CDF note 9414

Measurement of the Top Quark Mass using Quantities with Minimal Dependence on the Jet Energy Scale

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Abstract

We present two measurements of the top quark mass in the lepton plus jets channel with approximately $1.9fb^{-1}$of data using quantities with minimal dependence on the jet energy scale in the lepton plus jets channel at CDF. One measurement is of the transverse decay length of b-tagged jets (L2d), and the other is of the transverse momentum of the lepton (LepPt). Both these quantities are roughly linearly proportional to the top mass. Since the quantities have approximately the same statistical resolution in top mass determination, and since the quantities are approximately uncorrelated to one another, the statistical uncertainty in the mass measurement is significantly reduced by combining the results.

Using the L2d variable we measure a top mass of $(176.7^{+10.0}_{-8.9}(stat) \pm 3.4(syst))GeV/c^2$, using the LepPt variable we measure a top mass of $(173.5^{+8.9}_{-9.1}(stat) \pm 4.2(syst))GeV/c^2$, and doing a combined measurement using both variables we arrive at a top mass result of $(175.3 \pm 6.2(stat) \pm 3.0(syst))$ GeV/c^2. Since many of the systematic uncertainties are statistically limited, these results are expected to improve significantly if more data is added at the Tevatron in the future, or if the measurement is done at the LHC.

Preliminary Results for Summer 2008 Conferences

I. INTRODUCTION

This analysis grew out of a measurement of the top mass using the transverse decay length in the lepton plus jets channel. This decay length approach has previously been completed and published with 695 pb^{-1} of data [1]. Since the measurement relies almost exclusively on tracking, very different event information is used compared to conventional mass measurements, and so the result is expected to have small correlation with other measurements. Further, the jet energy scale systematic, which is the largest uncertainty on the world average top mass, should have a minimal effect on our results. However, the statistical resolution on the top mass is poor relative to more conventional, reconstruction based top mass measurements. Thus, much effort has been put into reducing the statistical uncertainty for this update of the measurement. In addition to incorporating more than double the data of the previous measurement, it was decided that a second variable should be included to further reduce the uncertainty.

The second variable selected was the transverse momentum of the leptons (transverse momentum for muons, and transverse energy as measured in the calorimeter for electrons), which are correlated to the top mass through the momentum of the W bosons. The use of this variable was originally proposed by C. S. Hill et. al. [2]. Measuring the top mass using the lepton transverse momentum has a similar statistical power to the the decay length technique, and has the advantage of being almost completely uncorrelated statistically.

Like the decay length technique, the lepton transverse momentum is approximately linearly correlated to the top mass. Since the distribution of both these variables is approximately that of a decaying exponential over most of their range, the shapes are largely specified by the mean of the distributions. Thus, to keep the analysis simple and minimize systematic uncertainties, only two quantities are actually used to determine the top mass: the mean of the L2d distribution, and the mean of the LepPt distribution.

We do not describe the CDF detector in this note. The detector is described in detail elsewhere [3].

II. DATA SAMPLE & EVENT SELECTION

This analysis is based on an integrated luminosity of 1.9 fb^{-1}. The data are collected with an inclusive lepton trigger that requires an electron or muon with E_T >18 GeV (P_T >18 GeV/c for the muon). From this inclusive lepton dataset we select events offline with a reconstructed isolated electron E_T (muon P_T) greater than 20 GeV. All leptons used in this analysis are required to be isolated and have a well resolved track in the central tracking chambers. Both electrons and muons are found with good efficiency out to a pseudorapidity of roughly 1.0.

The total missing transverse energy (MET) in the event is required to be greater than 20 GeV, and a minimum of three jets must also be identified with reconstructed transverse energies greater than 20 GeV. b-jets are identified (tagged) using the SecVtx algorithm [4]. This algorithm identifies tracks that are displaced from the primary vertex and attempts to reconstruct a secondary vertex from them. If the secondary vertex is well resolved and has a sufficient transverse decay length from the primary vertex, the jet is tagged as a b. Note that this decay length is the same as the one we use later to measure the top mass. In order for the event to pass selection, at least one jet must be tagged as a b for events with four or more jets of E_T greater than 20 GeV, and at least two jets must be tagged as a b for events with exactly three jets of E_T greater than 20 GeV.

III. EVENT COMPOSITION

A $t\bar{t}$ cross section measurement in the greater than three jet bin was performed to determine the numbers of signal and background events and uncertainties. Selected events are determined to be about 84% $t\bar{t}$ and the dominant backgrounds are found to be W+jets, and non-W QCD events. A small number of single top events also pass selection, but these are treated as signal rather than background via a shape parameterization over top mass. Together these distributions account for about 99% of events passing selection. The remaining 1% come from the electroweak diboson and Z+jets backgrounds. These backgrounds have not been directly evaluated. Instead the W+jets distributions are used in their place. The expected numbers for each subsample and total number of events observed in data are shown in Table I.

A. QCD background

QCD events may enter the selection when a jet fakes a lepton and the missing transverse energy is misreconstructed. The QCD background is evaluated from data by altering the lepton selection criteria to make the events much more likely to contain fake leptons. The new event selection is such that identified events do not overlap those selected

TABLE I: Expected event counts and observations in data

Distribution	Events	Recorded L2d Tags
W plus Jets	56.2 ± 11.5	71.3 ± 13.8
QCD	18.8 ± 12.7	20.2 ± 13.2
Electroweak	9.0 ± 0.4	11.3 ± 0.5
Single Top	8.4 ± 0.4	13.4 ± 0.7
$t\bar{t}$	478.3 ± 40.3	659.3 ± 45.5
Total Predicted	570.8 ± 44.3	775.5 ± 50.2
Data	576	783

from data for the analysis, and the selection is tuned to reduce bias in either L2d or LepPt compared to the standard event selection.

B. W+jets background

The W+jets sample represents the largest background. It is evaluated from ALPGEN events that are showered using Pythia. When heavy flavor quarks are produced from the Pythia shower, events are only kept if the opening angle between the quarks is less than 0.4 in $\eta\phi$ space. Similarly, events with heavy flavor production from ALPGEN are rejected if quarks from the heavy flavor pair have an opening angle that is less than 0.4. In this manner double counting of events between Pythia and ALPGEN is avoided.

C. Single Top Sample

As mentioned above, we do not really treat the single top distribution as a background. Rather, we parameterize the shape of the single top L2d and LepPt distributions according to top mass so that we can find the mean values for an arbitrary mass point. Distribution shapes were determined from the four mass points for which single top Monte Carlo samples were available ($m_t = 165, 170, 175$, and $180 GeV/c^2$). For each of these mass points we performed a Gaussian+Exponential fit to the L2d and LepPt distributions. The fit parameters are dependent on top mass, and they are fitted to parabolas over the full top mass range to determine shape parameterizations at any value of the top mass.

IV. CORRECTIONS TO THE $t\bar{t}$ SAMPLE

For background samples the L2d and LepPt values are evaluated directly from the Monte Carlo or the data (for the QCD fakes). These values are taken as is, with no correction, and cross checks with data are used to evaluate the systematic uncertainty. The $t\bar{t}$ Monte Carlo, on the other hand, is generated in Pythia, and various corrections must be applied. One necessary correction is due to inaccuracies in the Monte Carlo's modeling of the momenta of the colliding partons, which in turn propagate to the transverse momenta of the final state partons. The Monte Carlo for these samples use the CTEQ5L PDF. Since this PDF is LO it has limited accuracy. Using the LHAPDF set of parton distribution functions [6], we reweight our events according to the probabilities for the colliding partons to have the observed identities and momenta in order that the new results will be consistent with the CTEQ6M PDF. We further reweight the events so that the fraction of $t\bar{t}$ events produced from gluon fusion matches theoretical predictions for the appropriate top mass. CTEQ6M comes closer to the correct value, but still underestimates the gluon fusion fraction. These corrections are especially important to get the LepPt measurement correct.

Other corrections are applied to the signal sample for the L2d measurement. We estimate the average decay length in the Monte Carlo and in data in a dijet control region and use these results to determine the necessary correction (the decay length "scale factor") to the $t\bar{t}$ signal Monte Carlo. To evaluate the scale factor, we must work with a sample with a similar flavor distribution to our signal sample (that is, almost all b-jets). The same samples and almost the same event selection as for the b-tagging efficiency scale factor are used [4]. Back to back SecVtx tagged jets are required and one of the jets (the lepton jet) is required to contain a >= 8 GeV electron or muon. The lepton jets in this sample have been measured to be about 95 % b-jets, which is a similar flavor fraction to that of the tagged jets in the $t\bar{t}$ sample as required. This same selection is performed in both data and Monte Carlo, and the scale factor can be evaluated by dividing the mean decay length in data by the mean decay length from Monte Carlo. Only the

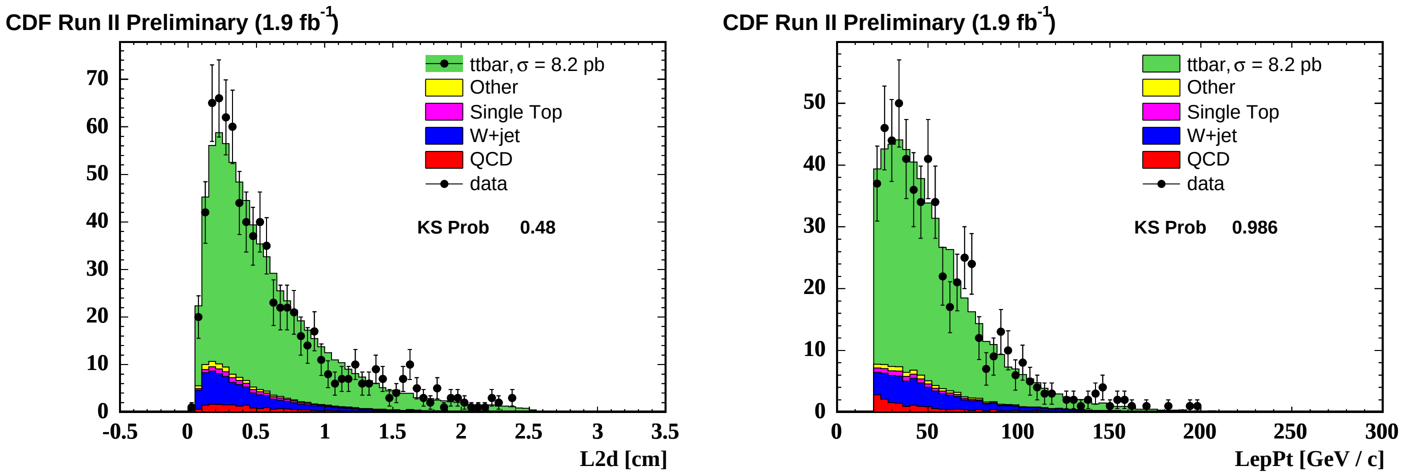

FIG. 1: Left: Signal, background, and data for the L2d distribution using hypothesis top mass M=178 GeV/c^2. Right: Signal, background, and data for the LepPt distribution using hypothesis top mass M=173 GeV/c^2.

lepton jets are used in this scale factor determination since it can be more easily proven that the Monte Carlo models the mistag rate correctly for these jets. Further, since this decay length scale factor drops slightly as the transverse energy of the tagged jets increases, it is necessary to correct the measured decay length to the higher kinematic range of $t\bar{t}$ jets. In the end, the mean decay length scale factor is evaluated to be 0.990 ± 0.011. Here, the uncertainty is dominated by the limited number of high E_T jets in the scale factor samples. Since this uncertainty contributes to the dominant systematic on the decay length measurement, it is reasuring to note that it will improve with increasing statistics.

V. METHOD

For each event passing selection, the LepPt value is recorded, as is the L2d of the two leading tagged jets. Signal and background distributions for top mass hypotheses similar to measured results are shown in Figure 1.

Pseudoexperiment events are drawn separately from signal and background samples and combined according to the measured cross section in the greater than three jet bins (8.2 fb^{-1}). This is done separately for 24 hypothesis top mass values ranging from a top mass of 130 GeV/c^2 to a top mass of 220 GeV/c^2. For each pseudoexperiment the total number of events is fixed to the value observed in the data. The total number of background events is fluctuated within statistical and systematic uncertainties, and the rest of the events are considered to be signal.

To evaluate the top mass results for each individual measurement (before L2d and LepPt are combined), the means and RMS's of the pseudoexperiment results are determined and are fit to quadratic polynomials as shown in Figure 2. Given the mean L2d and LepPt in data, the corresponding x-values of the central fit give us our expected mass, and the value of the shifted fits give us our ± one sigma asymmetric statistical uncertainties. It turns out that the decay length technique is more sensitive for small top mass values, however it loses sensitivity for large values of the mass. This is thought to be largely due to the loss of tagging efficiency from the larger fraction of b-jets which decay outside of the inner silicon layers for the more boosted jets from higher mass top quarks.

VI. COMBINATION

A joint top mass measurement using both the L2d and LepPt is also performed using pseudoexperiments. For each of the pseudoexperiments thrown for the L2d and LepPt measurements, the means are recorded. For a given top mass sample, these pseudoexperiments form an ellipse in the mean L2d versus mean LepPt plane. When results are measured in the data, they are compared to each of these hypothesis mass ellipses. The consistency of the mass hypothesis is evaluated based on the "distance" the data means are from the expected results (the ellipse center). This distance is evaluated from Equation 1:

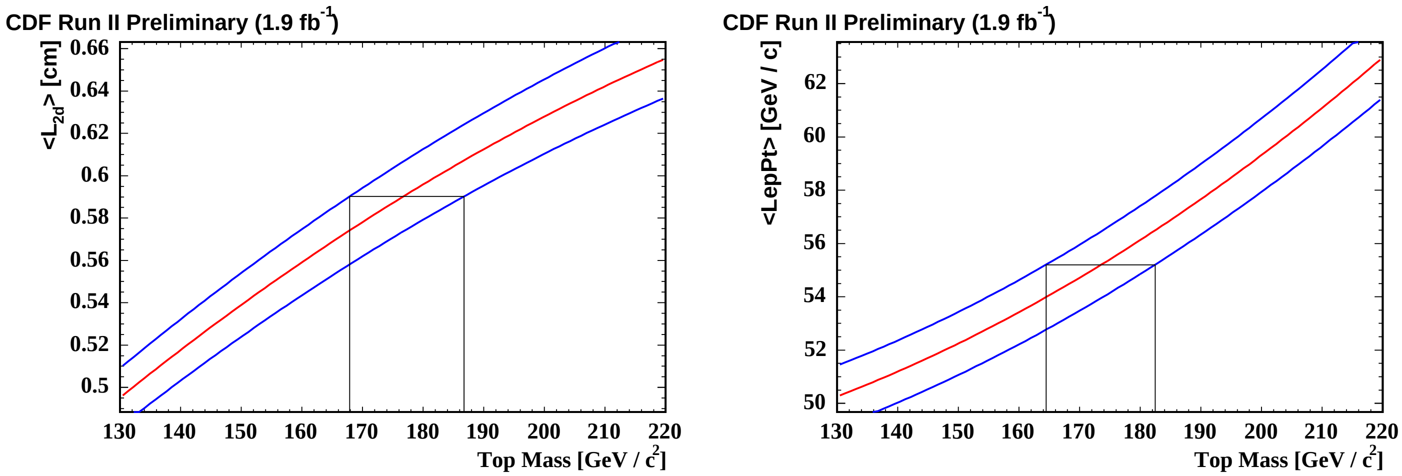

FIG. 2: Left: Expected central values and one sigma confidence intervals of L2d mean results depending on top mass. Right: Expected central values and one sigma confidence intervals of LepPt mean results depending on top mass. Solid lines show the plus and minus one sigma statistical uncertainties from data.

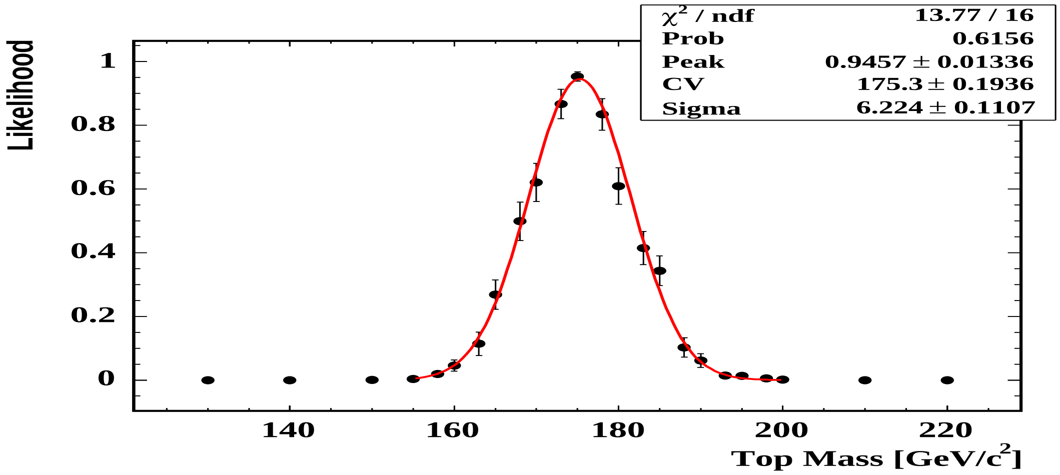

FIG. 3: Likelihood fit results for data under the combined measurement.

$$D = \sqrt{(\frac{\delta P_t}{\sigma_{Pt}})^2 + (\frac{\delta L_{2d}}{\sigma_{L2d}})^2} \tag{1}$$

Here, δP_t is the difference between the mean LepPt of the data and the hypothesis value, and σ_{Pt} is the size of the RMS of the hypothesis LepPt means from pseudoexperiments, etc. For a given mass hypothesis, some pseudoexperiments will have a smaller value of D than the data (less discrepant), and some will have a larger value of D (more discrepant). So, if the hypothesis represents the true value of the top mass, then the probability that a given pseudoexperiment will be at least as discrepant as the data is given by the fraction of pseudoexperiments with a larger value of D than the data. These fractions are evaluated along with uncertainties (dictated by number of pseudoexperiments and finite Monte Carlo statistics) and are fit to a Gaussian. Results for the data are shown in Figure 3. This fit provides us with our mass result and our statistical uncertainty.

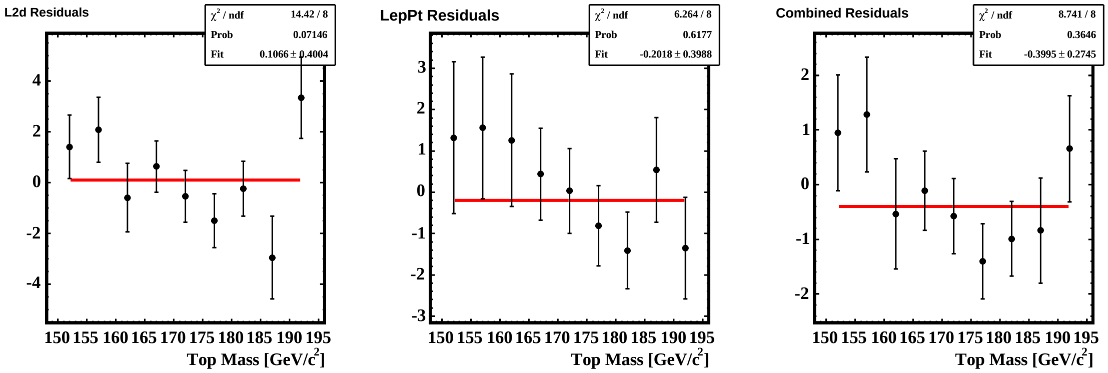

FIG. 4: Offset between input and output top masses (residuals) expected for L2d, LepPt, and in combination. Statistical errors are based on number of pseudoexperiments thrown and finite Monte Carlo statistics.

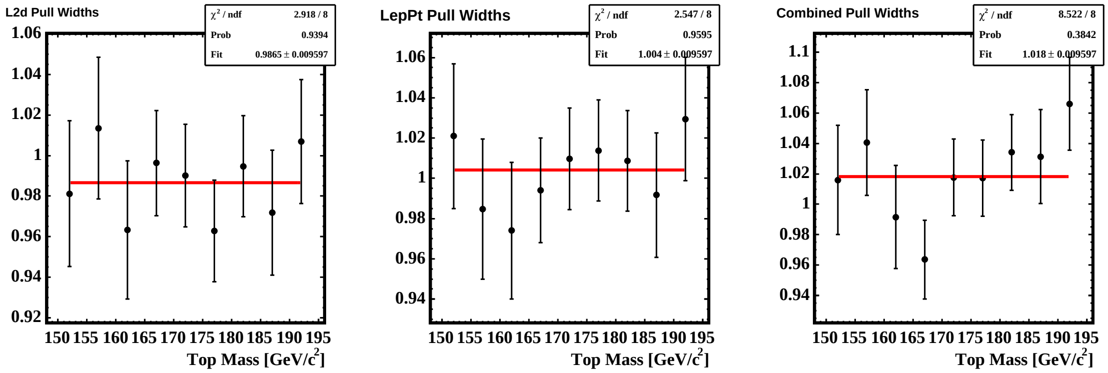

FIG. 5: Pull widths expected for L2d, LepPt, and in combination. Statistical errors are based on number of pseudoexperiments thrown and finite Monte Carlo statistics. For the L2d and LepPt measurements, for a given pseudoexperiment the "+" uncertainty is used if the measured result is above the expected value, and the "-" uncertainty is used if the measured result is below the expected value.

VII. SANITY CHECKS

To verify that our algorithms are unbiased, we choose nine mass points that were not included in the pseudoexperiments used to generate the curves of Figure 2. The mass results for these nine samples show no evidence of bias (as shown in Figure 4). The pull widths also come out close to 1.0 as expected as can be seen in Figure 5. Along with these nine samples, ten additional samples were run over where the true mass values were not known in advance. The mass results for these blind samples were correctly reproduced with approximately unity pull widths as well.

VIII. RESULTS

Using $1.9 fb^{-1}$of CDF data, we find 576 events passing our selection. From these events we measure a mean LepPt of 55.2 ± 1.3 GeV/c and a mean L2d of 0.596 ± 0.017 cm (after application of the L2d scale factor, PDF, and gluon fusion reweightings, as explained previously). The associated top mass results and uncertainties are shown in Table II. Under the combined measurement the fit result shown in Figure 3 returns us a top mass of $(175.3 \pm 6.2(stat))$ GeV/c^2. The consistency of the statistical error with expectations for a top mass of $175 GeV/c^2$ is shown in Figure 6.

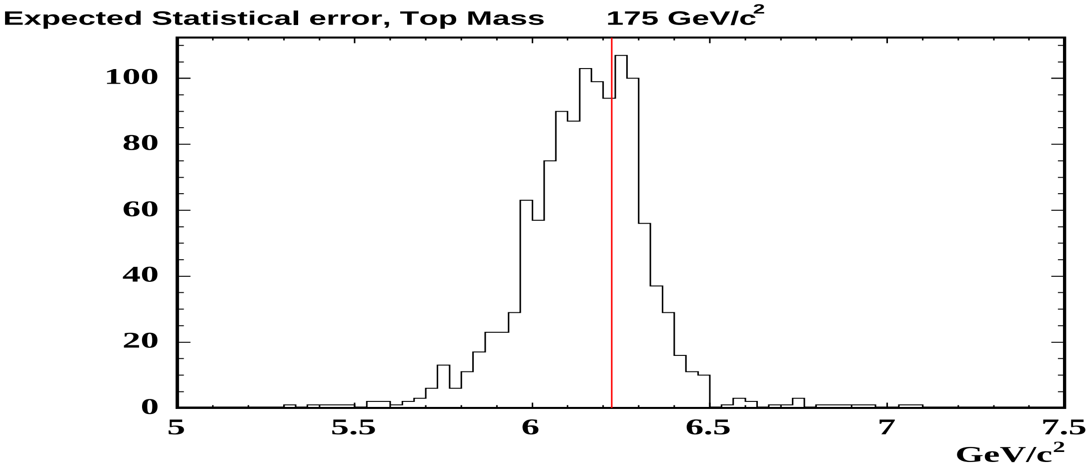

FIG. 6: Observed statistical error and expectations for a top mass of $175 GeV/c^2$.

TABLE II: Mass results and statistical uncertainties from the one-variable measurements

Variable	Measurement	Mass Results
L2d	$0.596 \pm 0.017\ cm$	$176.7^{+10.0}_{-8.9}\ GeV/c^2$
LepPt	$55.2 \pm 1.3\ GeV/c$	$173.5^{+8.9}_{-9.1}\ GeV/c^2$

IX. SYSTEMATIC UNCERTAINTIES

A. Background Uncertainty

To evaluate the systematic uncertainty on the background means, a direct comparison with the data was performed. We evaluate the mean LepPt and L2d for data and compare to our background estimations in the one and two jet bins as cross checks for our signal sample. These bins are both dominated by background events, in roughly the same proportion as in the signal region of our event selection. To be conservative, we take the larger of the shifts between the signal and the background for the one jet and two jet bins as our systematic uncertainty. The distributions in these cross check bins are shown in Figures 7 and 8.

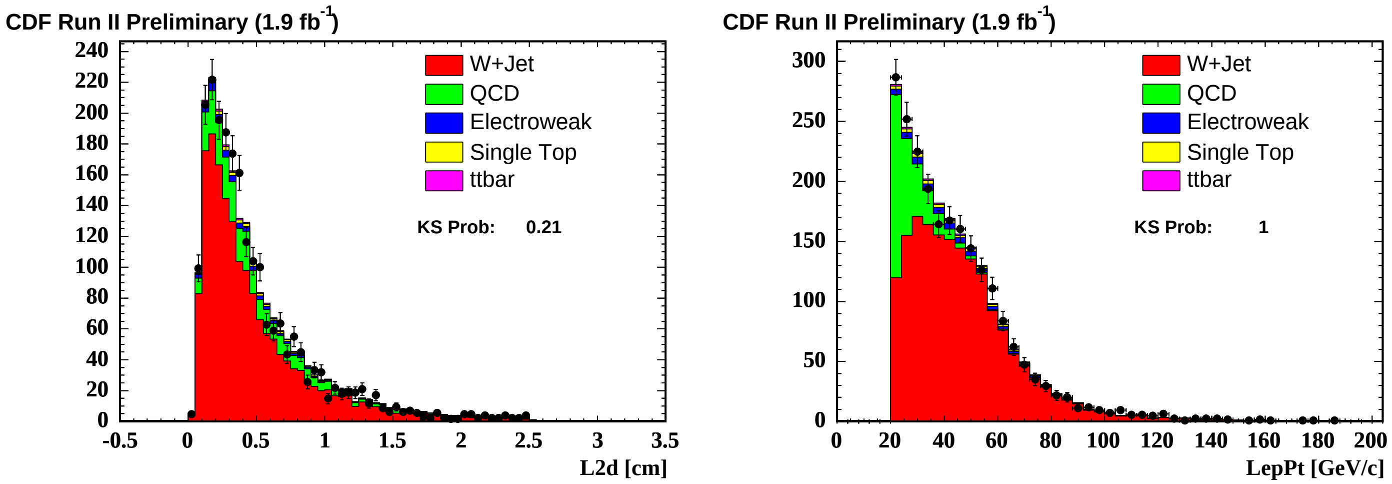

FIG. 7: Background predictions compared with data in the one jet control region for L2d and LepPt

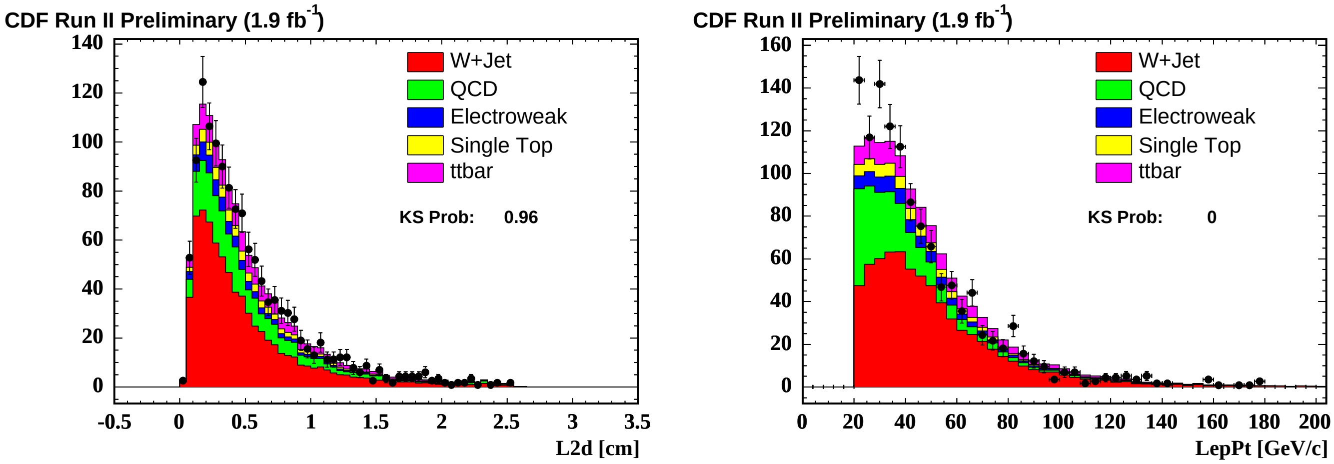

FIG. 8: Background predictions compared with data in the two jet control region for L2d and LepPt

B. Signal Uncertainties

Several different systematics are evaluated to determine the uncertainties on our signal distribution. To evaluate uncertainties on the QCD initial and final state radiation, new Monte Carlo samples are generated with more and less initial and final state QCD radiation. The larger of the shifts between the central value and the alternate radiation samples is taken as the systematic uncertainty. For the LepPt measurement this comes out to a surprisingly large number. This is because both the samples with more and less QCD radiation end up having a larger mean LepPt than the nominal sample. Thus, the QCD radiation systematic ends up being larger than the expected two sigma systematic that one would get by taking the difference between the results of the radiation up and the radiation down samples directly. In the end, this is a very conservative value for the systematic. Further work will be done to understand this and hopefully reduce it in the future.

There are many other possible inaccuracies in the Monte Carlo besides QCD radiation. To get a handle on these uncertainties the top mass is evaluated again using a Herwig Monte Carlo sample instead of Pythia. Using the Herwig sample alters a number of properties, including how the QCD fragmentation is performed, the QED radiation (which in Herwig is added in at the leptonic W-decay vertices using the PHOTOS algorithm [5]), the transverse Fermi motion of the colliding partons, and spin correlations between the top quarks. The full shift between the top mass results for the Pythia and Herwig samples is taken as a conservative estimate of the uncertainties due to these different generators.

As mentioned in IV, the Monte Carlo samples are generated with the CTEQ5L PDF [7] and then reweighted to the CTEQ6M PDF [8]. Then the parton distribution function systematic is evaluated by adding the twenty 90% CTEQ6M eigenvector shifts in quadrature (allowing gluon fractions to vary), which are also determined through the reweighting procedure. Since these eigenvectors do not account for uncertainties on the strong coupling constant, this extra uncertainty is determined by reweighting to the CTEQ6A and CTEQ6B PDFs [9]. These samples are similar to the nominal CTEQ6M NLO PDF, but with altered values of the strong coupling constant. Based on the trends observed from these PDFs, the corresponding systematic due to the uncertainty on the world average strong coupling constant is extracted and added in quadrature to the CTEQ6M eigenvector uncertainties.

As also mentioned in IV, the mean L2d results are corrected to account for differences in the Monte Carlo modeling of the decay length compared to data. There is a significant uncertainty on this correction which is limited by the current lack of statistics in data and Monte Carlo. A similar uncertainty is determined for the LepPt measurement due to uncertainty in the lepton transverse momentum. This uncertainty is found by fitting the Z-mass peak in data and Monte Carlo using electrons and muons separately. The observed shift is not corrected for. Rather, to be conservative, the weighted average of the full shift between the lepton types is taken as a systematic uncertainty.

The jet energy scale is the dominant uncertainty for many other top mass measurements. In our case, one expects this uncertainty to be small. Indeed, the only possible way for the jet energy scale to have any effect is in the way it changes event selection. Jets are considered only if their measured transverse energy is greater than 20 GeV. The missing transverse energy is also required to be above 20 GeV for all events, and jet energy scale fluctuations have an impact here as well. It turns out that the LepPt measurement is only minimally effected by the jet energy scale, however the L2d measurement suffers a larger shift. This is because jets near the 20 GeV threshold have a significantly lower than normal average decay length. Thus, when the Monte Carlo jet energy scale fluctuates up, more low decay-length jets pass selection, the L2d templates shift down, and the measured top mass goes up. At such low energies, the jet energy scale uncertainty is entirely dominated by uncertainties in out-of-cone effects on the jet energy scale. A breakdown of the contributions from different sources of jet energy scale uncertainty is show in Table III. Given that these two measurements do not use an in-situ W mass calibration, the jet energy scale uncertainty for both results comes out much smaller than for measurements which also do not use the in-situ calibration. Other approaches could also be used to improve this systematic as statistics improve. For example, we can cut into this systematic at the cost of statistics by imposing a stricter cut on the minimum value of L2d for the tagged jets we are considering. As fewer jets with lower decay length are used, there will be fewer jets with a transverse energy that is near the event selection cutoff.

TABLE III: Jet Energy Scale Mass Uncertainties

Systematic	L2d	LepPt	Combination
Level 1, Eta Dependent	0	0	0
Level 4, Multiple Interactions	0.1	0	0
Level 5, Absolute	0.2	0.1	0.1
Level 6, Underlying Event	0	0	0
Level 7, Out of Cone	1.0	0.2	0.6
Level 8, Splash out	0.1	0.1	0.1
Simultaneous	1.0	0.3	0.6

Final systematic uncertainty results are shown in Table IV.

TABLE IV: Systematic Results

Systematic	L2d	LepPt	Combination
QCD Radiation	0.9	2.3	1.5
PDFs	0.3	0.6	0.5
Generator	0.7	1.2	0.6
L2d Scale Factor	2.9	0	1.4
LepPt scale	0	2.3	1.1
Bkg Shape	1.0	2.3	1.6
Out of Cone JES	1.0	0.3	0.6
Total	3.4	4.2	3.0

X. CONCLUSION

We have performed two measurements of the top quark mass using variables with minimal correlation to the jet energy scale and combined them. Under an integrated luminosity of $1.9 fb^{-1}$ we measure a mean LepPt of $55.2 \pm 1.3\ GeV/c$ and a mean L2d of $0.596 \pm 0.017\ cm$ (after application of the L2d scale factor, NLO PDF, and gluon fraction corrections). Accounting for systematics, this results in a top quark mass of $(176.7^{+10.0}_{-8.9}(stat) \pm 3.4(syst))GeV/c^2$ using the decay length method, $(173.5^{+8.9}_{-9.1}(stat) \pm 4.2(syst))GeV/c^2$ using the lepton transverse momentum, and $(175.3 \pm 6.2(stat) \pm 3.0(syst))\ GeV/c^2$ in combination. If updated, the results of this method will improve, but will continue to be limited by statistics for the rest of Run II. However, if this analysis is done at the LHC statistics will no longer be an issue. Further, since some of the dominant systematics are statistically limited, the results of these techniques could well become competitive with conventional top mass analyses, and due to their reduced correlation with conventional top measurements they should help reduce the uncertainty on the world average top mass in a combination.

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