hamilton shows the critical bandwidth widening from 150 Hz at a signal tone duration of 400 msec to 200 Hz wide at 50 msec and 500 Hz at 25 msec. Van den Brink's data are similar in this region. Interpretations of van den Brink's data for shorter durations is complicated by interactions between the widening energy spectrum of the tone burst and the external and hypothetical internal filter pass-bands.

The narrower-than-critical-bandwidth portion of the data of these authors appears to be at variance with the results of Shafer et al. (1950) and also of Greenwood (1961), which showed that the threshold signal power was proportional to the filter width when the external filter was narrower than the critical bandwidth, as predicted by Equation 2. In the newer data the slope appears more gentle. Hamilton estimates a slope of 1.5 dB per octave instead of the 3 dB per octave slope required by the simple filter theory.

For Hamilton, the similarity between the signal spectrum and the masking noise spectrum offered an explanation for both these results-- the increase in the critical bandwidth for short durations and the lack of the expected improvement for the narrow bandwidth noise maskers. Short tones have a wider spectrum more similar to that of the noise, and narrow-band noises obviously have a spectrum more similar to that of the tones. He says that detection of the tones in noise of much wider bandwidth is improved because the tones stand out or have more salience. In support of this notion Hamilton mentions a study by Licklider et al. (1954) in which thresholds were measured for narrow-band noise stimuli as well as for the usual tone signals masked by wide-band noise. The detection threshold power of the narrow noise signal dropped as the bandwidth of this noise was narrowed to about 21 Hz. For this and narrower widths the threshold was the same as for pure tone signals. Hamilton thought the critical bandwidth was much wider than this and so he introduced the additional notion of salience without suggesting a mechanism by which it might work.

Van den Brink (1964) offers a similar explanation for these effects. The effect of signal duration he ascribes to an actual widening of the critical bandwidth. The idea that the critical bandwidth may be under the observer's control had been suggested before to explain the detection of signals of varying bandwidth (Green, Birdsall, and Tanner, 1957; Creelman, 1961). This idea came out of signal detectability theory, which shows that an optimal filter for the detection of a signal in wide-band Gaussian noise has a bandwidth equal to the bandwidth of the signal (Weiner, 1950; Peterson, Birdsall, and Fox, 1954). Van den Brink explains the apparent difference between the observed and theoretical slopes below the critical bandwidth in essentially the same way as Hamilton by saying that for noises wider than the critical bandwidth the observer is listening for the tone to appear in the noise, but that when the noise becomes as narrow as 10 Hz, the task becomes one of listening for the increase in the level of a fluctuating tone. The difference between the power prediction and the observed slope he labeled the "criterion effect" because he assumed it was due to this change in the nature of the task.

Before these last analyses, only the log-term average power was considered in the description of the stimulus. Hamilton and van den Brink mention fluctuations, but only to point up the different character of detection in narrow-band as opposed to wide-band noise. In terms of the filter-detector model one could say that the integration or averaging time of the power detector was assumed to be sufficiently long so that fluctuations in its output were unimportant.

Also, the idea of a sensory threshold that depended on the level of a stimulus but not its fluctuations was the usual basic assumption of psychophysical theory at the time of Fletcher. The object of a threshold measuring experiment was to find the signal level at which the tone would be heard half the time. It was assumed that when the tone was not present it could not be heard.

Signal detectability theorists have shown, however, that reasonable formal models of the noise-masked tone stimulus have the property that no detection system can distinguish perfectly between the noise plus tone and the noise alone. False detections of the tone when it is not present are an inevitable consequence of the random character of the noise (Peterson, Birdsall, and Fox, 1954). The presence of the tone only changes the probability distribution over the same set of possible detector outputs. It has been shown that 50% detection threshold levels can be strongly affected by manipulations of the effective pay-offs to the observer for his correct detections and his errors, but that nearly bias-free measures of sensitivity are possible when the false detections are used in conjunction with the correct detections to specify the observer's performance (Swets, 1961). In the Swets, Green, and Tanner (1962) study, this sort of analysis was done to specify the observer's performance, but the effects of varying the bandwidth of the noise upon the variability of power passing through the observer's internal filter were ignored.

Signal detectability theory suggests then that in our model we should not use Fletcher's rule that at threshold there is a constant ratio between the signal power and the noise power passed by the filter. It suggests that we should instead look at the statistical distributions of the output of the voltmeter in the two cases of noise alone and tone plus noise. Using a noise generator, a variable width band-pass filter, and a voltmeter, it is easy to demonstrate that when the average noise power level is held constant, the narrower the noise band passed by the filter, the larger the fluctuations in the needle on the meter.

De Boer (1962) argues that these fluctuations are important in the critical bandwidth experiments and that the narrower-than-critical-bandwidth part of the threshold curve must decline at a rate much less than 10 dB per decade (3 dB per octave) of filter width. Experiments measuring the discriminability of intensity changes in narrow-band noise led him to conclude that the slope could only be 4 dB per decade. De Boer drew a line with that slope through the previously collected critical bandwidth data and concluded that the data are consistent with such a slope. The intersection of the sloped line with the horizontal line is then moved toward the higher frequencies, indicating a critical bandwidth of over 200 Hz at a center frequency of 1000 Hz. This estimate of the critical bandwidth is in agreement with other kinds of critical bandwidth measurements that measure the amount of interaction between signals of different frequencies (Zwicker, Flottorp, and Stevens, 1957).

De Boer's argument was refined by Green and Swets (1966), who derived a formula for the threshold signal-to-noise ratio's dependence upon external filter bandwidth. Their derivation assumes that the observer's ability to tell whether of not the tone was added to the noise is limited only by the variability in the total energy output of the internal filter during the time the tone was on or might have been on. Their formula, which appears later in this section as Equation 3, is essentially the same as that derived earlier by Pfafflin and Mathews (1962) for a single-tuned filter. Simplifying assumptions are made by Green and Swets, resulting in the simple formula that when the noise is narrower than the observer's critical bandwidth the threshold signal-to-noise ratio should be proportional to the square root of the noise bandwidth, that is, the slope below the critical bandwidth should be 1.5 dB per octave or 5 dB per decade. They fit the same data that de Boer used with a line having this slope and a critical bandwidth estimate of 500 Hz (Figure 4).

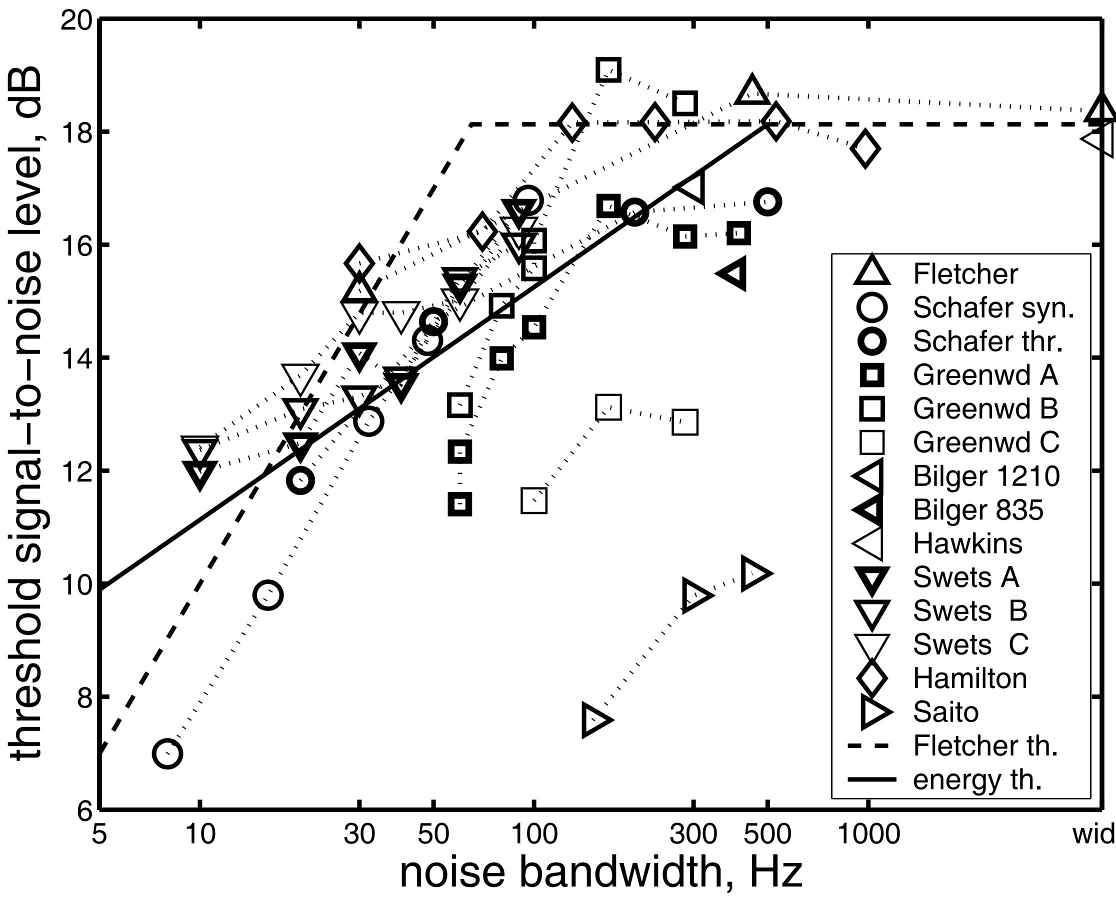


Figure 4. Comparison of Fletcher's model with the Green and Swets energy detection model. The energy detection model compared with data obtained in the original critical-band experiment and in several repetitions of it. The ordinate is the intensity in decibels of a sinusoidal signal (near 1000 Hz) that is just detectable in a band of noise whose width is indicated by the abscissa. The noise density is constant throughout any one series of experiments. The data indicate that as the noise band becomes narrower, the signal becomes easier to hear. The heavy solid line is the prediction of the energy-detection model, assuming that the internal filter is 500 Hz wide. The dotted line is Fletcher's approximation to his data. From Fletcher's curve one would assume that the internal filter is 60 Hz wide. (After de Boer, 1962).

At this point one can see that the experiments that compare the effects of external filters of different pass-bands have led to estimates of critical bandwidth ranging from 65 to 500 Hz. Also, the critical bandwidth estimates have been shown to vary with the duration of the signal tone, and there is no way of knowing that they are not affected by the width of the masking-noise spectrum. If this were the case, estimates based on the comparison of narrow and wide-band noise masking effectiveness obviously would be misleading. Van den Brink's description of the narrow-bandwidth masking task as intensity discrimination, whereas the wide-bandwidth task is described as more like pattern discrimination, suggests that different kinds of models might be appropriate in the two situations. Also, narrowing the noise bandwidth might be decreasing the observer's uncertainty as to the center frequency of the tone.

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Bandwidths Estimated from Detection Performance

Fletcher's original model offered a way out of this situation by providing a means of estimating the critical bandwidth using only the wide-band masking noise. If one knows the ratio at threshold between the tone-signal power and the noise power passed by the critical bandwidth, the observer's threshold ratio of signal power to noise power density can be divided by this masking ratio to give an estimate of the critical bandwidth. Fletcher thought this ratio (c in Equation 1) was about one, so the fact that and observer's threshold ratio of tone-signal power to noise power density is about 60 at a center frequency of 1000 cycles gives a critical bandwidth estimate of about 60 cycles. This critical ratio method of measuring the critical bandwidth was used by Hawkins and Stevens (1950) to plot frequency selectivity as a function of tone frequency.

Jeffress (1964) computed the theoretical performance of a narrow-band filter followed by an envelope-height detector. Assuming that detection is limited only by the variability in the envelope height, Jeffress estimates what he call the effective bandwidth as the width of a filter which when followed by an envelope-height detector would perform as well as the observer. That is, it would give the same percentage of correct detections of the tone when it was making the same percentage of false detections of the tone when it was absent. Jeffress applied this analysis to the wide band noise masking data of Green, Birdsall, and Tanner (1957) for a 1000 Hz tone of 250 msec duration and obtained a bandwidth estimate of only 20 Hz. Table 2 give values of effective bandwidth.

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Source of Estimate	**Source of Data**	**Tone Frequency in Hz**	**Tone Duration in seconds**	**Critical Bandwidth Estimate in Hz**	**Type of Detector**
Jeffress (1964)	Watson, Rilling, and Bourbon (1964)	500	0.25	30-49	envelope theory
Jeffress (1964)	Green, Birdsall, and Tanner (1957)	1000	0.1	50	envelope theory
	Green, Birdsall, and Tanner (1957)	1000	0.25	20	envelope theory

Jeffress (1964)	Hamilton (1957)	800	0.1	108	envelope theory
	Hamilton (1957)	800	0.4	37	envelope theory
Green and Swets (1966)	consensus	1000	0.1	500	energy theory
Sherwin et al. (1956)	Sherwin et al. (1956)	1000	0.3	40	electrical energy

Table 2. Critical bandwidths estimated by comparing observer's performance with theoretical or observed performance of energy and envelope detectors.

If the width of the filter is the reciprocal of the duration of the tone, the envelope height of the filter output at the end of the tone is an optimal statistic (Peterson, Birdsall, and Fox, 1954). However, when the filter width is much wider than this optimal value, averaging the height over the duration of the stimulus is much more efficient: performance is better for the same filter width.

In effect, the energy detection models presented by Pfafflin and Mathews and by Green and Swets perform this averaging, because the square of the envelope height passed by the filter is summed over the duration of the tone. Green and Swets assume that the filtered noise has a rectangular spectrum and that the tone signal is not affected by the filtering. The latter assumption is satisfied only when the signal duration is long compared with the reciprocal of the filter bandwidth. Their model allows the derivation of the means (m) and variances (s^2) of the energy statistic distributions in the tone-signal plus noise (SN) and noise only (N) situations. If we define a detectability index, d, by the formula

$d = (m_{SN} - m_N) / ((s_{SN}^2 + s_N^2)/2)^{0.5}$, (Equation 3)

Green and Swets show that for the energy detector,

$d = (E/N_0)/(W T + E/N_0)^{0.5}$, (Equation 4)

where E is the energy of the tone, N_0 is the power per unit bandwidth of the noise, T is the duration of the signal tone and of the observation interval when the tone is not present, and W is the width of the rectangularly-shaped band of noise.

Pfafflin and Mathews derive a formula for the case of a single-tuned filter which also assumes that the bandwidth is wide compared with the duration of the observation interval and which reduces to Equation 4 if in addition it can be assumed that the filter width is narrow compared with its center frequency, and if we replace W by (π/4) B, where B is the half-power bandwidth of the single-tuned filter.

Green and Swets estimate a bandwidth in the same manner as Jeffress, finding the width of a filter which when followed by an energy detector would perform as well as the observers. They estimate a critical bandwidth of 1000 Hz for a signal tone of 1000 Hz and 100 msec duration. Jeffress' effective bandwidth estimate for this level of performance is 70 Hz.

Green and Swets bring their estimate down to 500 Hz, the estimate they obtained from their analysis of the masking data with noise of varying width, by arbitrarily assuming that the observer is uncertain as to the time of occurrence of the tone and integrates the filter output over twice the duration of the tone.

These estimates of the critical bandwidth based on observer performance are based on comparisons of performances of observers with detection models that are perfectly consistent. If the same stimulus is fed into the model diagrammed in Figure 1a, the same response comes out. This is not the case for the human observer. Swets et al. (1959) and Green (1964) recorded the stimuli on tape so that the same stimulus could be presented more than once to the same observer. These investigators modified the detection model in a way equivalent to assuming that the criterion K in Figure 1a is a random variable. Both the improvement in detection performance afforded by multiple observations of the same signal measured by Swets et al., and the consistency of judgments for the same stimulus measured by Green indicated that the variance of the criterion K was about equal to the variance of the stimulus measure ϕ, that is, only about half the variability can be attributed to the stimulus. Watson (1962) verified this result by having three observers listen to the same signal.

Correcting performance-based critical bandwidth estimates for criterion variability results in narrower estimates of the critical bandwidth. For the Green and Swets energy detection model the bandwidth is approximately proportional to the variance of the stimulus energy statistic. If the estimates of this variance have been inflated by a factor of about two by criterion variability, so have the critical bandwidth estimates.

The bandwidth estimates based upon detection performance have as wide a range as the estimates based upon the data from the experiments in which the bandwidth of the masking noise was varied. The assumptions underlying the estimates are just as untenable. Clearly not all the variability in the observers' response is attributable to the stimulus. Also, even if the internal noise or criterion variability were accurately parceled out, only the product of the bandwidth and integration time could be estimated. A more direct approach is needed.

Introduction to the Experimental Design

The experimental part of this paper is an attempt to make a more direct measurement of the width of the critical bandwidth in the wide-band masking situation. The experimental plan is suggested by experiments seeking to test the adequacy of the energy detection model by seeing whether an energy detector and human observers classify the same samples of noise-masked tone the same way.

The first such attempt compared human observers with an electrical energy detector. Sherwin et al. (1956) tape-recorded randomly spaced 1000 Hz tone bursts mixed with continuous wide-band noise. Observers listened to the tape and responded when they thought a tone burst occurred. At the same time the signal from the tape was processed by an electrical detector, which consisted of an RLC (single-tuned) band-pass filter, a square law detector, and an exponential integrator, all followed by a chart recorder. Tapes for four signal durations were constructed (0.03, 0.1, 0.3, 1.0 seconds). The integrator time constant was set at half the signal duration to optimize the performance of the electrical detector. For all durations the electrical detector correlated positively with the percentage of observer detections of individual tone burst signals. This correlation was best for the 0.1 and 0.3 sec duration signals, so the authors concluded that the observers' effective integration times were near these values.

The half-power bandwidth of the electrical filter was 60 Hz. The observers, however, performed better than the electrical detector, making about half the number of false alarms for the same 60% level of correct detections. The 60 Hz filter would have to be narrowed about 30% so that a fixed threshold electrical detector would have the same performance as the observers. If the average of the observer ratings had been used, so that effects of criterion variability would have been suppressed, an even narrower filter would have been necessary to equal the observers' performance level. Sherwin et al. conclude that the observer's pass band is 30 Hz or less in width.

Watson (1962) and Pfafflin and Mathews (1966) have also correlated the output of band-pass filters followed by energy detectors with the responses of observers to noise-masked tone bursts. Watson improved the experimental situation by having the stimulus be either a gated burst of tone-plus-noise or a gated burst of noise alone. The false detections could thus be more exactly correlated with the stimulus that produced them. Watson used a 100 Hz-wide filter in his electrical detector. Pfafflin and Mathews used reproducible noises generated by an on-line computer. They show correlations between the observer responses and the energy passed by a 100 Hz-wide digital filter. Like Watson, they used gated bursts of noise so that the integration of filter output was over the entire duration of the stimulus. They mention that a 50 Hz-wide filter and a 100 Hz-wide filter gave about the same result, but that the output of wider filters does not correlate as well with the observer responses.

The aim of the present experiment was to follow the basic plan of these experiments, but to correlate the observers' responses with the output of a sequence of energy detectors varying only in the bandwidth of the filter. Stimuli were taped as in the Sherwin experiment. Pairs of observers responded to single noise bursts, as in the Watson experiment. And the electronic detector was simulated on a digital computer as in the

Pfafflin and Mathews study. The half-power bandwidth of the filter whose energy output correlates best with the observer responses should provide a reasonably direct measure of the critical bandwidth in this situation.

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Experimental Procedure

The apparatus for recording the stimulus tape is diagramed in Figure 5. The equipment manufacturers and models appear in Appendix Table A1. The noise source was attenuated 3 dB and mixed with the 30 dB attenuated output of the audio oscillator, whose frequency was set at 500 Hz with the aid of an electronic counter. This mixed signal fed into a band-pass filter set to pass frequencies from 250 to 750 Hz, and from there into an electronic switch set for a rise-decay time of 5 msec. The signal bursts from the switch were fed into the two-channel magnetic tape recorder, where they were recorded at 7.5 inches per second onto low-print magnetic tape.

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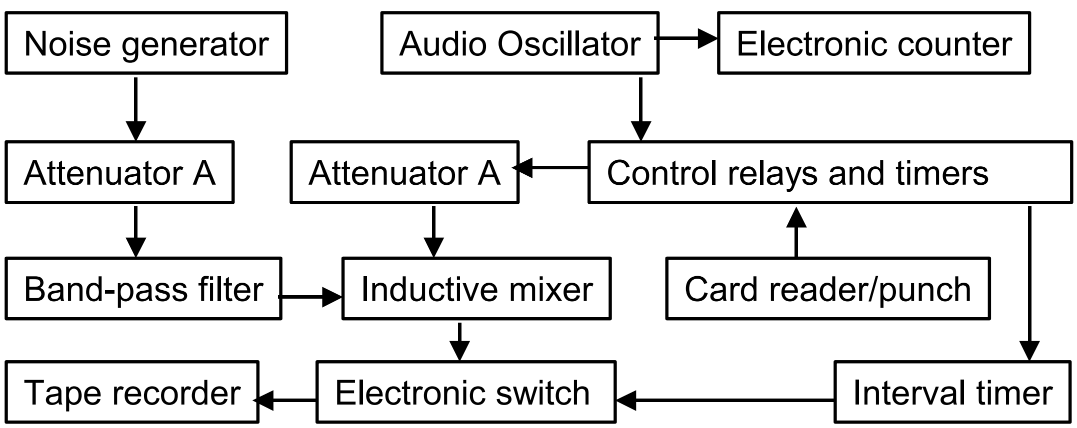

Figure 5. Diagram of the stimulus generating apparatus.

The summary punch read cards that controlled whether the oscillator was connected or shorted during the 100 msec stimulus interval. The sequence of 65 tone-no tone trials was punched into the cards according to a random number table using a procedure that made the probability of either kind of trial 0.5. The interval between trials was 7.2 seconds. The trial location and trial type were indicated by marker tones on the second channel of the tape.

The apparatus for playing back the tape to the observers is diagramed in Figure 6. Figure 7 shows the power spectrum of a continuous noise stimulus measured at the earphone terminals. The measurements of an audio frequency spectrometer and level recorder were corrected (-3 dB per octave) for the increasing width of the 1/3 octave band-pass filters. The stimulus channel was fed though an attenuator to two pairs of binaural earphones connected in parallel. Each observer responded by means of a knife switch, pressing to the left if he thought the stimulus burst contained a tone, or to the right if he thought only the noise was present. The observers had a 3.6 second interval in which to respond. The response interval was started by the experimenter as soon as he heard the stimulus burst through monitoring headphones. The response interval was indicated to the two observers by means of a white light on the wall in front of them. The observers were separated from each other by means of an opaque partition.

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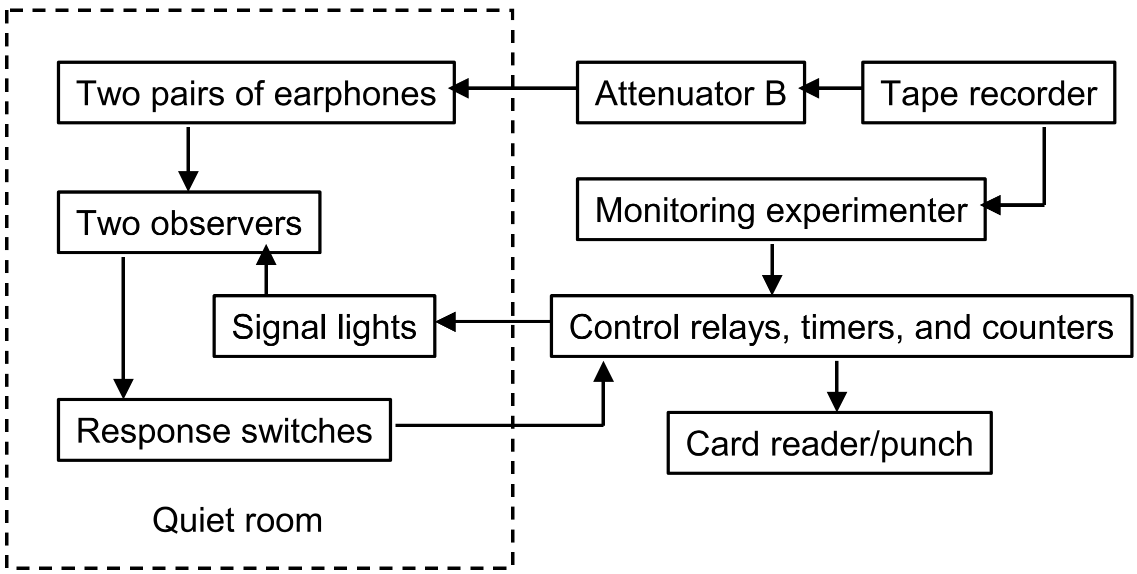

Figure 6. Diagram of the stimulus playback apparatus.

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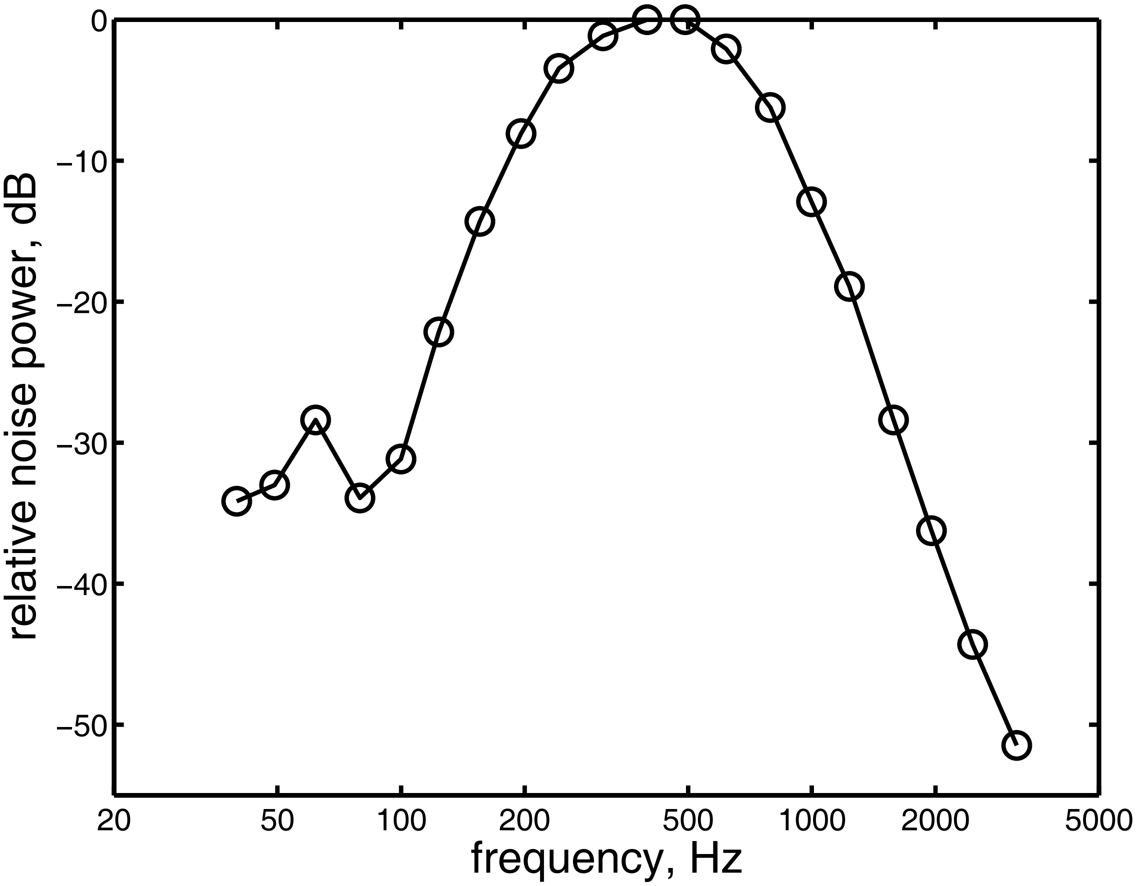

Figure 7. Power spectrum of the noise stimulus.

Three students and the experimenter served as observers. All had normal hearing as determined by a standard audiometric test. One of the observers was female (KD). None had previous experience in a noise-masked tone detection experiment, but all had served as subjects in other psychological experiments. AA and KD completed the experiment as a team before TA and RJ were run as a second team.

The observers were instructed to try to make as few errors as possible. In the initial training sessions, feedback lights indicated to the observers whether or not the noise burst contained a tone. Also, between each 65 trial session, both during training and the experimental runs, the observers returned to the equipment room and were shown counters displaying the number of correctly identified tones (hits), the number of tones missed, the number of plain noises called tone trials (false alarms), and the number of correct noise detections. If their pattern of errors showed extreme response bias or if they performed unusually poorly, they were so informed. Three to five sessions were run per day, with about 5 minutes between sessions. The tone-no tone sequences during training were all different and were generated in the same manner as the experimental sequences. The early training sequences used live stimuli and large signal-to-noise ratios; the final training sessions were similar in all respects to the experimental situation, in which the stimuli were on tape, no immediate feedback was given to the observers, and the signal-to-noise ratio in terms of E/N_0 was 10.7 dB with an overall stimulus sound level of 80 dB SPL. E/N_0 was measured as described in the Green and Swets (1966) appendix using a single-tuned filter with a 55 Hz half-power bandwidth. AA and KD received a total of about 2000 training trials before the experimental runs; TA and RJ received about 1500 such trials. Three experimental sessions were run on one day and two on another, so that each of the 65 stimuli had 5 judgments by each of the 4 observers. At least three of the observers gave no evidence of understanding the purpose of the experiment, even when some attempt was make afterward by the experimenter to explain it.

Before and after the experimental runs with the observers, both channels of the stimulus tape were converted from analogue to digital form by a Scientific Data Systems A/D converter controlled by a Scientific Data Systems model 930 computer. The conversion rate was 4000 samples per second. The digital representation consisted of 10 bits plus a sign bit. The stimulus actually processed by the filter program consisted of 192 msec (768 samples) of digital record approximately centered in time about the 100 msec burst of noise.

If we let X(i), i = 1, 768 represent a stimulus sequence, the filter program computed a filtered output sequence Y(i) according to the difference formula for the digital equivalent of a simple resonator (Rader and Gold, 1965):

$$Y(i) = K_1 Y(i-1) + K_2 Y(i-2) + LX(i),\ Y(0) = Y(-1) = 0,$$

where

$$L = w = 2\pi fT$$
$$K_1 = (1-w)\ 2^{0.5} \quad \text{(Equation 5)}$$
$$K_2 = -(1-w)^2$$

These are narrow-band approximations of the Rader and Gold coefficient formulas for the case where the center frequency of the resonator is 1/8 the sampling rate (1/T), and the nominal 3-dB bandwidth is 2f. The filter was allowed to ring (the filter output Y was computed for the input x equal zero) for an additional 2/f seconds following the stimulus. The energy passed by the filter was then computed as

$$E = {}_{i=1}\Sigma^{768+R}\ Y(i)^2, \quad \text{(Equation 6)}$$

where R is the greatest integer less than 8000/f. This energy output, E, was computed for nominal 3-dB bandwidths of 10, 20, 40, 80, 160, 250, 350, and 500 Hz.

The actual frequency responses of the filters appear in Figure 8. These were found by computing the difference-equation coefficients according to the narrow-band approximating formula and then computing the actual squared-amplitude response from the formula

$$A^2(f) = 1/[(1-K_1 \cos u - K_2 \cos 2u)^2 + (K_1 \sin u + K_2 \sin 2u)^2], \quad \text{(Equation 7)}$$

where f is the frequency in Hz, u = 2πfT, and 1/T is the sampling rate. For the 80 Hz and narrower filters, the narrow-band approximations were adequate. The wider nominal bandwidths resulted in even wider filters, with the maximum shifted slightly towards the low frequencies. The resulting 3-dB bandwidths were 10, 20, 40, 81, 176, 303, 494, and 720 Hz. The 720 Hz bandwidth is a high frequency 3-dB cutoff value as the low frequency response of this filter did not fall as far as 3 dB.

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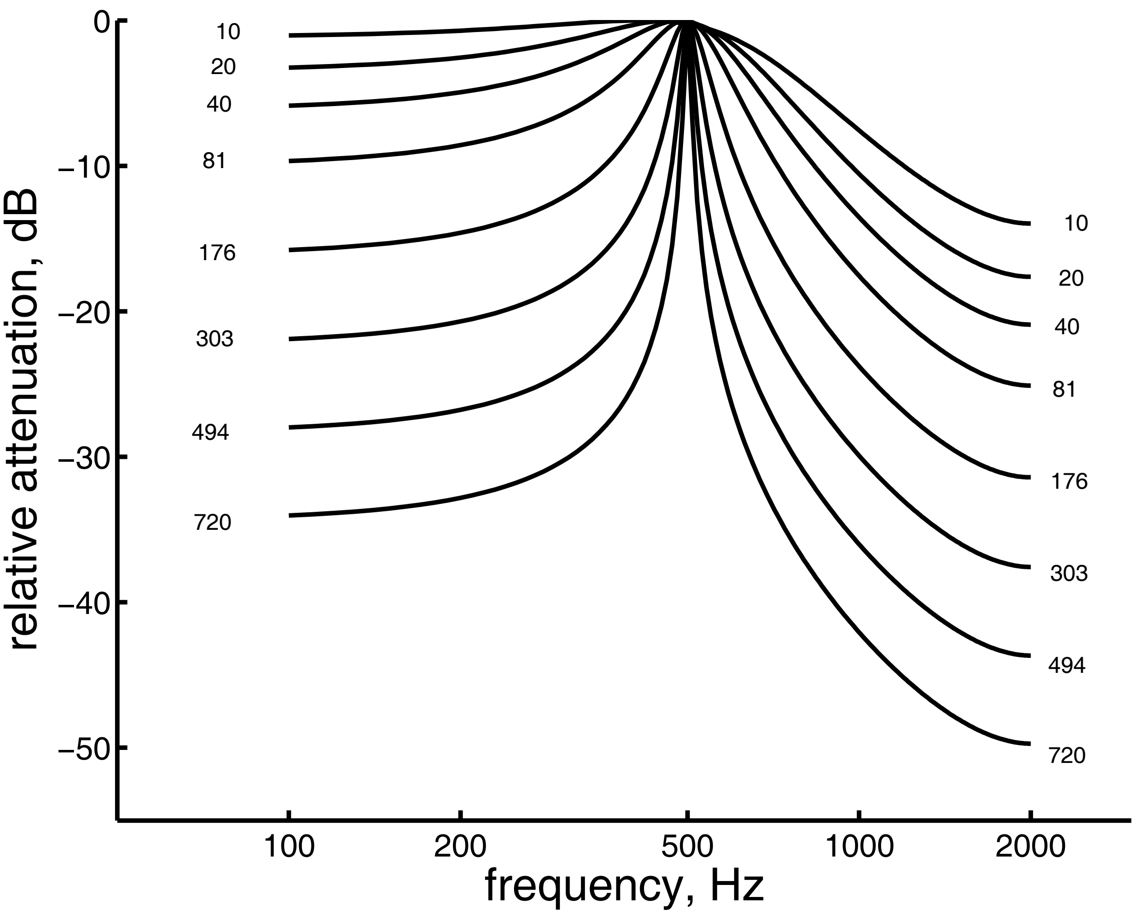

Figure 8. Digital filter amplitude responses in dB relative to the response at 500 Hz as a function of input frequency in Hz from Equation 7. The parameter is the 3-dB bandwidth of the filter in Hz.

The first five stimulus bursts were dropped from the analysis because the observers knew what the signal condition was on those trials. This left 33 tone-plus-noise trials and 27 noise-only trials. For each stimulus, totals were computed of the number of times (0-5) each observer said that a tone was present and of the number of times (0-20) any observer said a tone was present. Then, separately for the tone-plus-noise trials and the noise-only trials, the rank correlation between the energy statistic E and the number of tone responses was computed for each filter width. For comparison, the rank order correlations between the energy outputs of filters of different widths were also computed. These computations were done only for the results of the first digitization of the stimulus tape since the energy outputs of the second digitization ranked nearly perfectly with the outputs of the first for the 10 and 160 Hz wide filters (rank correlations >= 0.99).

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Results

Although the principal result comes from the correlation of the observer response totals with the digital filter outputs, to establish continuity with the prior investigations critical bandwidths are first estimated from the observer' detection performance.

The performance of the observers by sessions is collected in Table 3, which has the proportion of hits P_H, the proportion of false alarms P_F, and an index of detectability $d' = z(P_H) - z(P_F)$, where z() is the inverse cumulative normal distribution function. In computing d', proportions of 1 and 0 were replaced by 1-(1/120) and 1/120, respectively. The first trial performance levels are appropriate for estimating bandwidths from performance in the same manner as did Jeffress and Green and Swets. Table 4 gives these estimates. The effective bandwidths in dB are computed by finding the effective signal level in dB corresponding to the level of performance from Figure 5 in the Jeffress article, and subtracting this from the ratio of the signal power to the spectral power of the noise, which in this experiment was 20.7 dB. To obtain the estimates based on the Green and Swets analysis one solves Equation 4 for the bandwidth W.

$W = [((E/N_0)/d')^2 - E/N_0]/T$, (Equation 8)

where the stimulus duration T = 0.1 second, and $E/N_0 = 11.8$. The rough agreement with the results of other investigators shown in Table 2 indicates that the performance of the observers was comparable to those of other studies.

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	Run	**1**	**2**	**3**	**4**	**5**	**Average**	**Median d'**	**Response total based d'_e**
Observer									
AA	P_H	.727	.727	.697	.758	.697	.721		
	P_F	.148	.074	.000	.074	.000	.059		
	d'	1.65	2.08	2.58	2.18	2.58	2.13	2.18	2.34
KD	P_H	.606	.576	.667	.606	.727	.636		
	P_F	.259	.222	.370	.296	.444	.319		

	d'	0.92	0.95	0.77	0.80	0.76	0.83	0.80	0.99
TA	P_H	.939	.788	.727	.697	.758	.782		
	P_F	.593	.481	.407	.296	.407	.437		
	d'	1.32	0.86	0.84	1.05	0.94	0.92	0.94	1.14
RJ	P_H	.879	1.00	.939	.848	.788	.891		
	P_F	.519	.556	.370	.037	.148	.326		
	d'	1.12	1.90	1.88	2.79	1.84	1.67	1.88	3.16
Group	P_H	.788					.758		
	P_F	.380					.285		
	d'	1.11					1.26		2.32

Table 3. Proportions of correct tone detections, P_H, proportions of false detections, P_F, and detectability index, $d' = z(P_H) - z(P_F)$.

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Method of Computing Bandwidth		**Bandwidth estimate in Hz**				
		AA	**KD**	**TA**	**RJ**	**Group**
First run performance compared with theoretical envelope detector performance		42	87	59	74	74
First run performance compared with theoretical energy detector performance		393	1520	681	993	1010
Response total performance compared with theoretical envelope detector performance		25	81	69	16	26
Response total performance compared with theoretical energy detector performance		139	1330	950	66	144
Response total performance compared with computer-simulated energy detector performance		135	>720	>720	68	137
Response totals correlated with computer-simulated energy detector outputs	SN trials	20	10	41	20	10
Response totals correlated with computer-simulated energy detector outputs	N trials	81	81	303	176	176

Table 4. Critical bandwidths estimated from the current experiment (methods of estimation explained in the text).

Better estimates of observer bandwidths using these same formulas are obtained by using the observer's total yes responses to each stimulus to minimize the variability in the observer's responses not attributable to the stimulus, such as criterion variability. The response totals were analyzed using a procedure developed for measuring observer detection performance when the observer has been asked to give a several-category rating response instead of a yes or no response (Green and Swets, 1966). For each observer the total number of yes responses was computed for each stimulus and two cumulative distribution functions F(T) were constructed, one for the signal-tone-plus-noise (SN) stimuli and the other for the noise-only (N) stimuli. F(T) is the proportion of stimuli for which the number of yes responses was less than or equal to T, which had values 0, 1, ... , 5 for the individual totals and values 0, 1, ... , 20 for the group totals. The points $\{z[F_{SN}(T)], z[F_N(T)]\}$ were then plotted, giving the usual Receiver Operating characteristic (ROC) curve in normal coordinates (Figure 9). Points for which F(T) is 0 or 1 do not appear. Straight lines were drawn by eye through the data and the index of detectability d'_e was used to estimate bandwidths in the same way as the d' estimate based on the first run performance was used.

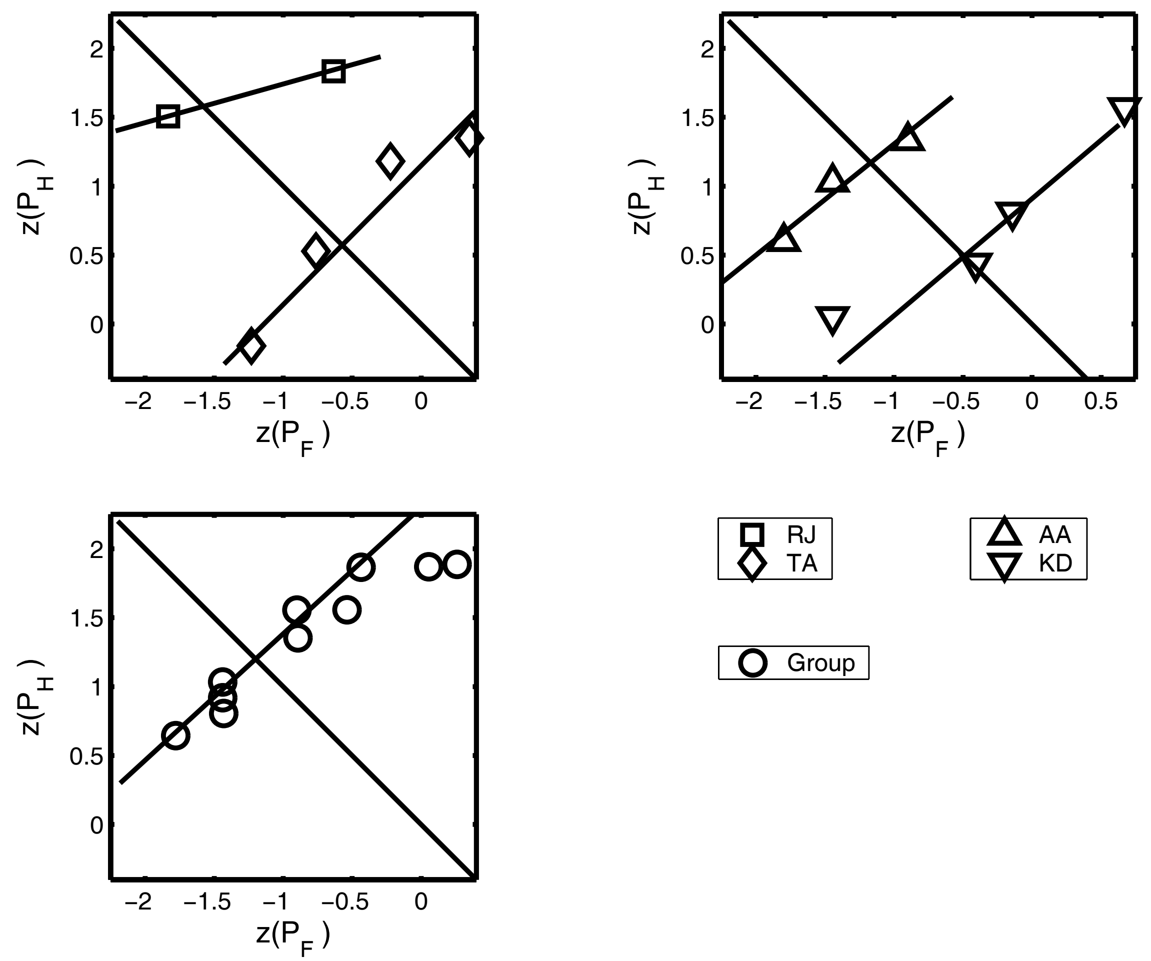


Figure 9. Receiver operating characteristic curves in normal coordinates generated from observer response totals. Ordinates are $z(P_H)$. Abscissas are $z(P_F)$. The corresponding d'_e values appear in Table 3.

The increase in the values of the detectability index d'_e in Figure 9 over the values of d' based upon the average values of P_H and P_F in the last column of Table 3 indicates the response totals do separate the tone-plus-noise trials from the noise-only trials better than do the responses from a single run of the stimulus tape.

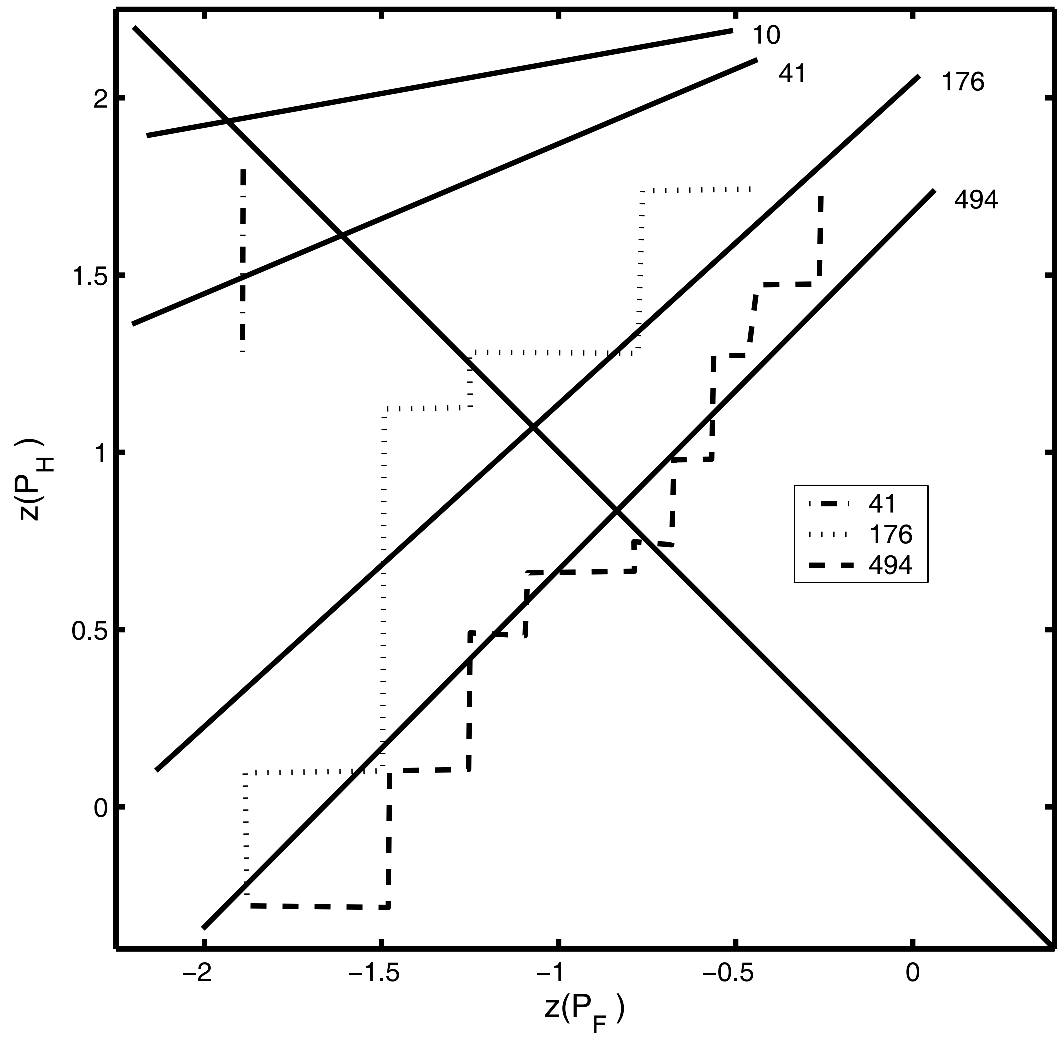
The resulting bandwidths, appearing in Table 4, indicate that two observers performed worse than an energy detector with no filter, since the noise was pre-filtered to a width of 500 Hz. The other two observers performed as well as energy detectors with bandwidths in the region of 100 Hz in agreement with the narrower critical bandwidth measurements.

Performance-based critical bandwidths were also estimated by comparing the performance of the observers with the actual performance of the digitally-simulated energy detector.

The performance of the model was evaluated in two ways. Points on the ROC curve were computed just as done above for the observers (Figure 10). No points appear for the two narrowest filters because the outputs for SN and N did not overlap. In addition, the means (*m*) and standard deviations (*s*) of the two energy output distributions were computed and the detection index was estimated as

$d'_e e = (m_{SN} - m_N) / ((s_{SN} + s_N)/2)$. (Equation 9)

The straight lines in Figure 10 are ROC curves for normal distributions having means and variances equal to those of the filter outputs. They intersect the negative diagonal at $d'_e/2$ and have a slope s_N/s_{SN}. The fit indicates that it matters little which method is used, except that the second method works throughout the range of filters.



10
41
176
494
41
176
494
z(P_H)
z(P_F)
2
1.5
1
0.5
0
−2
−1.5
−1
−0.5
0

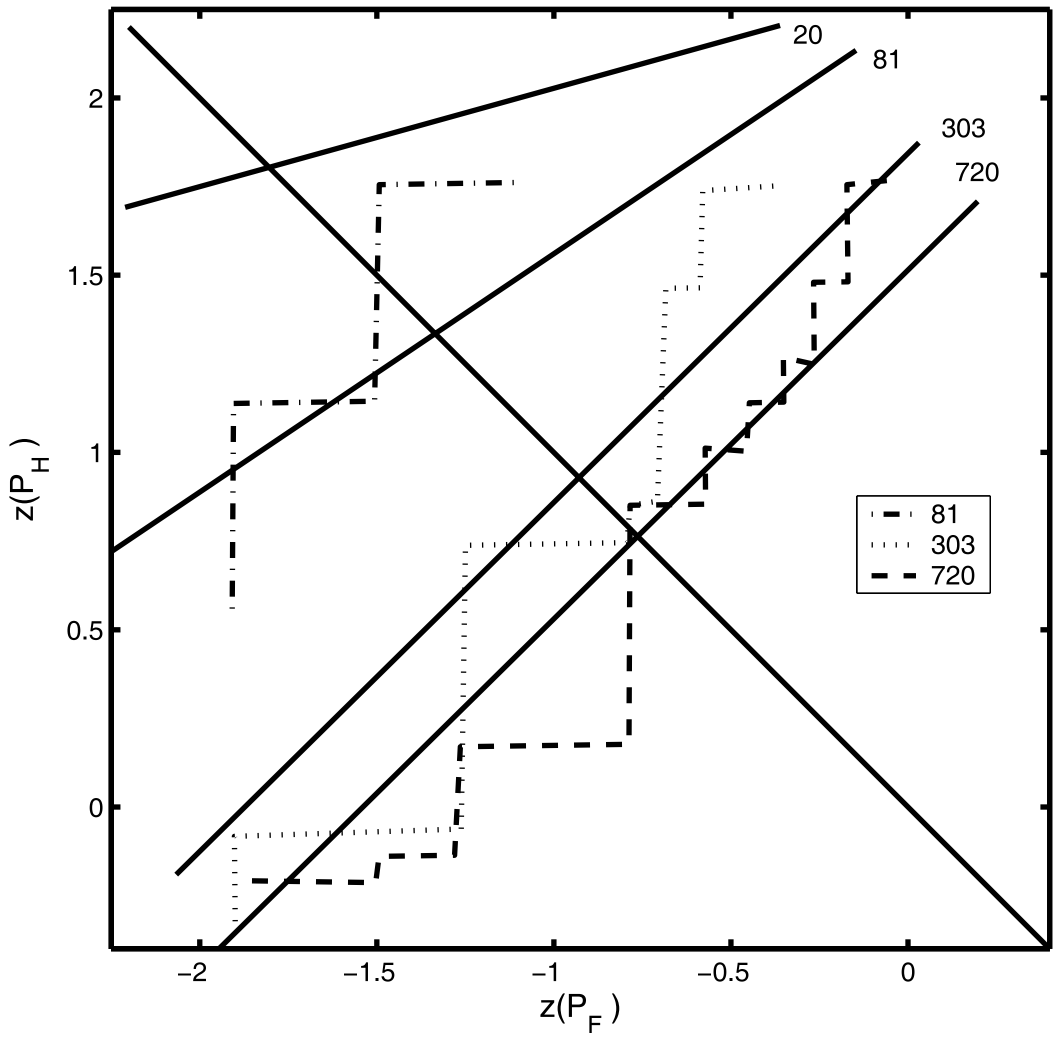


Figure 10. Receiver operating characteristic curves in normal coordinates based upon the digital-filter energy statistic. Ordinates are $z(P_H)$. Abscissas are $z(P_F)$. The stepped lines are computed as in Figure 9. The straight lines are curves for normal distributions with the means and standard deviations of the energy statistic distributions. The parameter is the 3-dB bandwidth of the filter.

The adequacy of the energy detection theory in predicting the performance of the computer-simulated energy detectors was evaluated by computing the detection index d for the filters as

$d'_e = (m_{SN} - m_N) / ((s_{SN}^2 + s_N^2)/2)^{0.5}$, (Equation 10)

and substituting this quantity for d' in Equation 8 to obtain bandwidth estimates. These estimates appear in Figure 11 together with wider estimates from the same equation modified for single-tuned filters and effective bandwidth estimates for comparison. The 10 Hz filter behaves more like the envelope detector because the impulse response duration of the filter is close to the duration of the stimulus burst. Final estimates of the observers' bandwidths based upon performance were made by plotting the performance (d'_e) of the filters as a function of their bandwidths (Figure 12) and finding the bandwidths corresponding to the observed detection performance of the observers. These estimates appear in Table 4 and agree roughly with the bandwidth estimates based upon Equation 8.

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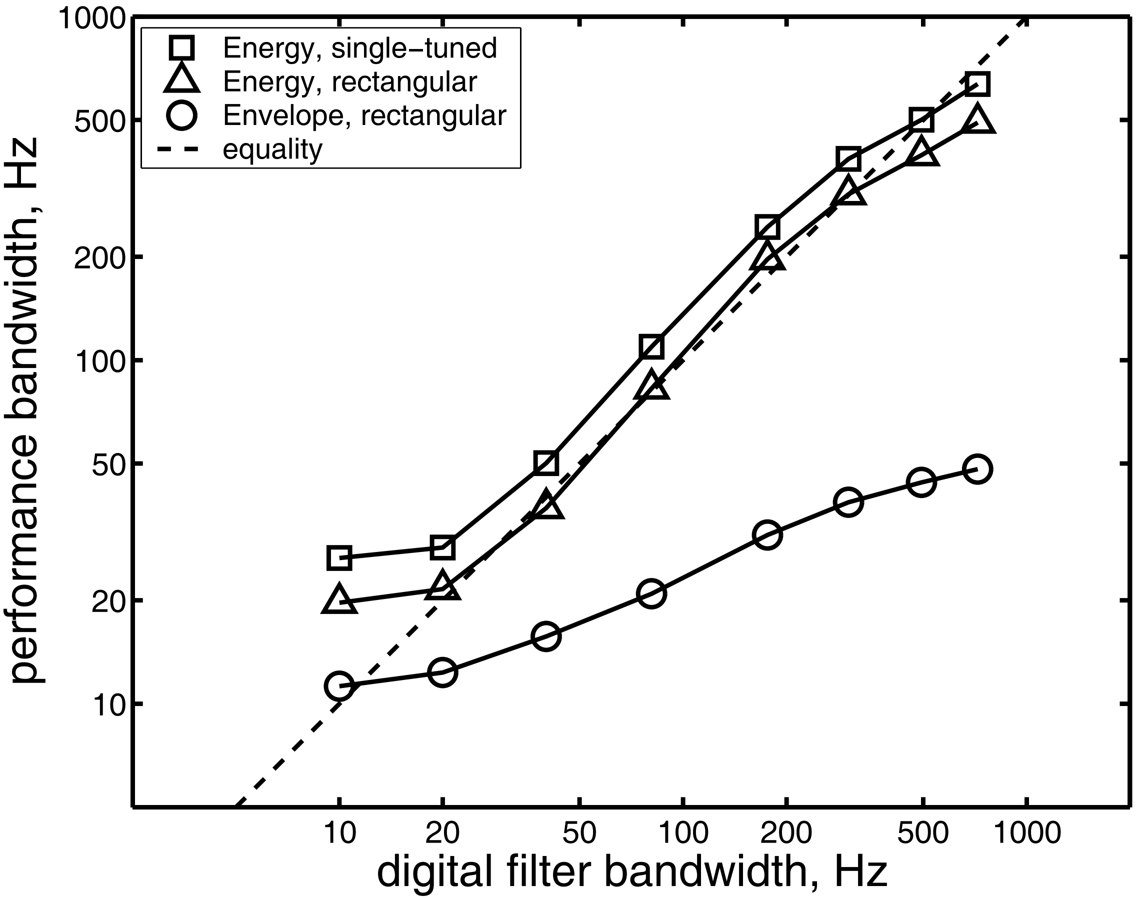

Figure 11. Bandwidths of the digital filters estimated from detection performance using energy and envelope detector theories.

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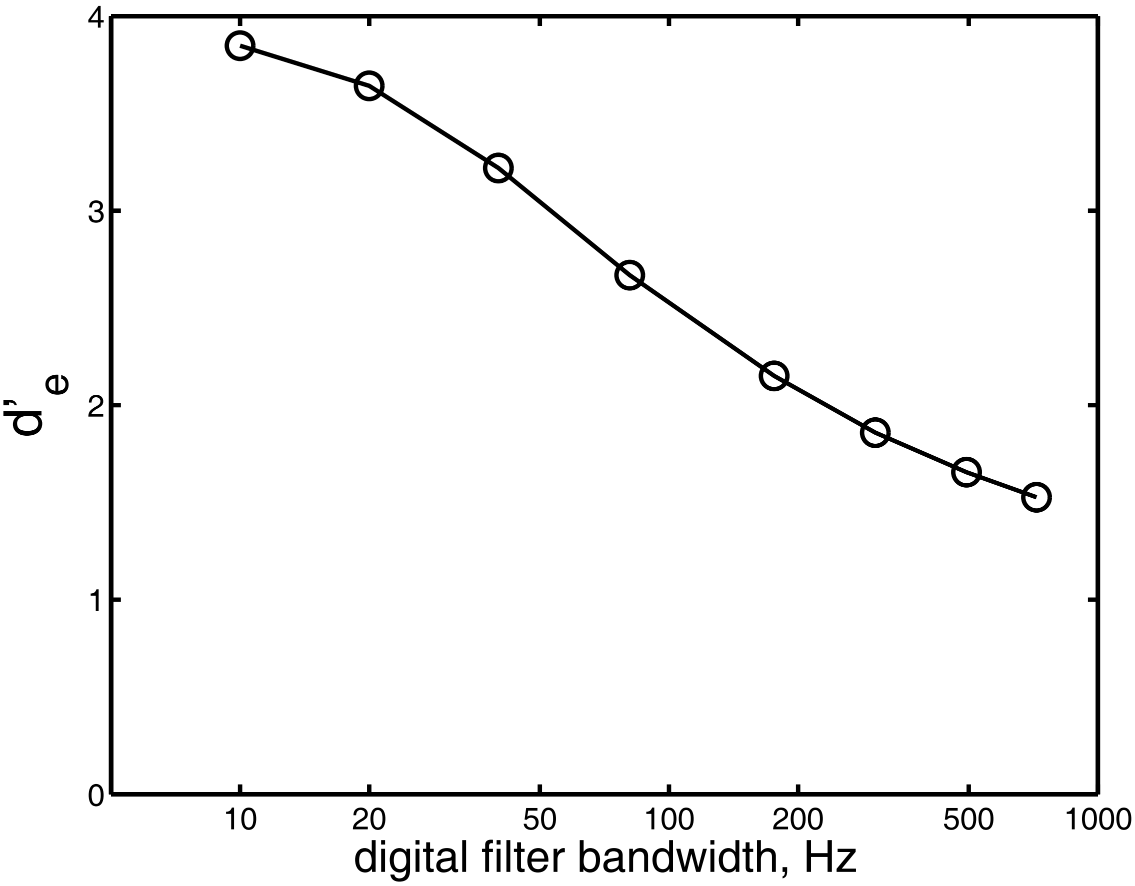

Figure 12. Detection performance (d'$_e$) of the digital-filter energy statistic as a function of a digital-filter bandwidth.

The Spearman rank correlation coefficient corrected for ties (Siegel, 1956) was computed between the observer's response totals and the filter outputs (Table 5) to find the width of the computer-simulated filter which best predicts the ordering of the observers' responses to the SN stimuli and the N stimuli separately (Figures 13 and 14). These curves are relatively flat because of the strong correlations between the filter outputs. The rank correlations of the filters with themselves appear in Figures 15 and 16 and may be regarded as predictions from the computer model for the shapes of the curves in Figures 13 and 14, respectively. The width of the filters which correlate best appear in Table 4 with the bandwidth estimates based on performance.

	Observer				
Filter width in Hz	**AA**	**KD**	**TA**	**RJ**	**Group**
33 SN (tone plus noise) Trials					

10	.744	.607	.408	.692	.837
20	.753	.598	.407	.703	.833
41	.709	.531	.437	.692	.795
81	.508	.444	.409	.594	.648
176	.255	.310	.434	.423	.455
303	.167	.257	.387	.363	.375
494	.135	.238	.364	.341	.342
720	.110	.251	.319	.318	.319
27 N (noise only) Trials					
10	.367	.439	.405	.350	.500
20	.463	.585	.351	.353	.549
41	.485	.742	.356	.446	.657
81	.530	.763	.385	.536	.721
176	.496	.751	.414	.572	.736
303	.482	.729	.444	.560	.735
494	.491	.708	.429	.527	.706
720	.513	.687	.431	.539	.705

Table 5. Rank correlations of computer-simulated energy detector outputs with observer's response totals.

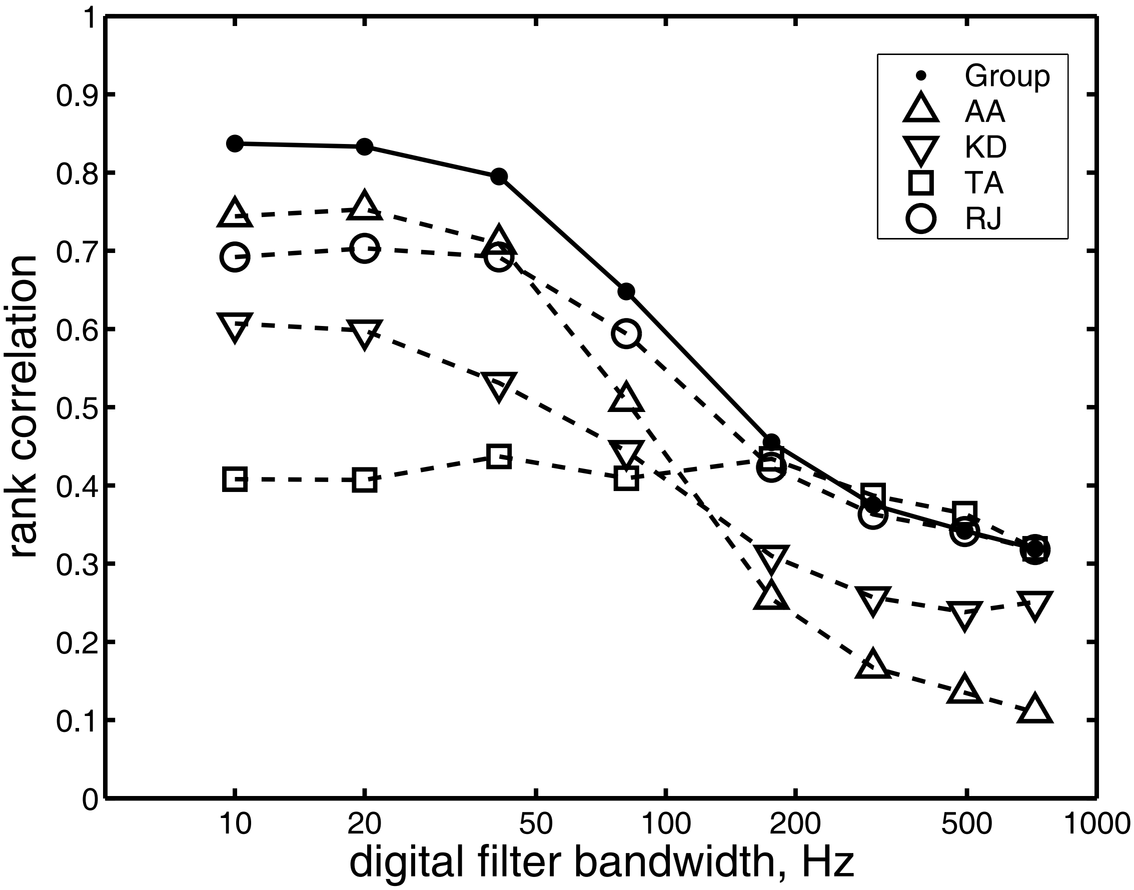

Figure 13. Rank correlation of observer response totals with digital-filter energy statistics (33 tone-plus-noise trials).

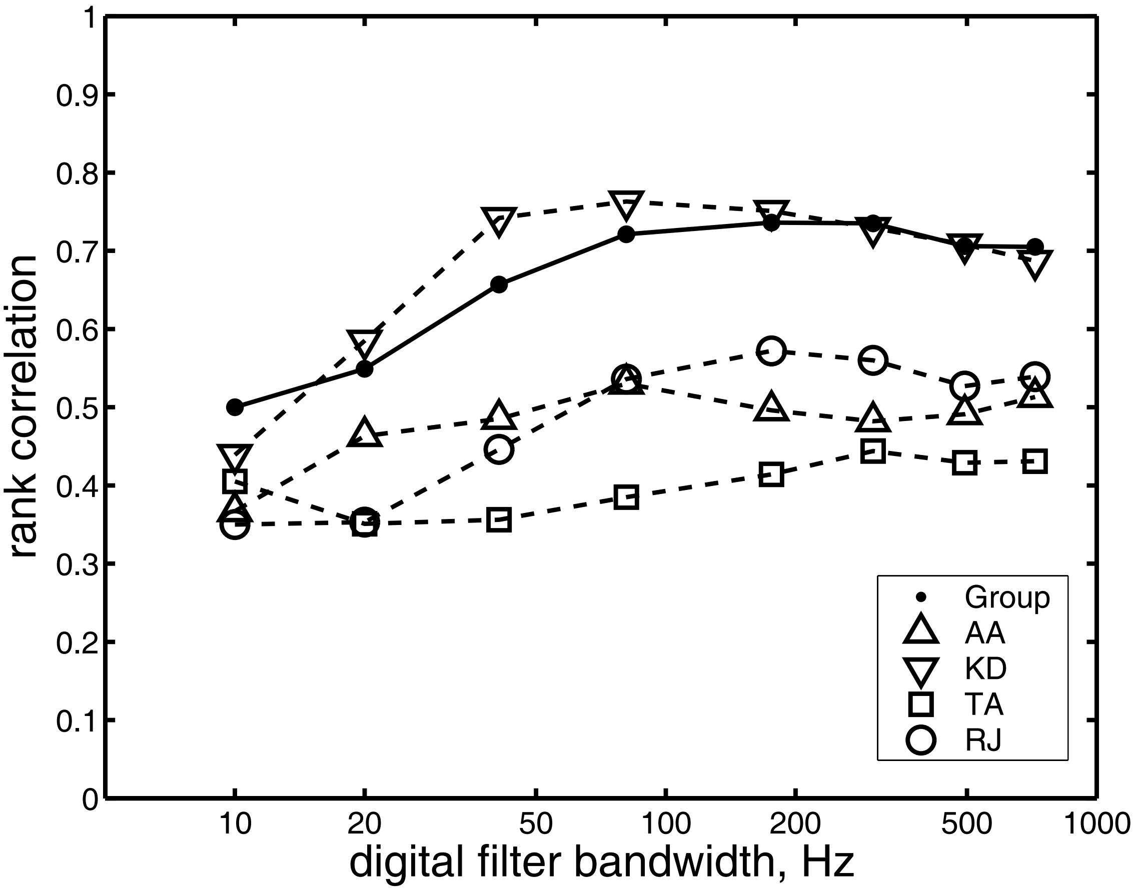

Figure 14. Rank correlation of observer response totals with digital-filter energy statistics (27 noise-only trials).

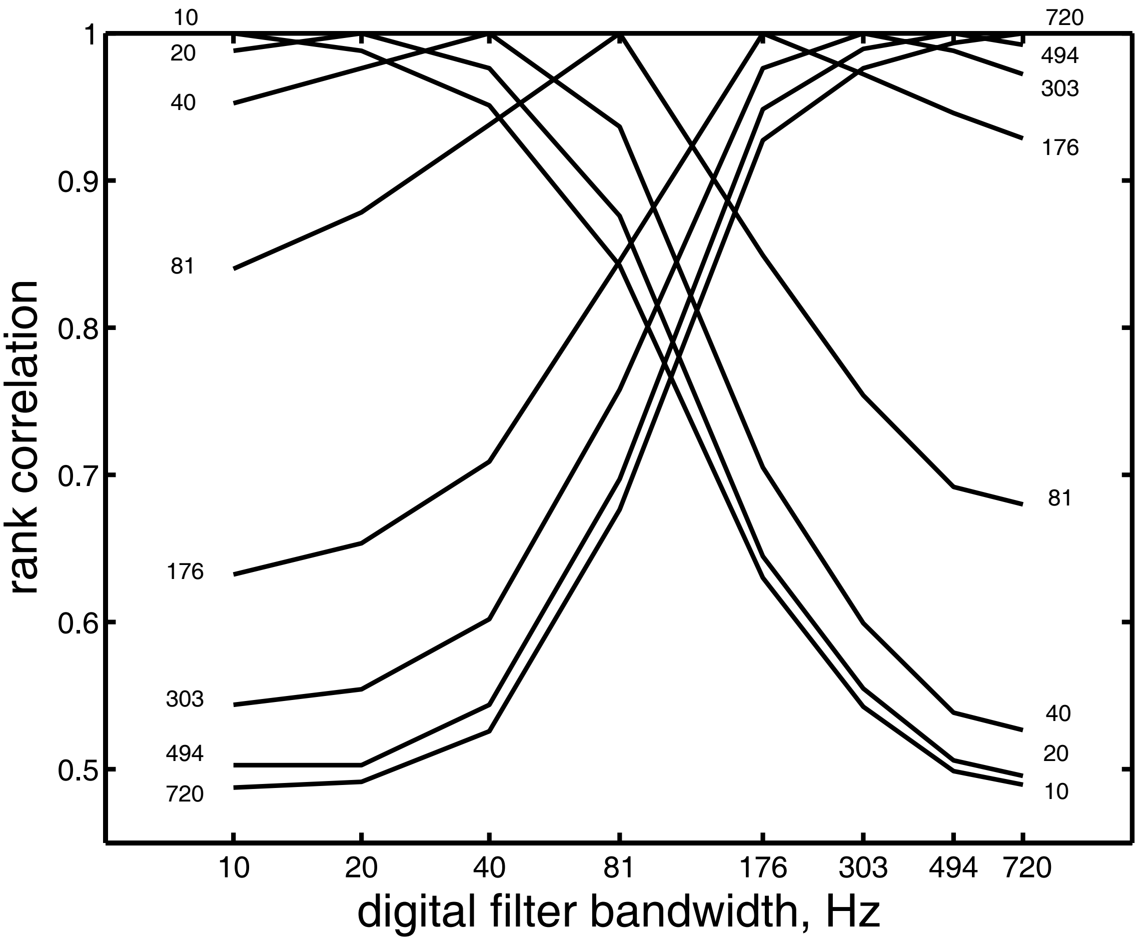

Figure 15. Rank correlations among filters of varying bandwidth (33 tone-plus-noise trials). The parameter is the 3-dB bandwidth of the filter.

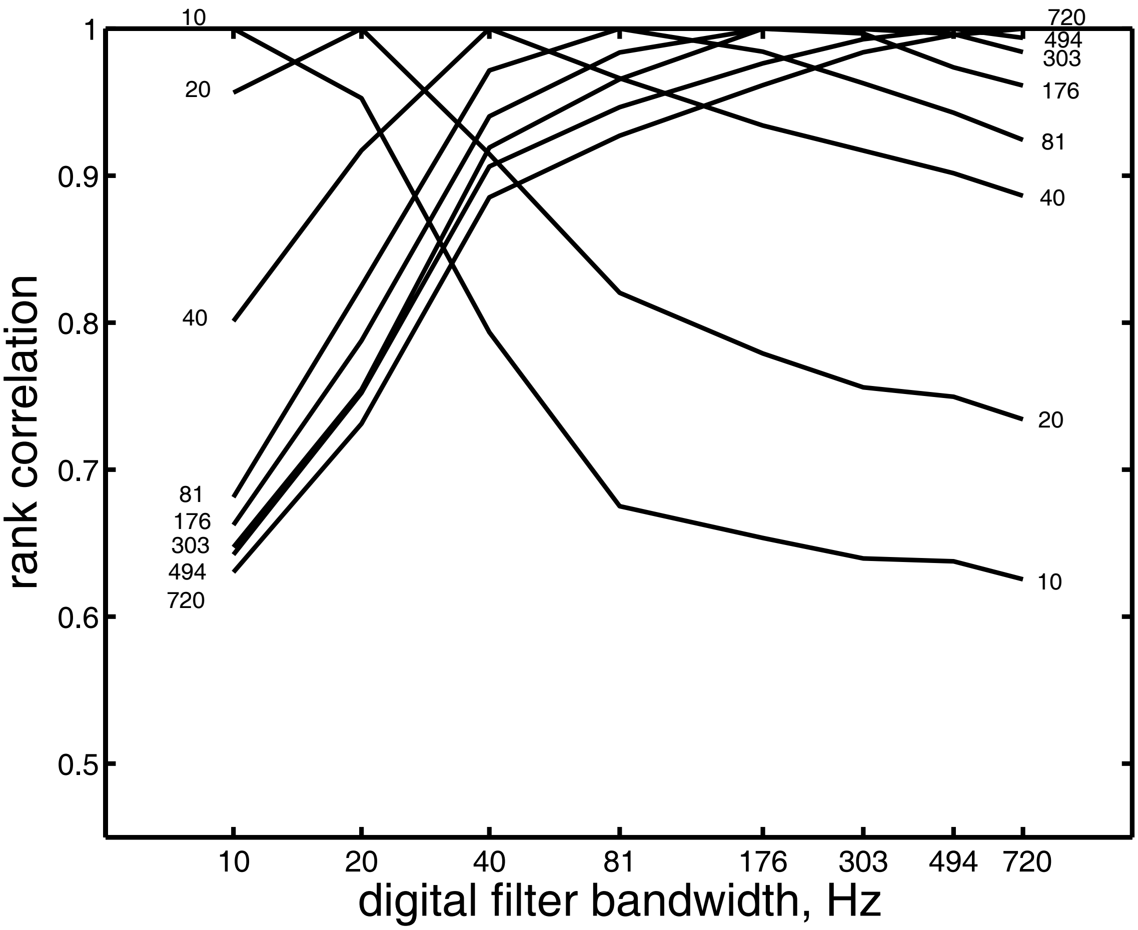

Figure 16. Rank correlations among filters of varying bandwidth (27 noise-only trials). The parameter is the 3-dB bandwidth of the filter.

Two important results come from this correlation analysis. That the 10 or 20 Hz filter output best predicts the observer's ordering of the signal trials shows that the observers are capable of a finer frequency selectivity than has been previously suggested or reported. In addition, the simple energy-detection model cannot explain the fact that a wider filter in the 100 to 200 Hz range predicts best the observer's ordering of the noise-only stimuli. The model, of course, predicts that the same width filter should correlate best with the responses to SN and N stimuli.

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Discussion

The results of the correlations indicate that the energy passed by a narrow filter is a good index of the detection performance when the signal tone is present, but not when it is absent. This is the sort of result one would expect if the observer is looking at a set of measures, only one of which represents essentially the energy in the narrow frequency region of the tone signal, but reports a tone signal detected if the maximum (i.e., if any) of the set of measures exceeds a criterion.

This kind of behavior would be exhibited by a filter-bank model that assumes that the observer is monitoring a set of narrow filters tuned to different frequencies near the tone-signal and responds if the largest output of the group is greater than some criterion. When the signal is indeed present the largest output would almost always be from the narrow filter centered at the tone frequency. If the signal is not present, any of the filters is equally likely to have the largest output. The responses on noise trials should thus correlate best with a filter as wide as the range of center frequencies of the filters being monitored. In the present experiment the rank correlation analysis gives estimates of about 20 Hz for the width of a filter in the bank and 150 to 200 Hz for the width of the bank.

This model gives an explanation for the apparent contradiction that narrowing the bandwidths of the masking noise appears to improve the observer's performance when the external filter is as wide as 200 Hz (van den Brink, 1964; Hamilton, 1957), but that observers can perform better than a filter only 60 Hz wide (Sherwin et al., 1956).

The model can also be given a physiological interpretation in terms of the demonstrated findings of frequency sharpening, and inhibition or masking effects in the neural stages of auditory signal processing (Galambos and Davis, 1943; Katsuki, 1961). Frequency sharpening appears in the physiological measurements of frequency tuning curves in that higher levels of neurons show narrower tuning curves. Neural inhibition is the presumed cause of the decrement in the neural response to a tone when a tone of a nearby frequency is introduced. The sharpening is interpreted as accounting for the narrowness of the filters, and the lateral inhibition is called upon to do the maximum detection. The obvious next research step is to see whether such a model can actually give an output that correlates better with the observer's responses than does the single-filter energy-detection model. Fletch attempted to use the noise-masked tone detection situation as a basis for measuring the bandwidths of the resonance curve of a single point on the basilar membrane. Von Bekesy (1943, 1949) actually measured these tuning curves on cadaver ears and obtained curves whose half squared-amplitude bandwidths are about 260 Hz and 420 Hz for the points tuned to 500 Hz and 1000 Hz, respectively. While the critical bandwidth measurements based on interactions of signals of different frequencies may be closely related to basilar membrane tuning, this study supports the notion that neural mechanisms for frequency selectivity are important in the noise-masked tone-detection experiment.

Of more general interest than the frequency selectivity problem is the problem introduced by the apparent differential frequency selectivity on tone and no-tone trials, the effect that the filter bank model was introduced to explain. The same effect probably occurs in the time domain when the signal tones occur in continuous noise. That is, if one correlated the output of the energy detectors with varying integration times with observer responses, one would probably find that on signal trials an integrator whose integration time is near the duration of the signal correlated best with the observers' responses, but that the longer duration integrators correlated best on noise-only trials. Similar results should also obtain in visual detection experiments along the dimensions of spatial or retinal position and temporal occurrence of a possible visual signal.

By analogy with the filter-bank model, models for these detection situations should involve parallel computation of many measures (i.e., statistics) of the stimulus. Each measure would be a nearly optimal measure for the detection of a signal similar to the actual signal. The measures should then be combined in a non-linear (maximum, for example) fashion to yield the final detection statistic. An additional complication which may be necessary to obtain an adequate quantitative description of the data would be a probability distribution over the sets of measures to be applied on particular trials.

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Appendix

Analog-to-digital Controller	Scientific Data Systems Model 930 Computer
Analog-to-digital Converter	Scientific Data Systems
Attenuator A	Hewlett Packard 350D
Attenuator B	Daven T-332-G
Audio Frequency Spectrometer	Bruel and Kjaer Type 3111
Audio Level Recorder	Bruel and Kjaer Type 2305
Audio Oscillator	Hewlett Packard 200AB

Band-Pass Filter	Kron-Hite 315A
Card Reader-Punch	IBM 526
Earphones	Permoflux PDR 600
Electronic Counter	Hewlett Packard 5512A
Electronic Switch	Grason Stadler 839E
Inductive Mixer	Cinema 7304-H3G2G
Interval Timer	Grason Stadler 471-1
Noise Generator	General Radio 1390A
Response Switches	Single pole double throw knife
Tape Recorder	Ampex PR-10

Table A1. Equipment manufacturers and models.