On a Strategy of Reduction of the Lack of Knowledge (LOK) in Model Validation

P. Ladevèze[a]*, G. Puel*[a] *and T. Romeuf*[b]

[a] LMT-Cachan (ENS Cachan/CNRS/Paris 6 University)
61, avenue du Président Wilson 94235 CACHAN Cedex FRANCE;

[b]European Aeronautic Defence and Space Company (EADS Space Transportation)
Route de Verneuil BP 96 78133 LES MUREAUX Cedex FRANCE.

E-mail: pierre.ladeveze@lmt.ens-cachan.fr, guillaume.puel@lmt.ens-cachan.fr, thierry.romeuf@launchers.eads.net

Abstract: The quantification of the quality of a structural mechanical model remains a major issue today, with the use of an increasing number of methods in order to validate a model in comparison with an experimental reference. This paper presents a new theory based on the concept of Lack of Knowledge combining convex uncertainty models with probabilistic features by introducing for each substructure two bounds of the strain energy as stochastic variables. A general strategy of reduction of the lack of knowledge is discussed and applied to academic as well as industrial cases.

Keywords: Lacks of Knowledge, reduction, model validation, structural dynamics

1. INTRODUCTION

Today, the problem of quantifying the quality of a structural mechanical model remains a major issue. As far as the comparison with an experimental reference is concerned, many approaches can be used to update a deterministic, dynamic model (stiffness, mass and damping) based on free or forced vibration tests [1],[2]. After this process, there may be some phenomena that still cannot be described properly: some uncertainties remain in the material properties, or the model of some parts (e.g. joints) may be simplified. In order to describe these uncertainties, the use of probabilistic methods has become increasingly popular: generally, these methods consist in studying the effects of the uncertainties which affect the input on the variability of the output. This can be done in various ways and has led to major improvements: for example, meta-models have been built by spanning the space of the most influential parameters and applying a specific technique to reduce the computational effort drastically [3],[4].

In [5], we introduced the concept of *Lack of Knowledge* (LOK), which combines convex uncertainty models [6],[7] with probabilistic features. The basic principle consists in globalizing the uncertainties on a substructure by means of an internal variable, called the *basic Lack of Knowledge* (*basic LOK*), which is included within an interval whose upper and lower bounds are stochastic variables. From these basic LOKs, one can derive, for the whole structure, the *effective Lack of Knowledge* (*effective LOK*) of a quantity of interest α, which leads to a stochastically bounded interval which can be compared with experimental values derived from a family of similar real structures. In [8], this theory was successfully applied to a simple problem, which proved its identification and prediction capabilities. In this paper, we establish the first bases of a general strategy of reduction of the lacks of knowledge and we present applications on academic as well as industrial cases.

2. BASIC LOKS

2.1. Basic concept

Each similar structure can be divided into several substructures; by the way, joints can also be treated as substructures. Only the errors concerning structural stiffnesses are considered, hence the use of substructural strain energies in the following definition; indeed, we associate to any substructure E a lack of knowledge m located *anywhere* within an interval whose the two bounds are two internal variables m_E^+ and m_E^- defined by

$$\left(1 - m_E^-\right) \overline{e}_E \leq e_E \leq \left(1 + m_E^+\right) \overline{e}_E, \tag{1}$$

where $\overline{e}_E$ and e_E are the strain energies associated respectively with the deterministic, theoretical model and with one of the real structures. m_E^+ and m_E^- are the *upper basic LOK* and the *lower basic LOK* respectively.

The basic LOKs m_E^+ and m_E^- are sampled using a probabilistic law; the nature of this law is chosen *a priori* and its characteristics are defined by two values $\overline{m}_E^+$ and $\overline{m}_E^-$:

- for example, if the distribution chosen is uniform, these two values include all possible sampled values of m_E^+ and m_E^-;
- in some particular cases of imperfect modelings (e.g. nonlinear joints represented with linear models), characterized by a severe lack of information, one cannot determine precisely the distribution of lack of knowledge and it can only be stated that m is somewhere within $[-\overline{m}_E^-; \overline{m}_E^+]$.

In the absence of specific information, it is reasonable to choose the previous description. We can also consider that a normal distribution is appropriate in cases in which the sources of errors are material uncertainties.

2.2. Illustration

Let us consider the case of a lack of knowledge of the material properties: for a family of similar real structures, we assume that the lack of knowledge m of a substructure E is defined by a centered normal distribution whose Probability Density Function (PDF) is written as follows:

$$m \in [-m_E^-; m_E^+] \text{ with PDF } p(m) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{m^2}{2\sigma^2}}. \tag{2}$$

The standard deviation σ can be associated to the values $\overline{m}_E^+$ and $\overline{m}_E^-$ by stating for example that $\int_{-\overline{m}_E^-}^{\overline{m}_E^+} p(m)\mathrm{d}m = 0.99$; the PDF is then set to zero below $-\overline{m}_E^-$ and beyond $\overline{m}_E^+$, and can be normalized again.

The probability of having m within a given interval $[-m_E^-; m_E^+]$ is

$$P(-m_E^- \leq m \leq m_E^+) = \int_{-m_E^-}^{m_E^+} p(m)\mathrm{d}m. \tag{3}$$

Since the basic LOKs are defined on both sides of the theoretical model, this occurrence can be described by two independent events:

- $m \in [0; m_E^+]$, *i.e.* the event $(m_E^- = 0, m_E^+)$ occurs with probability $P^+(m_E^+)$;
- $m \in [-m_E^-; 0]$, *i.e.* the event $(m_E^-, m_E^+ = 0)$ occurs with probability $P^-(m_E^-)$.

Of course, one has $P^+(\infty) + P^-(\infty) = 1$. In this special case of a centered distribution, one even has $P^+(\infty) = P^-(\infty) = \frac{1}{2}$. This situation is depicted in Figure 1. This case illustrates how the basic LOKs should be sampled: depending on the value of m obtained, one gets two distinct types of intervals: $[0; m_E^+]$ and $[-m_E^-; 0]$.

2.3. Definition of an Interval Probability

Since the use of two distinct probabilities P^+ and P^- is rather impractical, we developed in [9] some mathematical tools in order to circumvent this difficulty.

Let us consider a family of intervals $[-m_E^-; m_E^+] \ni m$ with $m_E^+ + m_E^- = L$. An interval $[-m_E^-; m_E^+]$ is called a *standard interval* $I(L)$ if, for a given interval length L, the probability of m being in $I(L)$ is the greatest of all such intervals of length L, *i.e.*

$$I(L) = \arg \max_{\substack{[-m_E^-; m_E^+] \\ m_E^+ + m_E^- = L}} P^+(m_E^+) + P^-(m_E^-). \tag{4}$$

From this definition, we can introduce the concept of *interval probability* $P(L)$ by stating that for a given length L, $P(L)$ is the probability of having m in $I(L)$, *i.e.*

$$P(L) = P(m \in I(L)) = \max_{\substack{[-m_E^-; m_E^+] \\ m_E^+ + m_E^- = L}} P^+(m_E^+) + P^-(m_E^-). \tag{5}$$

One interpretation of these definitions is that if one wants to determine an interval such that m has a given probability P of being inside, one has to select the standard interval $I(L)$ whose probability interval $P(L)$ is equal to P, and one can show that this interval is the smallest interval $[-m_E^-; m_E^+]$ such that $P^+(m_E^+) + P^-(m_E^-) = P$, *i.e.*

$$I(L) = \arg \min_{\substack{[-m_E^-; m_E^+] \\ P^+(m_E^+) + P^-(m_E^-) = P}} m_E^+ + m_E^-. \tag{6}$$

One can also prove that the bounds of $I(L)$ verify the equality: $p(m_E^+) = p(m_E^-)$. These remarks are summarized in Figure 2 in the case of a non-centered normal law.

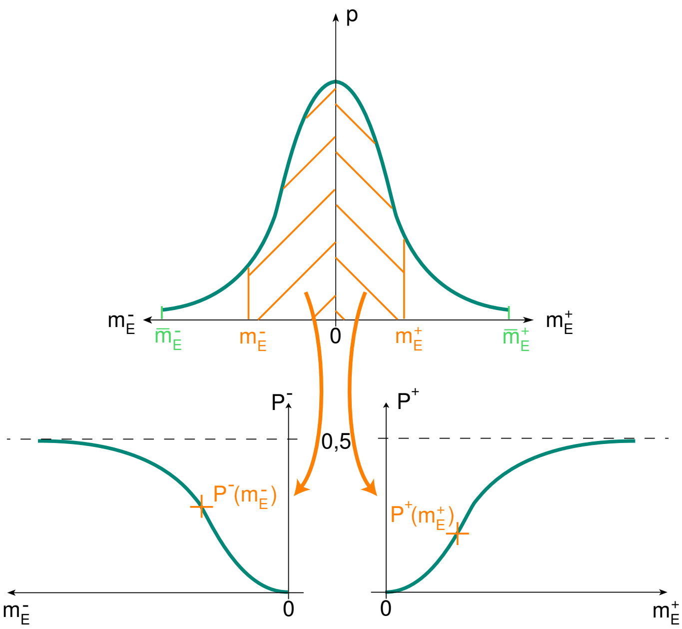

Figure 1. Example of a centered normal law: (top) PDF of m; (bottom) P^- and P^+.

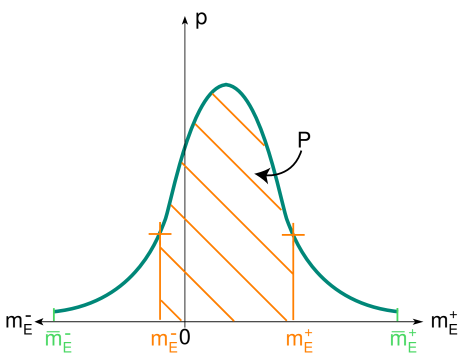

Figure 2. Illustration of the concepts of standard interval and interval probability.

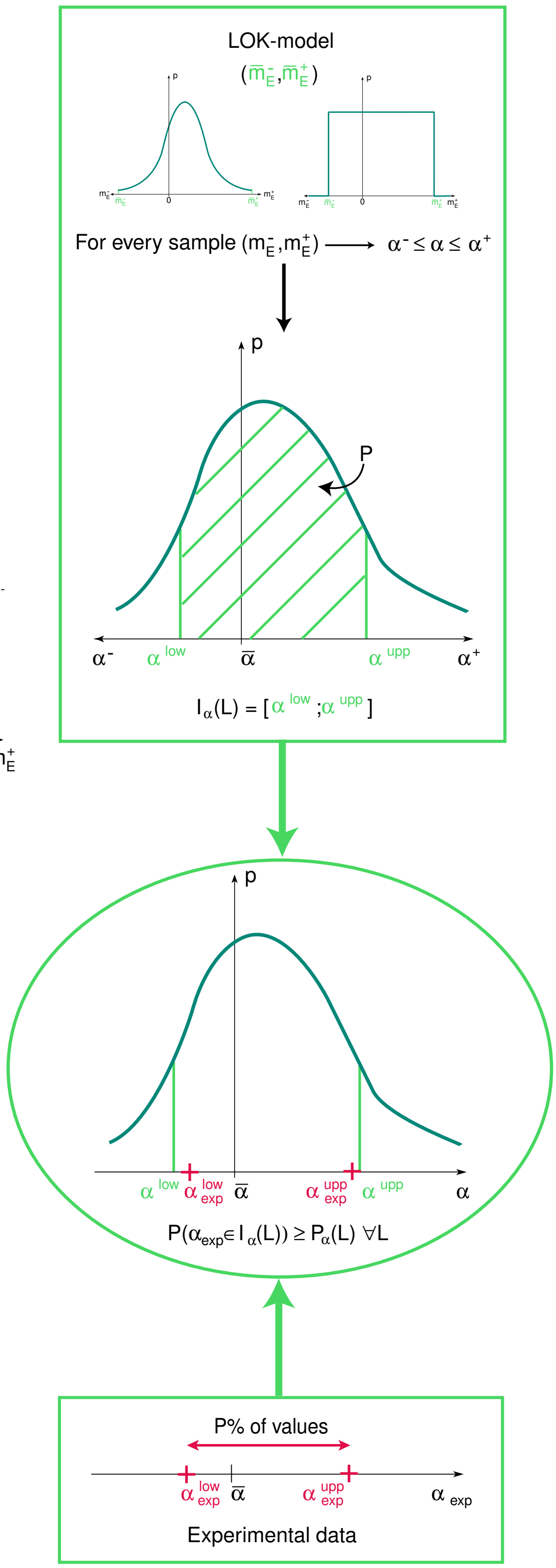

Figure 3. Illustration of the concept of Lack of Knowledge.

3. THE USE OF LOK

3.1. Principle

Let us consider a certain quantity of interest α.

- for every sample of $(m_E^-, m_E^+)_{E\in\Omega}$, one can calculate two bounds α^- and α^+ of the quantity of interest α_{mod} relative to the model, as will be shown in Section 3.2. If one knows the stochastic laws for the basic LOKs, one can obtain the probabilistic distribution of these bounds α^- and α^+ by means of an interval probability $P_\alpha(L)$ such that $P(\alpha_{mod} \in I_\alpha(L)) = P_\alpha(L)\,\forall L$. Remembering the previous interpretation of an interval probability, one can get for a given probability value P the associated standard interval $I_\alpha(L)$ such that $P(\alpha_{mod} \in I_\alpha(L)) = P$. We will refer to the two bounds of this interval as the *effective Lack of Knowledge* (*effective LOK*), and denote them α^{low} and α^{upp}.

- based on the similar real structures, one can derive in the same way two bounds α_{exp}^{low} and α_{exp}^{upp} which include P% of the experimental values α_{exp}.

The experimental data and the values obtained from the LOK model are then compared as in Figure 3. In order for the model to be conservative, the basic LOKs should be such that

$$P(\alpha_{exp} \in I_\alpha(L)) \geq P_\alpha(L)\,\forall L. \tag{7}$$

This means that one should have $\alpha^{low} \leq \alpha_{exp}^{low} \leq \alpha_{exp}^{upp} \leq \alpha^{upp}$ for any given probability value P. Note that this last interpretation is a generalization of the 99%-bounds described in [5] and [8].

3.2. Effective LOKs

The comparison between the results of the model and reality is made using quantities which are standard in the field of modal analysis: in this paper, we use free-vibration tests; therefore, our quantities of interest α are eigenfrequencies and eigenmodes. The previously defined pair of quantities α^{low} and α^{upp} is called the *effective LOK* and the corresponding values for eigenfrequencies and eigenmodes are reviewed below.

3.2.1. Effective LOK of an Eigenfrequency

If the modes $\overline{\phi}_i$ are mass-normalized, a first-order approximation ($\phi_i \simeq \overline{\phi}_i$) gives

$$\omega_i^2 - \overline{\omega}_i^2 = \phi_i^T K \phi_i - \overline{\phi}_i^T \overline{K\phi}_i \simeq \overline{\phi}_i^T (K - \overline{K})\overline{\phi}_i = 2\sum_{E\in\Omega} \left(e_E(\overline{\phi}_i) - \overline{e}_E(\overline{\phi}_i)\right). \tag{8}$$

From relationship (1), one has for a given sample $(m_E^-, m_E^+)_{E\in\Omega}$

$$\omega_i^{2-} \leq \omega_i^2 \leq \omega_i^{2+} \tag{9}$$

with

$$\omega_i^{2-} = \overline{\omega}_i^2 - 2\sum_{E\in\Omega} m_E^- \overline{e}_E(\overline{\phi}_i), \tag{10}$$

$$\omega_i^{2+} = \overline{\omega}_i^2 + 2\sum_{E\in\Omega} m_E^+ \overline{e}_E(\overline{\phi}_i) \tag{11}$$

Thus, for a given probability value P, one can derive the two bounds $\omega_i^{2\,low}$ and $\omega_i^{2\,upp}$ of the associated standard interval $I_{\omega_i^2}(L)$, i.e. the effective LOK of an eigenfrequency.

3.2.2. Effective LOK of an Eigendisplacement

For small values of the basic LOKs, we can approximate the variation of an eigendisplacement (defined as the value at a Degree of Freedom of an eigenmode) by writing

$$\phi_{ki} - \overline{\phi}_{ki} \simeq U^T \Delta K \overline{\phi}_i = \sum_{E\in\Omega} U^T \left(K_E - \overline{K}_E\right)\overline{\phi}_i \tag{12}$$

where U is a given vector. Using $U^T K_E \overline{\phi}_i = \frac{1}{2}e_E(U+\overline{\phi}_i) - \frac{1}{2}e_E(U-\overline{\phi}_i)$ and relationship (1), one gets for a given sample $(m_E^-, m_E^+)_{E\in\Omega}$

$$\phi_{ki}^- \le \phi_{ki} \le \phi_{ki}^+ \tag{13}$$

with

$$\phi_{ki}^- = \overline{\phi}_{ki} - \frac{1}{2}\sum_{E\in\Omega}\left\{m_E^-\overline{e}_E(U+\overline{\phi}_i) + m_E^+\overline{e}_E(U-\overline{\phi}_i)\right\}, \tag{14}$$

$$\phi_{ki}^+ = \overline{\phi}_{ki} + \frac{1}{2}\sum_{E\in\Omega}\left\{m_E^+\overline{e}_E(U+\overline{\phi}_i) + m_E^-\overline{e}_E(U-\overline{\phi}_i)\right\} \tag{15}$$

Thus, for a given probability P, one can derive the two bounds ϕ_{ki}^{low} and ϕ_{ki}^{upp} of the associated standard interval $I_{\phi_{ki}}(L)$, i.e. the effective LOK of an eigendisplacement.

4. DETERMINATION OF THE BASIC LOKS

The purpose of determining the basic LOKs is to find the values of m_E^+ and m_E^- which are the most representative of the dispersion. The process we introduce here is based on the idea that the more abundant the experimental data, the better we can reduce the LOK-level within the structure. Therefore, the first step of the process consists in setting initial, overestimated values of the basic LOKs for all the substructures; this can be done by applying one's *a priori* knowledge or experience of the structure being studied. Indeed, it is not vital to use accurate estimates; the most important point is to use overestimated values $(\overline{m}_E^{+\,0}, \overline{m}_E^{-\,0})_{E\in\Omega}$ of the basic LOKs for each substructure.

The reduction process consists in using relevant experimental data to reduce the LOK-level individually for each substructure. Let us consider a given substructure E^*. One has to find smaller values of $\overline{m}_{E^*}^+$ and $\overline{m}_{E^*}^-$, which, in terms of interval probabilities, yields the following relationship:

$$P_{E^*}^0(L) \le P_{E^*}(L) \quad \forall L. \tag{16}$$

This reduction should be carried out with the constraint created by the experimental information selected:

$$\alpha^{low} \leq \alpha^{low}_{exp} \leq \alpha^{upp}_{exp} \leq \alpha^{upp}. \tag{17}$$

In fact, as one is interested in the minimization of the lack of knowledge of Substructure E^*, one intends to take into account the worst happening case concerning all the other substructures. We can write formally for each given sample $(m^-_E, m^+_E)_{E\in\Omega}$:

$$\alpha^{worst\,+} = \overline{\alpha} + S_{E^*}\Delta\alpha^+_{E^*} + \sum_{E\neq E^*} \Delta\alpha^{worst\,+}_E \tag{18}$$

$$\alpha^{worst\,-} = \overline{\alpha} + S_{E^*}\Delta\alpha^-_{E^*} + \sum_{E\neq E^*} \Delta\alpha^{worst\,-}_E. \tag{19}$$

This worst-case analysis is completed by the introduction of a coefficient quantifying whether the experimental information is more or less representative of the behavior of the structure; this value $S_{E^*} \in]0;1]$ is called *test severity coefficient* for Substructure E^* and is maximal when the test fits perfectly the global mechanics of the structure. Then we can associate to these bounds $\alpha^{worst\,+}$ and $\alpha^{worst\,-}$ an interval probability $P_{\alpha^{worst}}(L)$ and derive the two bounds $\alpha^{worst\,upp}$ and $\alpha^{worst\,low}$ of the associated standard interval $I_{\alpha^{worst}}(L)$ for a given probability P.So the following constraints are introduced:

$$\alpha^{worst\,low} \leq \alpha^{low}_{exp} \leq \alpha^{upp}_{exp} \leq \alpha^{worst\,upp}. \tag{20}$$

So as a summary, the problem consists in finding

$$\max P_{E^*}(L, \overline{m}) \quad \forall L \tag{21}$$

with the previous constraints, and for several given values of L.

5. APPLICATION TO A SIMPLE PROBLEM

5.1. Definition of the Structure

5.1.1. Deterministic Theoretical Model

The structure being considered is a plane truss similar to that studied in [8]; it consists of six bars connected by spherical joints, as shown in Figure 4. We assume that the bars are solicited only in traction-compression and that the connections between the ground and the structure at Nodes 1 and 2 are perfectly rigid links. The material properties of the associated theoretical model are given in Table 1.

5.1.2. Experimental Data

A family of such actual trusses is simulated and their eigenfrequencies and eigenmodes constitute the data which is then used in the reduction process described in Section 4: the ‘experimental’ data are simulated by using the theoretical model and introducing some stochastic distributions in the stiffness characteristics of the substructures; these changes are summarized in Table 1. Note that material “X” is considered to be imperfectly known; hence the uniform law chosen for the simulation. For each of these ‘real’ structures, we are able to calculate eigenfrequencies and eigenmodes and, thus, derive experimental distributions of the eigenfrequencies or eigendisplacements associated to the real structures (see for example the distribution of ω^2_{exp} for Mode 6 in Figure 5).

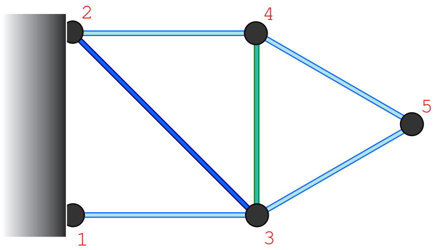

Figure 4. Plane truss example

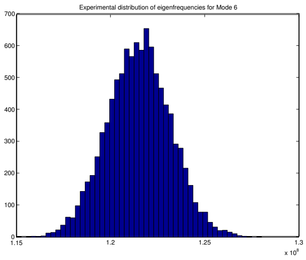

Figure 5. Experimental distribution of ω_{exp}^2 for Mode 6

Table 1. Properties of the deterministic plane truss and of the simulated structures, with Bars 1-3, 3-5, 4-5 and 2-4 constituting Group g1, bar 2-3 as Group g2 and bar 3-4 as Group 3.

Gp	Material	Young's modulus	Density	Law	Mean/Range	Simulated stiffnesses
g1	aluminium	$\overline{E}_{g1} = 72GPa$	$2700kg/m^3$	normal	0%/5%	$\in [0.95\overline{K}_{g1}; 1.05\overline{K}_{g1}]$
g2	steel	$\overline{E}_{g2} = 210GPa$	$7800kg/m^3$	normal	−5%/10%	$\in [0.85\overline{K}_{g2}; 1.05\overline{K}_{g2}]$
g3	"X"	$\overline{E}_{g3} = 10GPa$	$1500kg/m^3$	uniform	5%/15%	$\in [0.90\overline{K}_{g3}; 1.20\overline{K}_{g3}]$

5.2. Reduction of the Basic LOKs of the Structure Being Considered

The reduction process is carried out by assuming *a priori* an initial LOK-level of 50% for each substructure, which guarantees overestimated starting values. We also assume a normal LOK-distribution for the aluminum and steel bars and select a uniform LOK-distribution for the "X" bar. From the measured distributions of eigenfrequencies, we decide to keep in mind the values $\omega_{i\,exp}^{2\,upp}$ and $\omega_{i\,exp}^{2\,low}$ that include 99% of the experimental eigenfrequencies; this means that we do not care any more about the distribution of these experimental eigenfrequencies within the two 99%-values. If we wanted a more precise description, we could also take the 50%-values in order to have an estimation of the standard deviation of the experimental values.

Next, it is important to select the most relevant experimental tests to carry out the successive processes. An effective method consists in using the fact that the sensitivities of the effective LOKs to the basic LOKs are directly related to the modal strain energy of the theoretical, deterministic model (see Section 3.2 for more details). The most relevant modal tests for reducing the basic LOK of Substructure E^* are those in which the modal strain energy is contained mainly within Substructure E^*. As we are interested with experimental eigenfrequencies, the modal strain energies $\overline{e}_E(\overline{\phi}_i)$ are considered for Modes 1 to 6 and are listed in Table 2 where the largest substructural energies are emphasized.

Table 2. Modal strain energies for Modes 1 to 6.

$\overline{e}_E(\overline{\phi}_i)$	i=1	i=2	i=3	i=4	i=5	i=6
E=g1	$\mathbf{3.3.10^5}$	$\mathbf{1.3.10^6}$	$\mathbf{7.6.10^6}$	$3.8.10^6$	$\mathbf{2.5.10^7}$	$\mathbf{6.0.10^7}$
E=g2	$\mathbf{1.4.10^5}$	$6.7.10^4$	$9.9.10^3$	$\mathbf{1.0.10^7}$	$2.0.10^6$	$1.7.10^5$
E=g3	$\mathbf{2.5.10^5}$	$\mathbf{1.7.10^6}$	$6.1.10^5$	$4.7.10^5$	$6.9.10^4$	$1.9.10^5$

The reduction is achieved by selecting as relevant experimental tests $\omega_{i\,exp}^{2\,upp}$ ans $\omega_{i\,exp}^{2\,low}$ derived from Modes 6, 4 and 2 for Groups 1, 2 and 3 respectively, and by considering that these data are representative of the global behavior of the structure (test severity coefficients equal to one). The results come out as

$$\overline{m}_{g1}^{+} = 0.032 \qquad \overline{m}_{g2}^{+} = 0.034 \qquad \overline{m}_{g3}^{+} = 0.205$$
$$\overline{m}_{g1}^{-} = 0.034 \qquad \overline{m}_{g2}^{-} = 0.092 \qquad \overline{m}_{g3}^{-} = 0.101.$$

In this very special case, with a first-order assumption, these results are to be compared directly with the stiffness distributions introduced into the deterministic model to simulate the experimental data: $[(1-0.05)\overline{K}_{g1}; (1+0.05)\overline{K}_{g1}]$, $[(1-0.15)\overline{K}_{g2}; (1+0.05)\overline{K}_{g2}]$ and $[(1-0.10)\overline{K}_{g3}; (1+0.20)\overline{K}_{g3}]$. We can conclude that the agreement is rather good. The choice of the relevant experimental data is crucial; if one tried to reduce the LOKs of any group using Mode 1, the minimization process would not lead to any reduction because the influence of the other two groups is not small enough.

5.3. Capacity of Prediction

With the values just obtained, we are able to calculate the effective LOKs for the three other modes (1, 3 and 5) in order to evaluate the results of the reduction process. The basic LOKs are sampled with the values determined and the probabilistic laws chosen; the corresponding calculated 99%-values are listed and compared with the experimental 99%-values in Table 3 below. The constraints are successfully respected for Modes 1, 3 and 5, which shows the consistency of the results obtained with Modes 2, 4 and 6.

Table 3. Comparison of eigenfrequencies and eigendisplacements (99%-values) for Modes 1, 3, 5.

i	$\omega_i^{2\,low}$	$\omega_{i\,exp}^{2\,low}$	$\overline{\omega}_i 2$	$\omega_{i\,exp}^{2\,upp}$	$\omega_i^{2\,upp}$	ϕ_{ki}^{low}	$\phi_{ki\,exp}^{low}$	$\overline{\phi}_{ki}$	$\phi_{ki\,exp}^{upp}$	ϕ_{ki}^{upp}
1	$1.36.10^6$	$1.35.10^6$	$1.43.10^6$	$1.53.10^6$	$1.54.10^6$	0.85	0.88	0.95	0.99	1.01
3	$1.58.10^7$	$1.58.10^7$	$1.64.10^7$	$1.71.10^7$	$1.70.10^7$	−1.00	−0.98	−0.95	−0.91	−0.90
5	$5.28.10^7$	$5.29.10^7$	$5.51.10^7$	$5.68.10^7$	$5.69.10^7$	−0.74	−0.72	−0.68	−0.62	−0.62

6. STUDY OF A STRUCTURE WITH A MODELLING ERROR

6.1. Presentation of the Structure

6.1.1. Deterministic Theoretical Model

In this example, we want to study the ability of our theory to describe a modelling error in the theoretical model. The studied structure is a beam clamped at one end; we are interested with its bending vibrations. The theoretical model consists of 100 standard Bernoulli-Euler elements based on a cubic interpolation of displacements.

6.1.2. Experimental Data

The experimental structure is simulated by inserting a joint in the middle of the beam: the two corresponding ends of the two half beams are linked by two linear springs: one concerning the vertical translation ($k = 7.10^7 N/m$) and a rotational one ($K = 1000 N.m/rad$), as in Figure 6. We then compute the eigenfrequencies and eigenmodes of this structure.

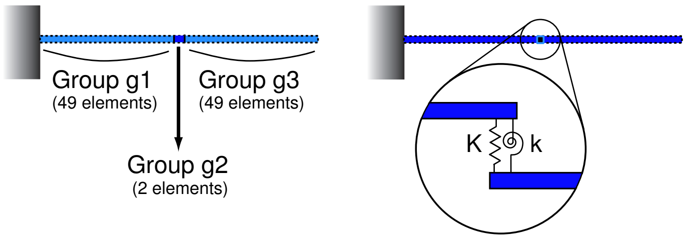

Figure 6. Theoretical model and experimental simulation of the clamped beam.

6.2. Calculation of the Basic LOKs

Before determining the Lacks of Knowledge, the model is updated with the first 15 modes of the experimental structure according [2]. This method leads to the correction of the stiffnesses of the two elements at both sides of the joint (with a factor of 0.41). After updating, we still have a global residual error of 4%, and no further improvement can be made, which means that the model cannot represent the experimental data in a better way.

The determination of the Lacks of Knowledge is achieved on three different groups: Group g2 corresponds to the two elements at both sides of the joint, Group g1 and Group g3 to the other elements located before and after the joint respectively, as indicated in Figure 6. We use the eigenfrequencies of Modes 4, 8 and 12, by considering them as extreme values describing the distribution coming from the reality; moreover, the test severity coefficients are set to one. With an initial LOK-level of 50% and a normal law assumption for each group, we obtain the following results:

$$\overline{m}_{g1}^{+} = 0.003 \qquad \overline{m}_{g2}^{+} = 0 \qquad \overline{m}_{g3}^{+} = 0$$
$$\overline{m}_{g1}^{-} = 0 \qquad \overline{m}_{g2}^{-} = 0.040 \qquad \overline{m}_{g3}^{-} = 0,$$

which means that the actual structure is perfectly described by the theoretical model, excepted in the neighborhood of the joint where we find a lack of knowledge of 4%. This example shows that the theory of the Lacks of Knowledge is useful to indicate the areas where the model is not good enough to represent the global behavior of the whole structure, and gives an estimate of its accuracy.

7. STUDY OF A REAL CASE

7.1. Description of the Structure

We will now present the application of the method to an actual, industrial structure: the Sylda5 satellite support developed by the EADS company is capable of carrying two individual satellites and is represented in Figure 7. Vibration tests were performed by IABG for DASA/DORNIER under contract with CNES: the test setup consisted of 5 exciters and 260 sensors. The model proposed by EADS represents both the support itself and a payload simulating the presence of a satellite; it consists of 38 substructures with various materials, including orthotropic sandwiches, aluminum and steel. The first tests have shown that it was essential to take the ground into account in the model; this was done using 3 rotational springs, one translation spring and a rigid-body-movement constraint for all the bottom nodes. In the end, the model consists of 27648 DOFs and 9728 elements.

We consider as experimental data the extreme values of the eigenfrequencies and eigenmodes measured from a series of tests, without caring about their distribution.

7.2. Determination of the Basic LOKs

First, the model is updated with the first 12 modes using the method described in [2]. At this point, we want to describe the remaining lacks of knowledge. In order to do that, we divide the whole structure into 4 main groups of substructures, depicted in Figure 8:

- Group g1 is associated with the payload substructure;
- Group g2 represents the junction between the payload substructure and the Sylda support itself;
- Group g3 is the Sylda support;
- Group g4 is associated with the ground model.

Figure 7. The Sylda5 satellite support.

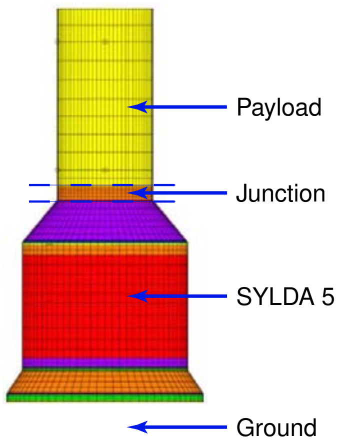

Figure 8. Model associated to Sylda5.

The objective is to carry out the reduction of the most influential lacks of knowledge. An initial value of 50% is assumed for each pair $({\overline{m}_E^+}^0, {\overline{m}_E^-}^0)$, where $E \in \{g1, g2, g3, g4\}$. In the first 8 modes, only Group 1 and Group 3 have significant modal strain energy levels and, thus, they are for the moment the only ones involved in the reduction process. We consider as experimental data the extreme values of the eigenfrequencies measured from a series of tests, without caring about their distribution.

With such values from Modes 7 and 8 on the one hand, and from Modes 4 and 5 on the other hand, we reduce the basic LOKs of Group 1 and Group 3 respectively to

$$\overline{m}_{g1}^{+} = 0.154 \qquad \overline{m}_{g3}^{+} = 0.001$$
$$\overline{m}_{g1}^{-} = 0.009 \qquad \overline{m}_{g3}^{-} = 0.012,$$

by using test severity coefficients equal to one. As a conclusion, we can stress that these results corroborate the quality of the updated Sylda support model (Group 3) and give an estimation of the accuracy of the model used to describe the payload (Group 1).

8. CONCLUSION

We showed in this paper some applications of the theory of the Lacks of Knowledge which combines convex uncertainty models with probabilistic features. The method is able to quantify local uncertainties by using quantities of interest defined on the whole structure and it can also be useful to the estimation of modelling accuracy. The reduction process that we introduced in this paper consider experimental data as information usable to reduce the overestimated basic LOKs assumed for each substructure. This approach should lead the way to the development of a general method for reducing the lacks of knowledge for predetermined families of parameters by designating what tests should be performed or which substructure models should be improved.

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