



## Measurement of the $B^+$ lifetime, using a new simulation free method to correct for trigger induced biases.

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URL <http://www-cdf.fnal.gov>  
(Dated: June 25, 2008)

This note describes the first measurement using a new method of B Hadron lifetime measurement, applicable to data collected with an impact parameter based trigger. The method corrects for the intrinsic bias of the trigger using information from data only. This removes uncertainties due to Monte Carlo and data agreement. We present here a measurement of the  $B^\pm$  lifetime using this method. The decay mode used is  $B^+ \rightarrow \bar{D}^0(K\pi)\pi^+$  and the events analysed were produced in  $\bar{p}p$  collisions at  $\sqrt{s} = 1.96 TeV$  at the Fermilab Tevatron and collected by the CDF II detector. The charge conjugate is assumed. We measure

$$\tau(B^\pm) = 1.662 \pm 0.023(stat.) \pm 0.015(syst.)ps \quad (1)$$

*Preliminary Results for Summer 2008 Conferences*

This note describes a new method for B hadron lifetime measurement using data collected by a impact parameter based trigger. This method does not use MC to correct for trigger biases, using instead information available in the data. Ultimately this results in lower systematic errors as it is unnecessary to consider how the MC and data agree. We present here a measurement on the charged B meson lifetime which is used to test and validate the method. This new method will be used to measure lifetimes at CDF and also has potential at future experiments with impact parameter based triggers.

Lifetime measurements are a test of non spectator effects in hadronic decays and test HQE theory which makes predictions on lifetime ratios and the lifetime hierarchy.

This note describes a measurement of the  $B^+$  lifetime in  $\bar{p}p$  collisions at  $\sqrt{s} = 1.96$  TeV with the CDF detector at the Fermilab Tevatron. The CDF detector is described in detail in [1].

## II. DATA SAMPLE & EVENT SELECTION

This analysis is based on an integrated luminosity of  $1.0 \text{ fb}^{-1}$  collected with the CDF II detector between February 2002 and March 2006. The data are collected using an impact parameter based trigger. This trigger requires that two tracks individually have a  $P_T > 2 \text{ GeV}$  and impact parameter between 120 and 1000 microns. In combination the track pair must also have an opening angle between 2 and 90 degrees in the transverse plane and the intersection must be 200 microns from the B meson interaction point. There are three separate trigger path that differ slightly in their requirements for the charges of tracks and  $\Sigma P_T$ . There separate paths switch on at differing luminosities.

From this dataset events are selected offline with reconstructed B candidates that pass the trigger requirements. Further selections cuts are chosen to reduce background. In particular requirements that the directional distance in the transverse plane between the B vertex and the interaction point be greater than 350 microns and the directional distance of the D vertex and interaction point be greater than -100 microns remove very little signal but large amounts of combinatorial background. Other cuts relate to the  $P_T$  of the B meson greater than 5.5 GeV and  $P_T(D^0) > 2.4 \text{ GeV}$  and standard track quality cuts also remove background. A lower mass cut at 5.23 GeV is imposed to remove partially reconstructed B meson decays. In total after selection cuts,  $24,200 \pm 200$  signal events are reconstructed. Over the full mass range S/B=2.7. Over the signal region which lies in the mass range 5.23-5.37 GeV S/B = 4.8. The mass distribution is shown in Figure 1

## III. METHOD FOR LIFETIME MEASUREMENT

### A. Correcting for the trigger bias

The trigger cuts on pairs of tracks with large impact parameters and this is effectively selection for displaced secondary vertices. This method of selection is intrinsically biasing to the lifetime distribution as the trigger is more likely to accept a longer lived particle. The trigger cuts applied can be viewed as indirect cuts on lifetime. A B particle decays; the decay products have particular momentum and direction. The information of the kinematics of a particular decay is sufficient to calculate what those indirect cuts on lifetime applied by the trigger are for that particular event. A trigger that applies no bias could have recorded that event had it decayed at any time. To correct for the trigger bias all that needs to be calculated is what time could the event have decayed with those particular kinematics and still have passed the trigger. The times when the event can pass the trigger is referred to as the acceptance function. The calculation is best described in the series of diagrams shown in Figure 2. The diagrams only depict impact parameter cuts explicitly but all other cuts by the trigger and analysis cuts are also implied.

To calculate the acceptance function the event is slid along its path of decay and at each time we consider whether or not the trigger and analysis cut conditions are satisfied. From the first diagram it is seen that this is not the case as there are not 2 tracks with the required impact parameters. As we slide further eventually we reach the point where all trigger cuts are satisfied and the acceptance turns on. As the B continues to move away from its production point there may be a point where 2 possible track pairs could fire the trigger. At this point the acceptance function increases as the probability of finding 2 tracks out of three is higher than the probability of finding two tracks out of two. There will then come a time when the trigger conditions are no longer met and at this point the acceptance returns to 0. The diagrams illustrate the cuts on impact parameter only, the actual calculation considers also all other cuts. The acceptance function is therefore dependent only on the decay kinematics and the single track finding efficiency which determines its height. The track finding efficiency is assumed to be flat as a function of impact parameters between 0 and 1mm, track momentum above 2GeV, track  $\eta < 1.1$ , and its value is fit simultaneously with the lifetime. To

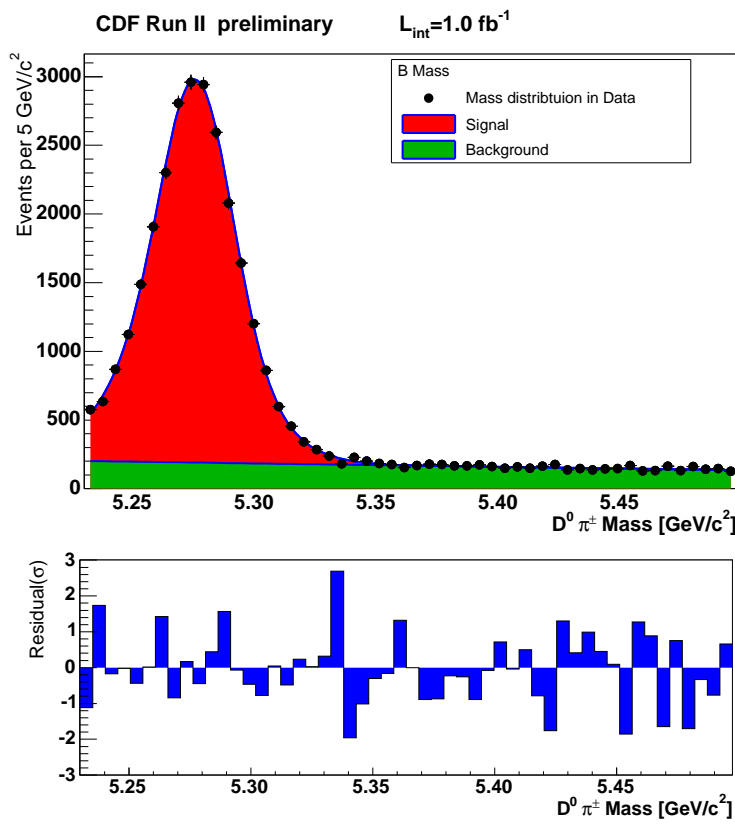


FIG. 1: The mass distribution of selected events. We find  $24,200 \pm 200 B^+ \rightarrow D^0(K\pi)\pi$  events.

correct for the trigger bias all that is required is to normalise the standard lifetime decay exponential convoluted with detector resolution over the acceptance function. The acceptance function will be unique to each event, as it is dependent on event kinematics.

The full PDF is for signal lifetime is given by

$$P(t_0) = \frac{P(\text{trk}|\varepsilon_s) \frac{1}{\tau} e^{-\frac{t_0}{\tau} + \frac{1}{2} \frac{\sigma^2}{\tau^2}} \text{F}\left(\frac{t_0}{\sigma} - \frac{\sigma}{\tau}\right)}{\sum_{\substack{i=\text{all} \\ \text{intervals}}} \text{poly}_i(\varepsilon_s) \left[ -e^{-\frac{t}{\tau} + \frac{1}{2} \frac{\sigma^2}{\tau^2}} \text{F}\left(\frac{t}{\sigma} - \frac{\sigma}{\tau}\right) + \text{F}\left(\frac{t}{\sigma}\right) \right]_{t=t_{\min i}}^{t=t_{\max i}}} \quad (2)$$

The terms involving  $\varepsilon_s$  and  $P(\text{trk})$  simply govern the height of the acceptance function by taking into account combinations of tracks that could have passed the trigger. The term  $\text{poly}(\varepsilon_s)$  is the sum over possible track combinations that pass the trigger and is the correct normalisation factor. The rest of the numerator is simply an exponential decay convoluted with the detector resolution. For this measurement the resolution is a single Gaussian width 26 microns. In the expression the function F is the Frequency function defined as

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{y^2}{2}} dy \quad (3)$$

The acceptance function is split into parts of differing heights, which leads to the summation and so the denominator of this expression is simply the integral of the numerator over the time when the acceptance function is non zero. The likelihood can be constructed and minimised to find the best fit lifetime. This method has been tested in a variety of full detector simulation MC using different decay modes and the returned best fit lifetimes are consistent with the input truth value. This is a demonstration that the trigger bias is corrected for. Sample results are shown in Table I for tests on full detector simulation MC. Studies have also been carried out using toy MC which confirm no remaining bias on the fit result.

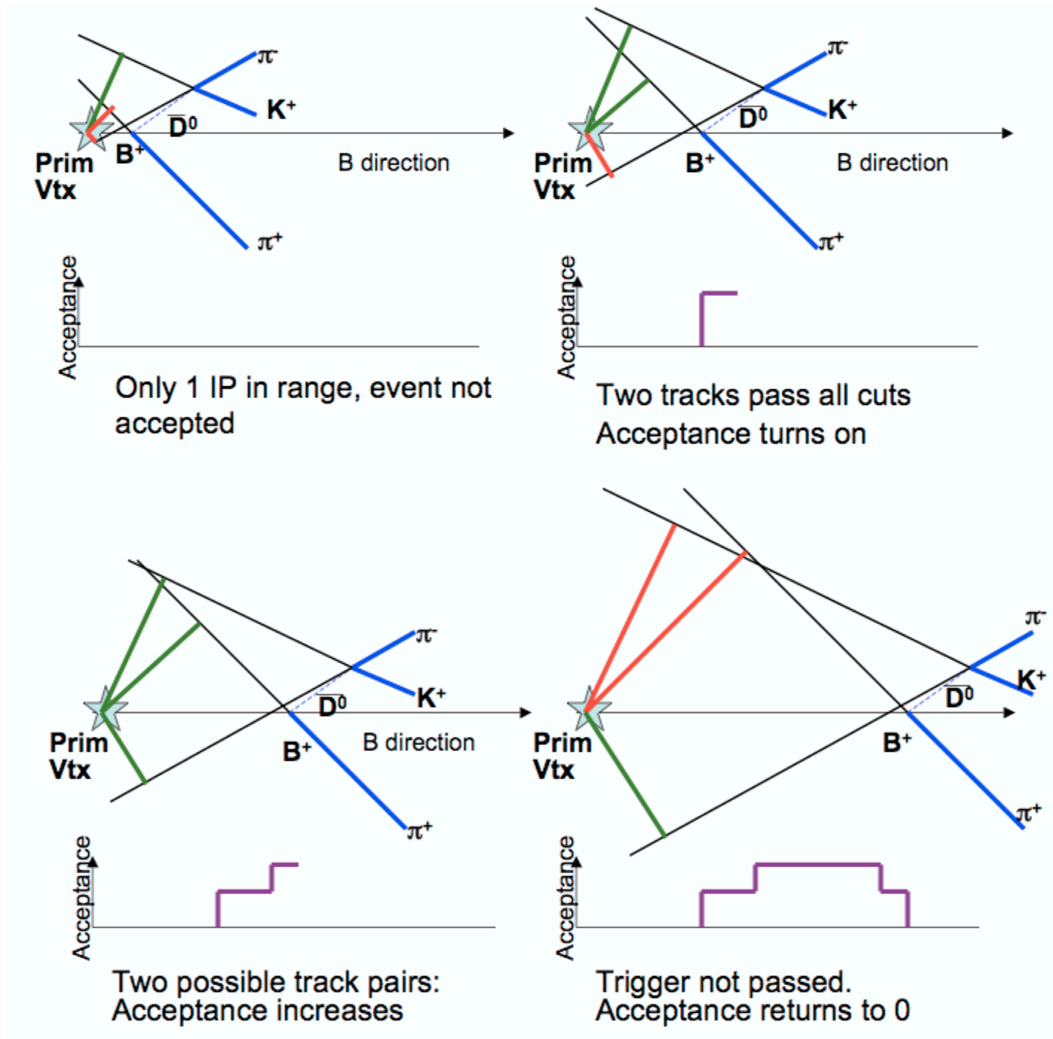


FIG. 2: The diagrams above depict the calculation of the acceptance function. This function is unique to the event and is dependent only on decay kinematics.

Decay Mode	$N^0$ of events	Input Lifetime( $\mu m$ )	Fitted lifetime( $\mu m$ )
$B^+ \rightarrow D^0(K\pi)\pi^+$	75K	496	$493.3 \pm 3.2$
$B^0 \rightarrow D^-(K\pi\pi)\pi^+$	71K	464	$467.8 \pm 2.8$
$B_s \rightarrow \phi(KK)\phi(KK)$	35K	438	$443 \pm 5.0$
$\Lambda_B \rightarrow \Lambda_c(pK\pi)\pi$	24K	323	$319 \pm 6.0$

TABLE I: Fit results using full detector simulation MC

## B. Addition of Background

The addition of background causes a severe complication that must be overcome. The distribution of acceptance functions in signal and background are different and because the acceptance is used as an event by event quantity the full and complete PDF contains the terms  $P(s|Acc)$  and  $P(b|Acc)$  which are the probability that an event is signal (background) given the particular acceptance function. The difference in the distribution of acceptance function is demonstrated in Fig 3 which shows the mean signal and background acceptance. Ignoring this term and replacing it with an overall signal fraction results in a bias of approximately 6 microns determined from a study of toy MC.

The PDF requires a term that describes the signal fraction as a function of the acceptance. The complication arises



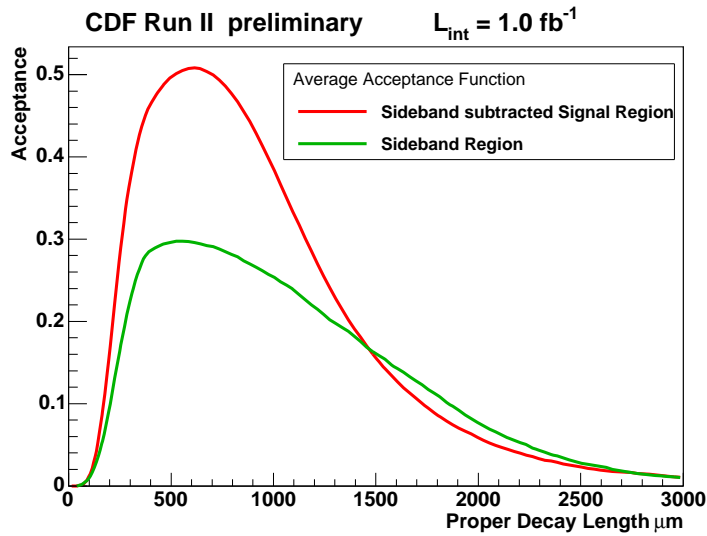


FIG. 3: The plot shows the mean acceptance function for signal and background.

as the Acceptance is a function and not a scalar variable. There are a number of techniques and functions available to fit scalar variables but the same is not true for functions. To solve this the acceptance function is characterised by a single scalar variable, which is easier to handle. The transformation from function to scalar variable is done via a fisher discriminant analysis. We choose this transformation in particular as it preserves the most information about the differences between signal and background acceptance function distributions.

The variables that go into the fisher discriminant analysis are the heights of the acceptance in different bins of  $ct$ . For fisher analysis a mean set of variables must also be provided for signal and background. To do this we use the sideband in data to represent background and use a sideband subtracted signal region to represent signal. The fisher discriminant analysis leaves us with a scalar variable for every event, which in essence characterises the acceptance function. The probability that an event is signal(background) given its acceptance function can now be fit for simultaneously with lifetime. The distribution of this scalar variable in data and the fitted function are shown in Figure 4 and 5. The fitted function is crosschecked by binning the events in regions of fisher scalar and performing a standard mass fit for those events and extracting a signal fraction from this fit. There is good agreement between the two determinations of signal fraction as a function of fisher scalar. Details of general Fisher Discriminant analysis can be found in [3], and details of Lagrange interpolating polynomials are found in [2].

### C. Additional Fit Functions

The full likelihood is given by

$$P = P(mass|s)P(t|Acc, s)P(s|Acc) + P(mass|b)P(t|Acc, b)(1 - (P(s|Acc))) \quad (4)$$

Discussed in this subsection are fit functions for mass and the background lifetime distribution. The signal lifetime and  $P(s|Acc)$  are described above.

Background from partially reconstructed B decays is removed by the lower mass cut. The remainder of the background in the sample is combinatoric. For signal the basic function for fitting the lifetime is an exponential which is then normalised over the acceptance function. A function that is normalised over the acceptance is also required for background. A sum of exponentials as used in other lifetime analyses is found to give a poor fit. Instead an interpolating exponential function is used. This function is called  $y(t)$  and is given by

$$y(t) = e^{a_j + \left(\frac{a_{j+1} - a_j}{t_{j+1} - t_j}\right)(t - t_j)} \quad \text{for } t_j \leq t \leq t_{j+1} \quad (5)$$

where  $a_j$  are fit parameters and  $t_j$  are chosen such that they are concentrated at low  $t$  where the lifetime distribution changes most rapidly.  $y(t)$  represents the lifetime distribution before the trigger in the same way as an exponential

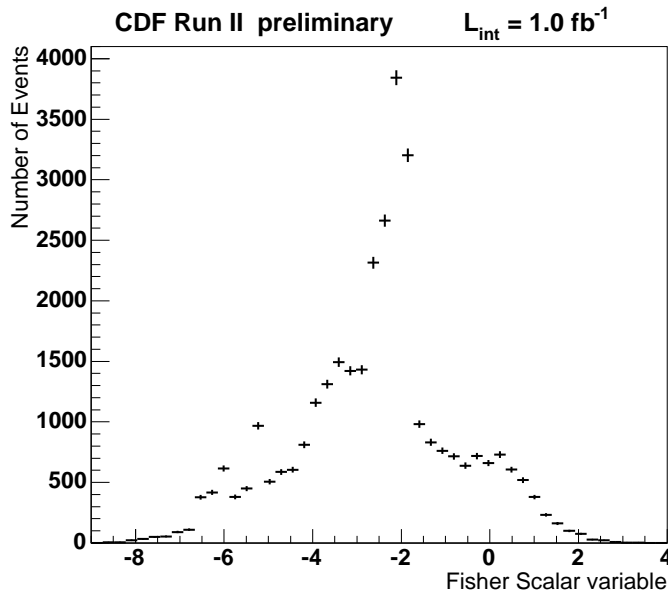


FIG. 4: The distribution of fisher scalar for event in the sample

represents the distribution of signal lifetime before the trigger.  $y(t)$  is normalised over the acceptance function, and this gives a good fit to the lifetime distribution after the trigger.

The mass model for background is a first order polynomial. The choice of this function is driven by the wrong sign distribution. For signal we use a sum of two Gaussian, this choice is driven by full detector simulation MC although no parameters are taken from MC or the wrong sign distribution, but are determined instead by floating them in the likelihood fit.

#### D. Fitting procedure

There are three steps to determine the best fit lifetime.

- A fit to the mass distribution only. The mass parameters are fixed in subsequent parts of the fit
- Using the mass fit, sideband and signal regions are defined allowing calculation of fisher scalar for every event
- Simultaneous fit to lifetime and  $P(s|fish)$ .

## IV. SYSTEMATIC UNCERTAINTIES

A detailed fast MC simulation of the decay, trigger and simple detector geometry was created to understand the effect of assumptions in the method that lead to systematic errors.

Systematic uncertainties are dominated by assumptions of the single track finding efficiency. The calculation of acceptance assumes that a track finding efficiency is independent of impact parameter (between 120 and 1000 microns) and  $P_T$  (above 2GeV). In real data we see variations, the track finding efficiency starts to drop at higher impact parameters and rises with higher transverse momenta. To study the effect of these variations Fast MC was generated containing these variations and the mean bias is observed to be 3.1 and 1.8 microns for variations dependent on impact parameter and transverse momentum respectively. The other leading source of systematic error comes from neglecting the correlation between mass and lifetime in the background. The systematic error for this is also determined by studies involving Fast MC. The other sources of systematic error are found to be small in comparison to the ones already detailed. They arise from the background lifetime parameterisation, the uncertainty in the silicon detector alignment, a residual bias from correcting for the difference in acceptance function distributions, the resolution of the measured lifetime for each event and also the variation in the track finding efficiency as a function of  $\eta$ . It should

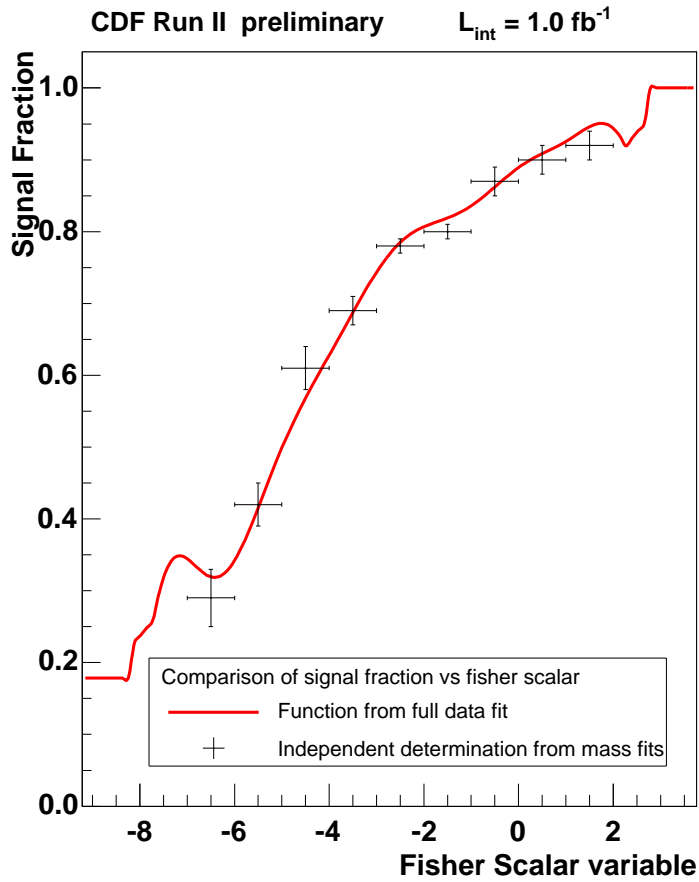


FIG. 5: Signal fraction as a function of fisher scalar as determined a) by the simultaneous lifetime and fisher fit and b) by independent determination from mass fits.

be noted that improvements to the method that would take into account variations in track finding efficiency and correlations are possible and will reduce the systematic error further. However, given that lifetime measurements with current available samples will remain statistically limited it is not critical to improve the method at this time.

The systematic uncertainties are summarized in. Values given for errors in terms of  $ct$ (microns) and  $t$  (pico seconds). Table II.

Source of Sys. Uncert.	Error (microns)	Error(ps)
Track finding efficiency dependence on impact parameter	3.1	0.0103
Track finding efficiency dependence on $P_T$	1.8	0.0060
Mass-Lifetime correlation in background	2.5	0.0083
Background lifetime parameterisation	0.8	0.0027
Silicon Alignment	0.4	0.0013
Fitter Bias	0.4	0.0013
Lifetime resolution model	0.3	0.0010
Track finding efficiency dependence on $\eta$	0.3	0.0010
<b>Total Systematic Uncertainty</b>	<b>4.5</b>	<b>0.015</b>

TABLE II: Summary of systematic uncertainties.

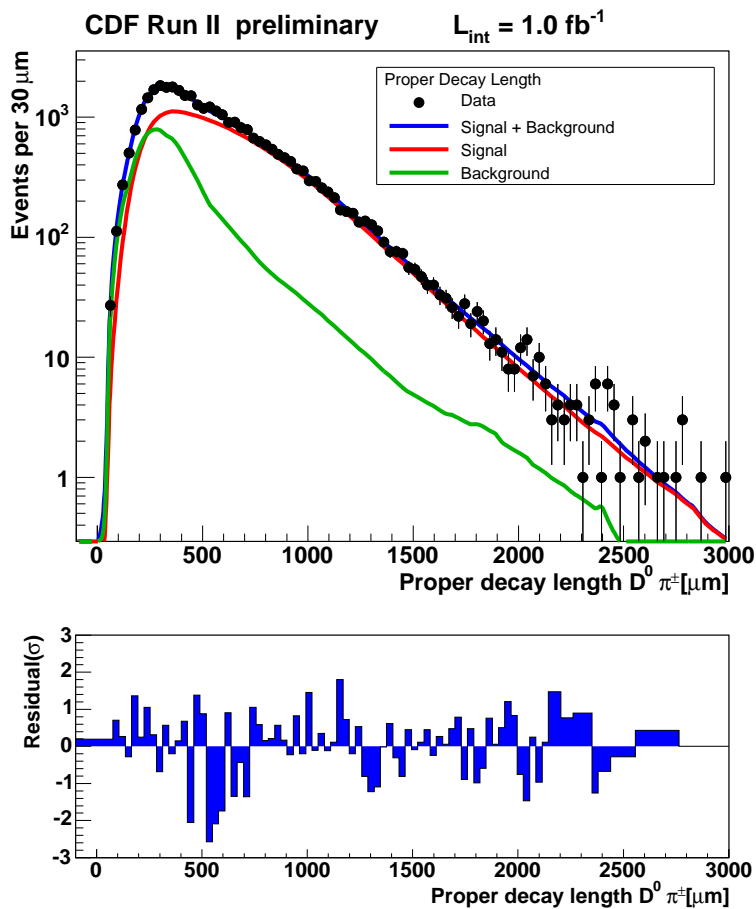


FIG. 6: Lifetime projection

## V. RESULTS

We measure the  $B^+$  lifetime as

$$\tau(B^\pm) = 1.662 \pm 0.023(\text{stat.}) \pm 0.015(\text{syst.})\text{ps} \quad (6)$$

or

$$c\tau(B^\pm) = 498.2 \pm 6.8(\text{stat.}) \pm 4.5(\text{syst.})\mu\text{m} \quad (7)$$

The lifetime fit projection is shown in Figure 6. This measurement is in agreement with the PDG value of  $1.638 \pm 0.011$  ps. It is also in agreement with other CDF measurements from an unbiased trigger path, which is  $1.630 \pm 0.016 \pm 0.011$  ps. This measurement demonstrates that correcting for the trigger bias using this analytical method results in a measurement that has systematic errors not much larger than using data from an unbiased trigger. The measurement of the  $B^+$  lifetime has been carried out as a demonstration that the trigger induced bias can be corrected for using information from the data alone, and that the measurement has a small systematic error. This simulation free method of trigger bias correction can now be used to measure other B hadron lifetimes that are less well known.

## Acknowledgments

We thank the Fermilab staff and the technical staffs of the participating institutions for their vital contributions. This work was supported by the U.S. Department of Energy and National Science Foundation; the Italian Istituto

Nazionale di Fisica Nucleare; the Ministry of Education, Culture, Sports, Science and Technology of Japan; the Natural Sciences and Engineering Research Council of Canada; the National Science Council of the Republic of China; the Swiss National Science Foundation; the A.P. Sloan Foundation; the Bundesministerium für Bildung und Forschung, Germany; the Korean Science and Engineering Foundation and the Korean Research Foundation; the Science and Technology Facilities Council and the Royal Society, UK; the Institut National de Physique Nucleaire et Physique des Particules/CNRS; the Russian Foundation for Basic Research; the Ministerio de Educación y Ciencia and Programa Consolider-Ingenio 2010, Spain; the Slovak R&D Agency; and the Academy of Finland.

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