Simulation Error Models for Improved Reservoir Prediction

A. O'Sullivan, M. Christie

Institute of Petroleum Engineering, Heriot-Watt University,
EH14 4AS, Edinburgh, Scotland;
E-mail: alannah.osullivan@pet.hw.ac.uk, mike.christie@pet.hw.ac.uk

Abstract: Successful reservoir prediction requires an accurate estimation of parameters to be used in the reservoir model. This research focuses on developing error models for simulation error within the petroleum industry, enabling accurate parameter estimation. The standard approach in the oil industry to parameter estimation in a Bayesian framework includes inappropriate assumptions about the error data. This leads to the parameter estimations being biased and over confident. An error model is designed to significantly reduce the bias effect and to estimate an accurate range of spread. A 2D viscous fingering example problem will be used to demonstrate both construction of the error model, and the benefits gained in doing so.

Keywords: Error model, parameter estimation, simulation error, likelihood, viscous fingering

1. INTRODUCTION

The accuracy of reservoir forecasting is limited by both the accuracy of the input data and of the method chosen for solving the system. If the data is accurate, it is expected the use of a detailed geological model to solve the flow equations representing the system will provide more accurate results compared with a reduced model. The sheer size of a reservoir makes it impossible to physically measure the properties, such as porosity and permeability in each cell for a detailed model. However an accurate description of the reservoir is needed to make an accurate forecast, leaving the true parameter values to be estimated. This problem of accurate parameter estimation is common to many applied scientific disciplines, such as weather forecasting, contaminant transport and reservoir simulation. This paper focuses on developing simulation error models for accurate parameter estimation research in the oil industry.

History matching in a Bayesian framework allows likely regions of parameter space to be identified. By simulating oil production with initial estimates from possible parameter values and comparing the simulated results with early production data, Bayesian analysis provides a formal framework for revising the parameter estimate values, [1]. Coarse models are favoured in the history matching process for time saving purposes, however this produces poor estimates for parameter space as simulation errors are introduced from using an approximate model. To over come this problem of simulation error, the idea of using an error model has been introduced. An error model is based on using simulation results from a fast model, such as an up-scaled model, together with statistical error data which is collected using a limited amount of detailed model data. The goal of an error

Sensitivity Analysis of Model Output
Kenneth M. Hanson and François M. Hemez, eds.
Los Alamos National Laboratory, 2005; http://library.lanl.gov/

model is to produce parameter estimations with an accuracy comparable to that of a full field model, yet with a speed similar to that of a coarsely gridded model.

This paper is organised as follows. We start with a review of how parameter estimation is typically carried out under a Bayesian framework, and examine the associated assumptions. In section 3, a viscous fingering example will be set up to show the importance of an error model. This problem will also be used to show how an error model is constructed and demonstrate the benefits gained in doing so. Finally, factors input to the error model are examined in more detail to optimize results with respect to the amount of computation.

2. HISTORY MATCHING IN A BAYSIAN FRAMEWORK

Bayesian analysis is applied in the history matching process to determine parameter estimates for use in reservoir simulation and prediction. Bayes theorem, (1) is used to find appropriate regions of parameter space.

$$p(m|O) = \frac{p(O|m)p(m)}{\int p(O|m)p(m)} \tag{1}$$

The posterior probability, $p(m|O)$, is calculated from a combination of the prior and the likelihood function, $p(O|m)$. The prior is set by initial knowledge or beliefs of the parameters. Correctly calculating the likelihood function, (2), is key in defining an accurate posterior distribution. In the oil industry, the likelihood commonly assumes a Gaussian distribution for the error data.

$$p(m|O) = \exp(-M) \tag{2}$$

The probability p for the observation O to occur assuming the model m is correct, is measured in terms of likelihood function. The misfit, M in Equation (2), measures the mismatch between observed and simulated data (3).

$$M = \langle o - s|C^{-1}|o - s\rangle \tag{3}$$

For a given property to be estimated, o represents the observed data, and s the simulated data. Together with the inverse covariance C^{-1} the misfit is defined. The inverse covariance constructed in full represents spread of both data and simulation error, C_d and C_s respectively. Tarantola [2], shows under Gaussian assumption the observational error and simulation error can be combined by addition of the covariance matrices, (4).

$$C = C_d + C_s \tag{4}$$

In this paper we are concerned with quantifying simulation error only. Studies regarding simulation error have been recorded in the literature, [3],[4],[5],[6].

2.1. The Misfit Function

Typically in the oil industry, the misfit is simplified to a least squares model (5). This approach is characterised by a sum of squares of the difference between observed and simulated data and has a single value, σ^2, to represent the variance of this difference.

$$M = \sum_{i=1}^{n} \frac{(o_i - s_i)^2}{2\sigma^2} \tag{5}$$

From Gaussian statistics, this least squares method assumes the data values are independent, that is, the error is randomly spread.

Measurement of rock properties are limited by both the equipment and the user, which contributes error to the final solution. These types of data error are time independent so the least squares approach is valid for this case. Finite difference methods used in simulation progress using the solution from previous steps with an adjustment involving a time and flux combination. Thus the solution follows an evolution in time, and as such, the simulation error will also [7]. As simulation error is correlated in time the least squares method does not hold when used in conjunction with a coarse model.

By using a single value for σ^2, time dependent variance is not represented. Too small a value for σ^2 can give an over confident prediction in parameter estimation. Using an approximate reservoir model can give consistently wrong results causing heavily biased parameter predictions when using a standard least squares misfit.

3. VISCOUS FINGERING EXAMPLE

The following viscous fingering example is a relatively simple yet realistic problem for demonstrating parameter estimation. This 2D model has a set of injector wells spanning the left hand side of the reservoir, and a set of producers down the right hand side. Gas is injected into the reservoir, forcing the oil toward the producer wells. Due to the difference in mobilities between the two fluids, the injected gas fingers through the oil, Figure 1. These fingers cause early breakthrough, reducing recovery, [8],[9],[10].

The challenge is to determine a probability distribution for the viscosity of the oil given the concentration data, Figure 2, of the fluids output at the producer wells. The concentration data represents measured history data although the reservoir in this example is synthetic, thus the data is also. The data is treated as if the viscosity used in the simulation is unknown. The prior assumption is the viscosity value lies in the range $\mu = 5 \ldots 25$.

There are two main ways to solve this challenge. Using a fine grid model of the reservoir, we expect to find an accurate estimation for the oil viscosity. However, a fine grid model is too time consuming to be practical, so we compare this method with using a coarse grid model. The coarse grid model introduces errors due to using a limited amount of information but is extremely fast and simple.

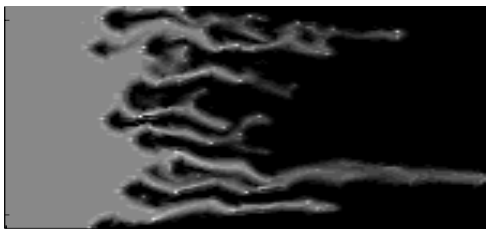

Figure 1. Viscous fingering.

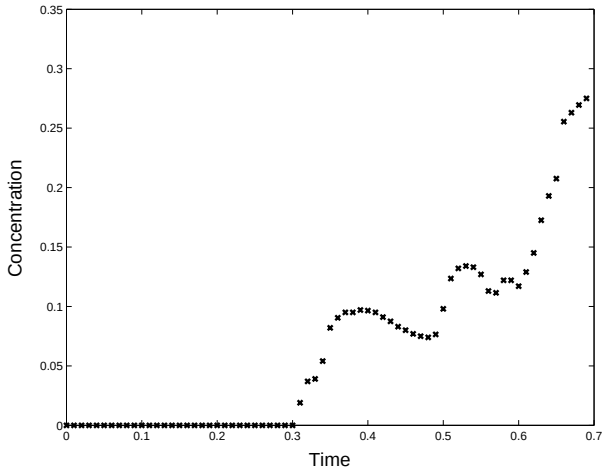

Figure 2. Concentration data for unknown viscosity value.

3.1. The Fine Grid Solution

The exact permeability field for the reservoir is unknown, thus a number of fine grid simulations are used, obtaining a mean solution with some degree of uncertainty. This method produces reasonable results, however the simulation time is long taking of the order hours on a standard work station, depending on the level of detail used in the model.

Figure 3 shows three realisations each (from sets of 20) for viscosity of $\mu = 5$, 10, and 15, as well as the unknown viscosity data. Although the results vary, Figure 3, the unknown viscosity looks to be somewhere between $\mu = 10$ and 15. At this stage, if we were to obtain a smaller range for the true viscosity value, we would proceed with further fine grid simulations, trying viscosity values in the range $\mu = 10 \ldots 15$. For each viscosity value chosen for simulation, each of the 20 realisations would need to be used in the simulations. It is easily seen that this method of parameter estimation is too time consuming and is thus impractical in an industry based situation.

For this viscous fingering example, the true value of the 'unknown' viscosity is $\mu = 13$. The fine grid solution above, although time consuming, has proven accurate.

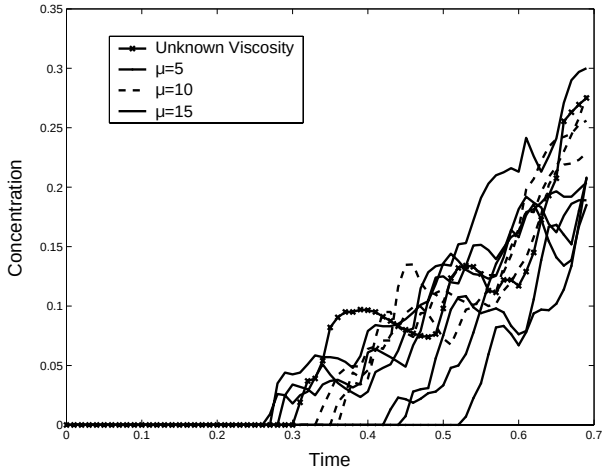

Figure 3. Fine grid approximation to the unknown data.

3.2. The Coarse Grid Solution

A coarse grid model usually consists of an up-scaled version of the fine grid model. The averaging behaviour of the up-scaling process means the full details of the reservoir are not available, hence different results are generated when compared with the fine grid solution in this viscous fingering example. Here the Todd & Longstaff model is used to approximate the average concentration at the production well in lieu of an up-scaled model [11],[12].

The Todd & Longstaff model is a well-known model which approximates the average response of the viscous fingering in the reservoir. Like up-scaled models, the Todd & Longstaff model is also subject to inaccuracies. The fractional flow in the Todd & Longstaff model is found simply from equation (6), where c is the average concentration and M_{eff} is the effective viscosity ratio.

$$f(c) = \frac{c}{c + (1-c)/M_{\text{eff}}} \tag{6}$$

M_{eff} is defined by equation (7), where M is the true viscosity ratio, μ_o/μ_s.

$$M_{\text{eff}} = (0.78 + 0.22M^{\frac{1}{4}})^4 \tag{7}$$

In this case, $\mu_s = 1$. The effective viscosity is found from using coefficients which are determined by fitting to Blackwell's experiment [13]. This model is extremely fast to solve and very simple to use.

The solution from the Todd & Longstaff model needs to be in a comparable form to the fine grid solution, which is given as concentration with time. Concentration is input to equation (6) as a linear drop across the reservoir from 1 to 0, defining the fractional flow. The derivative of the fractional flow describes the speed at which the fingers travel, [11], and can therefore be expressed as,

$$\frac{df}{dc} = \frac{x}{t}. \tag{8}$$

The Todd & Longstaff model is scalable, giving $x = 1$ at the producer wells. This enables t to calculated as $1/(df/dc)$. This construction of the Todd & Longstaff model does not contain the details of the permeability field. This is evident in Figure 4 where the Todd & Longstaff model fails predict a value for viscosity near the true value, which was noted in section 3.1 to be $\mu = 13$.

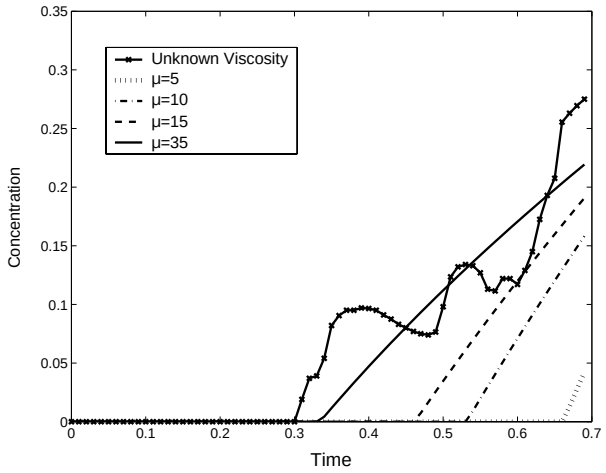

Figure 4. Todd & Longstaff model.

4. THE ERROR MODEL

The goal of an error model is to reduce bias and over confidence in predictions introduced from using a coarse model in the history matching process, so that accurate parameter estimates can be obtained by using limited fine grid simulations. The proposed way to achieve these goals is to include the mean error and time dependent covariance in the history matching process. An error model is incorporated into a solution by way of the misfit function defined in Bayes theorem. In this paper, three misfit definitions will be studied. The first is the least squares misfit, which has already been described as inappropriate for simulation error,

$$M = \sum_{i=1}^{n} \frac{(o_i - s_i)^2}{2\sigma^2}.$$

The second takes the same form as the least squares misfit, and also includes the mean error,

$$M = \sum_{i=1}^{n} \frac{(o_i - s_i - \bar{e})^2}{2\sigma^2}. \tag{9}$$

The third definition includes both mean error and full covariance,

$$M = \frac{1}{2}(o - s - \bar{e})^T C^{-1}(o - s - \bar{e}). \tag{10}$$

If the fine grid simulation is taken as truth, then the simulation error (11), can be calculated from the fine grid simulation minus the coarse grid (Todd & Longstaff) result,

$$e_j = FG_j - CG_j. \tag{11}$$

In equation (11), j is the realisation number. Figure 4 shows 20 fine grid simulations

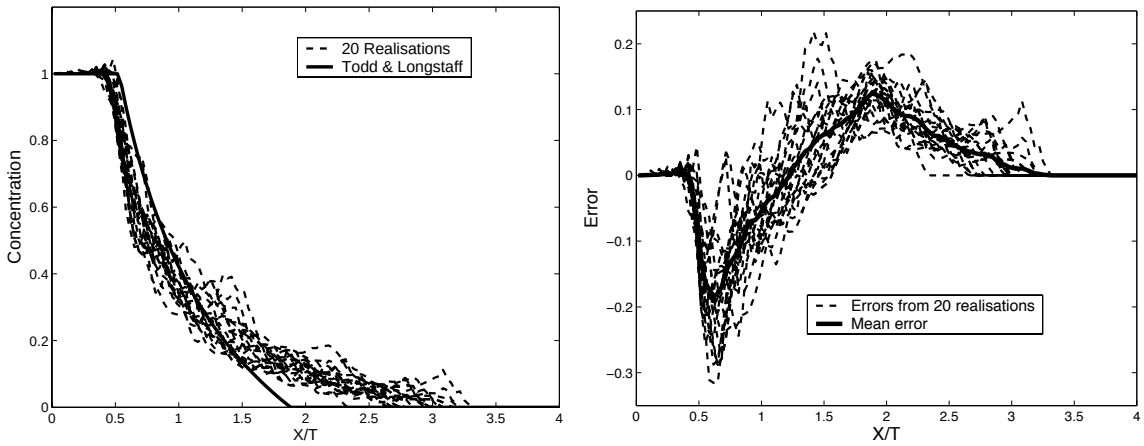

Figure 5. The Todd & Longstaff approximation and mean error.

and the Todd & Longstaff approximation. Todd & Longstaff has an early break through point and is late in sweeping all the oil to the producer well. However it approximates the average behaviour of the concentration quite well. Figure 5 shows the error plots, that is, e_j from equation (11). The curve in bold is the mean error (12), for the chosen viscosity. The covariance is also calculated from the 20 realisations, (13), [14]. Three sets of mean error data and covariance data for error are calculated for viscosity values μ =5, 10 and 15 and these points are known as the *base points* for this example.

$$\bar{e}(\xi) = \frac{1}{n}\sum_{j=1}^{n} e_j(\xi) \tag{12}$$

$$C(s,t) = \frac{1}{n-1}\sum_{j=1}^{n}(e_j(t) - \bar{e}(t))(e_j(s) - \bar{e}(s)) \tag{13}$$

4.1. Interpolated Data

The next stage in constructing the error model is to approximate the mean and covariance between the base points, limiting the fine grid realisations to just the base points of the error model. This means Bayes theorem can be applied over regions for which no fine grid simulations have been run. In this case, MATLAB's linear interpolation function was used. Figure 6 shows the interpolated errors at integer values between the base points.

Observing the peaks and troughs of the interpolated error data, it is seen that this scheme is not ideal. However outside the peaks and troughs, the interpolation scheme performs well, and is for now, used in the model. The interpolation scheme will be reviewed in section 6. For viscosities greater than $\mu = 15$ the data has been set to that of $\mu = 15$, satisfying the prior, 3. Now we have sufficient data for the error model.

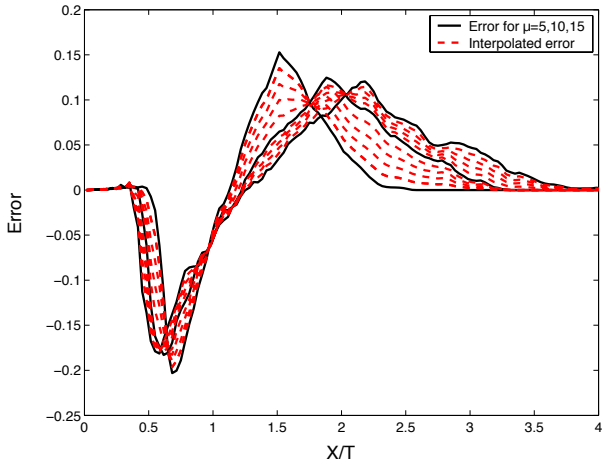

Figure 6. Interpolated data.

5. RESULTS

The three misfit definitions defined in the previous section, will be used to define the likelihood functions, giving 3 probability distribution curves. Recall the true value of the viscosity of the 'unknown data' is $\mu = 13$. The three definitions are referred to as

- LSQ
  Least squares model, (5)
- PME
  Least squares with mean error included, (9)
- FEM
  Full error model, which includes mean error and covariance, (10)

The prior model states the viscosity is given by a uniform probability distribution function in the range $\mu = 5 \ldots 25$. Using the data with 'unknown' concentration the three likelihood functions are calculated, predicting the probability distribution functions shown in Figure 7. The likelihoods were obtained by the coarse grid model and the interpolated fine grid model being defined at 0.1 viscosity increments up to 0.7 pore volumes injected. The LSQ misfit predicts a maximum likelihood of $\mu = 25$ for viscosity. The true value is $\mu = 13$, showing the LSQ model has given a heavily biased result. The other two misfit models show the bias effect to be almost completely removed.

The main difference between these two distributions is the variance, or width of the probability distribution. The PME model used a guessed variance of $\sigma^2 = 0.01$, which gives a fairly wide likelihood curve. The FEM model predicts close to the actual truth with a tighter distribution.

A benefit to having a more exact range for variance lies in the history matching process. When using a single value for variance, parameter space maybe defined either unnecessarily large, extending computation time as the prior contains unnecessary information,

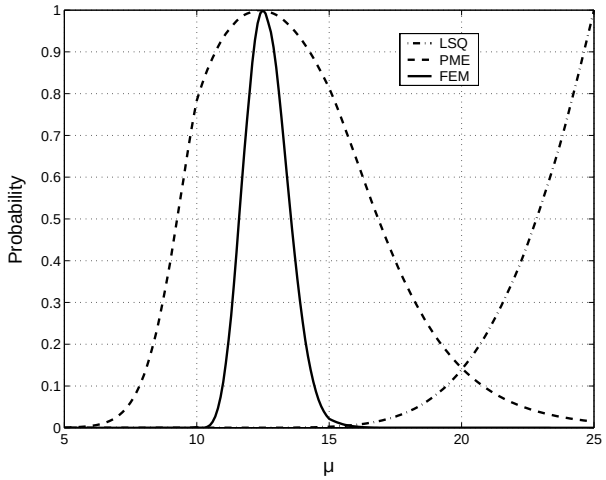

Figure 7. Comparison of likelihood functions, mean of 20 realisations.

or too narrow so that some important parts of parameter space are neglected. Using a correct variance we can home in on the correct region of parameter space more quickly.

5.1. Overall Performance

Figure 8 shows the overall performance of the different misfit functions. The x-axis shows the true viscosity that was input to the Todd & Longstaff model, and the y-axis shows the predicted value or maximum likehood prediction. The least squares approach over predicts the viscosity value in each case and quickly reaches the prior restriction set in the model, $\mu = 25$ for viscosity. The remaining two misfits perform better than the least squares approach, lying closer to the ideal case. Figure 8 shows the FEM to predict only slightly closer to the ideal case than PME, that is, it appears there is no noticeable advantage in using the full error model. If this is the case, it would make sense to use PME misfit to reduce the computing time. Figure 9 shows error bounds (p10 and p90 estimates) on the maximum likelihood predictions for the two mean error models. Also plotted on both graphs is the ideal case. It is clear the full error model shows a smaller range of viscosity predictions than the PME case, making the full covariance calculation worthwhile.

5.2. Compared with Detailed Simulation

The averaged set realisations provided as production data was accurately shown to have a maximum likelihood value of near $\mu = 13$ when tested with the FEM. The likelihood function for this realisation can be compared with the likelihood function created from using detailed simulation as simulation data.

Fine grid simulations were run for viscosities at integer values between $\mu = 5 \ldots 15$, and the error computed (11). The errors at these integer values are then interpolated with MATLAB's linear interpolation function at 0.1 increments to produce fine grid data over a range of viscosity values. It has been shown in section 3.2 that using a coarse

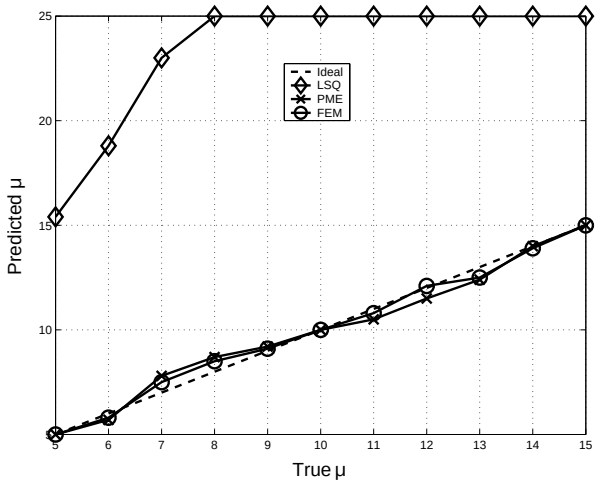

Figure 8. Overall performance of the misfit functions.

grid model in simulation gives a significant model error which is correlated in time. This correlation stemmed from the coarse grid model predicting early break through and being late in sweeping the oil to the producer well. When fine grid simulation is used as the simulated data this error correlation does not exist, as for this example the observed data and the simulated data were created with the same grid size and set of statistics. The likelihood for the detailed simulation case has therefore been created with least squares model alone as an error model for simulation is not required. The LSQ likelihood function for fine grid simulation is shown in Figure 10 with the FEM likelihood result.

A calibration curve is plotted to demonstrate how close the two likelihoods are in their predictions. This is done by calculating the cumulative probabilities from the likelihood curves. Figure 10 plots the cumulative data for the likelihood belonging to the FEM on the x-axis, and the cumulative data for the detailed simulation on the y-axis. The resulting curve is close to the ideal case, which is shown as the straight line. If the two likelihood functions were indistinguishable the calibration plot would show this as a straight line. This is represented in the calibration plot Figure 10 as the ideal case. The actual calibration plotted in the figure does not stray too far from the ideal case.

6. ISSUES

The results from the previous section are promising and the rest of the paper is dedicated to studying details of the error model. We investigate whether similar results can be obtained for less computation effort. Also other aspects are investigated such as the interpolation method.

6.1. Base Points

The first part of the error model to be revisited is the number of base points. Previously 3 base points were chosen, at $\mu = 5$, 10 and 15. As the base points are the part of the model that requires some fine grid simulation, it is the most computationally expensive

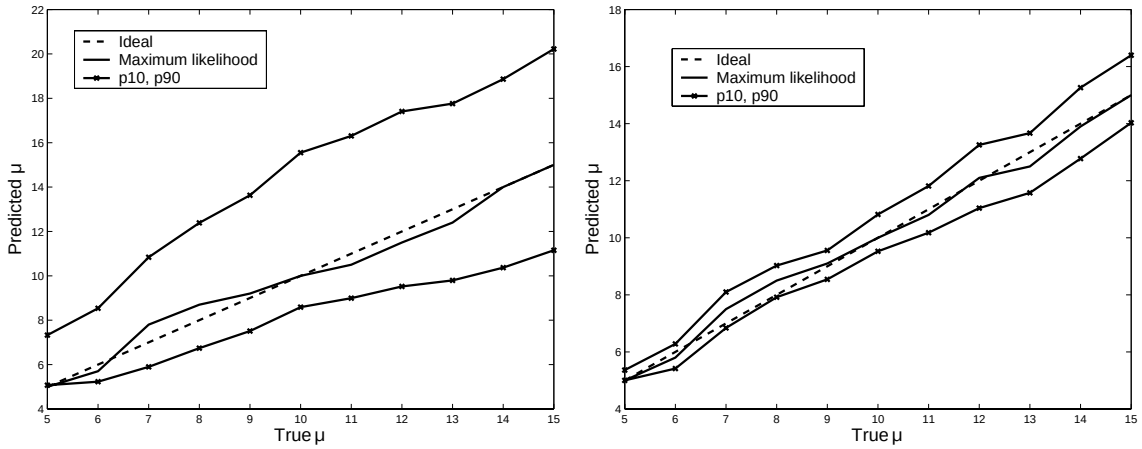

Figure 9. Error bounds for PME (left) and FEM (right).

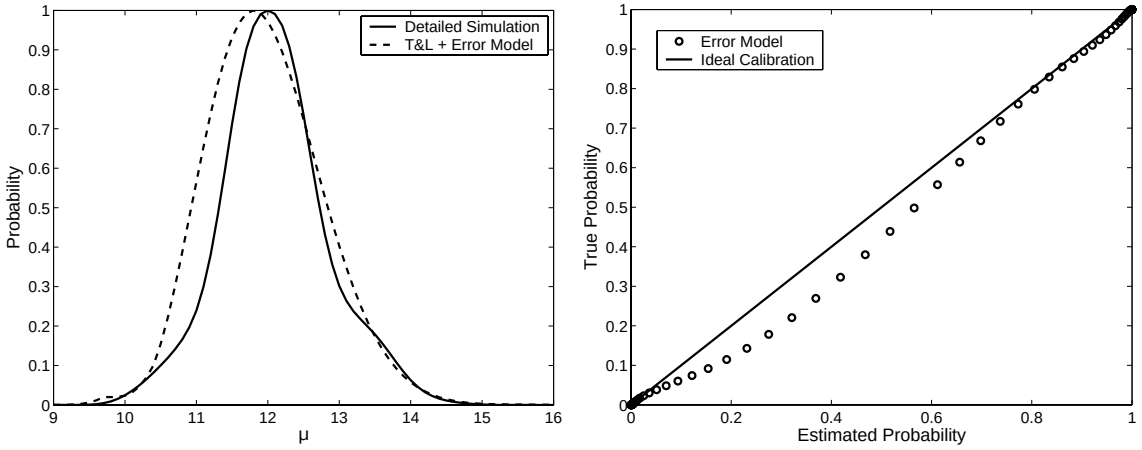

Figure 10. Comparison of Todd & Longstaff model and detailed simulation likelihoods, mean of 20 realisations (left) and calibration plot based on mean of 20 realisations (right).

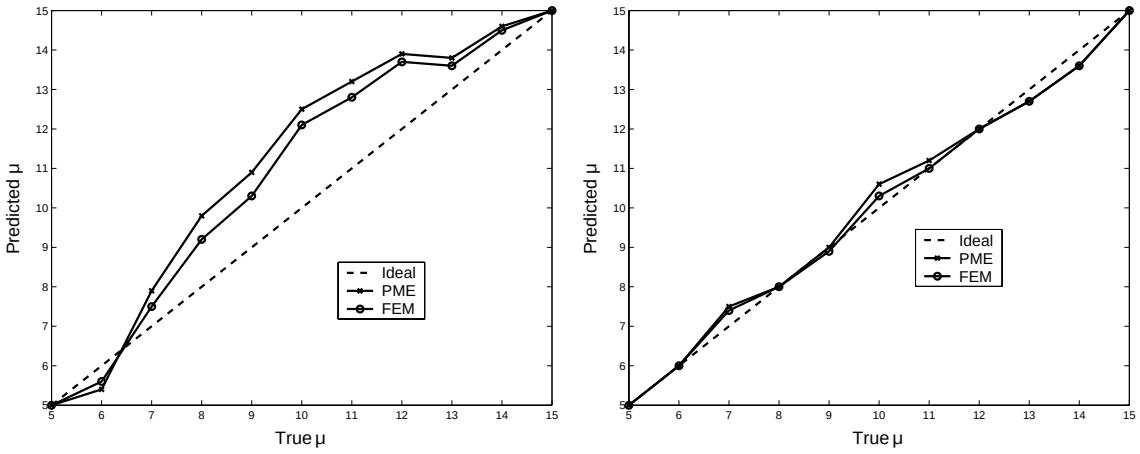

Figure 11. Overall performance: two base points (left) and four base points (right).

part. For this reason, including more or fewer base points than 3 is studied to see the effect on parameter estimation.

First two base points, at $\mu = 5$ and 15 are examined, Figure 11. Only the PME and FEM are compared, as the LSQ model is not competitive against these two models. Near the base points, PME and FEM are able to predict viscosity well. Away from the base points, both likelihoods begin to show signs of bias. The two models have similar behaviour and again as in the case with 3 base points, the FEM is slightly more accurate in predicting the maximum likelihood value. While bias effects are visible away from the base points, they are not as extreme as in the LSQ model.

Figure 11 shows predictions from both models from using 4 base points, at $\mu = 5, 8, 12$ and 15. Again the FEM narrowly obtains more accurate predictions. Overall there doesn't appear to be much gained in using 4 base points over 3. As the fine grid realisations are expensive to compute, it would be advised not to compute them unless necessary.

Table 1 shows results for PME and FEM with 2, 3 and 4 base points. All scenarios correctly predict at the base points, as that is how the error model is defined. Between the base points, typically the likelihoods slightly over predict rather than under predict and this is most noticeable with just two base points.

6.2. How Many Realisations Should be Used?

Different realisations created by a random permeability field generator give rise to differing predictions. This study has used a set of 20 realisations to compute the likelihoods at each base point. Although the results are not presented in this paper, it was found that using different sets of 20 realisations gave differing results for parameter predictions. As a consequence, we investigate how many realisations are required in simulation for optimum results. Figure 12 shows likelihoods created by using increasing numbers of realisations. The first likelihood curve has used 5 realisations in the error model, the second 10, and so on. As more realisations are used, the maximum likelihood is converging to a value,

Table 1. Comparison of 2,3 and 4 base points.

	PME			FEM		
True	2	3	4	2	3	4
5	5.0	5.0	5.0	5.0	5.0	5.0
6	5.4	5.7	6.0	5.6	5.8	6.0
7	7.9	7.8	7.5	7.5	7.5	7.4
8	9.8	8.7	8.0	9.2	8.5	8.0
9	10.9	9.2	9.0	10.3	9.1	8.9
10	12.5	10.0	10.6	12.1	10.0	10.3
11	13.2	10.5	11.2	12.8	10.8	11.0
12	13.9	11.5	12.0	13.7	12.1	12.0
13	13.8	12.4	12.7	13.6	12.5	12.7
14	14.6	14.0	13.6	14.5	13.9	13.6
15	15.0	15.0	15.0	15.0	15.0	15.0

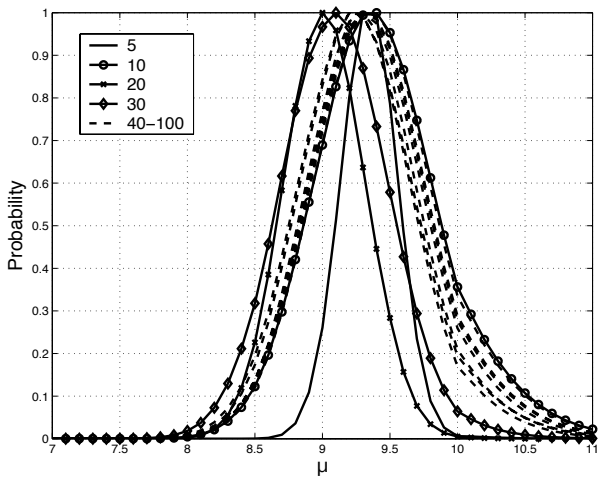

Figure 12. Likelihoods for increasing numbers of realisations.

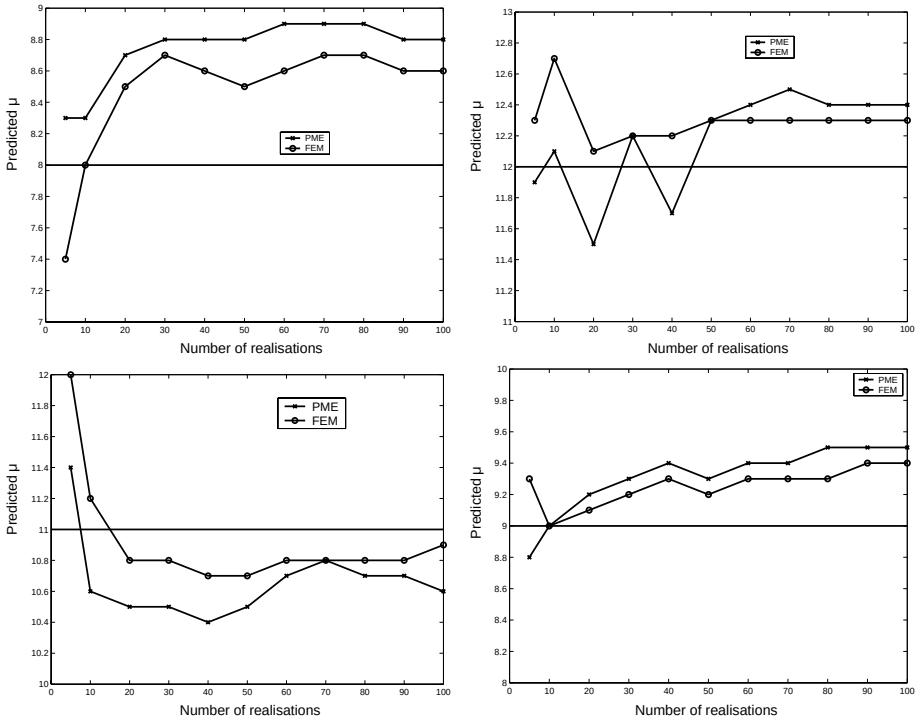

Figure 13. Convergence of likelihood functions, $\mu = 8, 9, 11$ and 12.

however it is not to the true viscosity, showing there is still a slight presence of bias. Figure 13 shows as the number of realisations used increases, the predicted viscosity tends toward some value. Although some bias is still present, there is not much variation in prediction with the full error model after 20 realisations. 20 realisations seems an appropriate number in this case.

6.3. Interpolation Scheme

As stated earlier, the interpolation scheme used thus far is MATLAB's linear interpolation scheme, Figure 14. This interpolation scheme did not adequately represent the peaks and troughs for the error data. An interpolation scheme has been created to produce a fair representation for the expected form of the error, Figure 14. This interpolation scheme focused on 3 points. The first point being where the mean error plot left zero, A. The second at its maximum, B, and the third, the end point, C. Between $\mu = 5$ and 10, the necessary number of evenly separated points were added, giving new A, B, C for intermediate viscosities. To link the points, the shapes of the two outer curves were followed. Some smoothing was necessary. The same was done for between $\mu = 10$ and 15 and the final effect is shown in the Figure 14.

The next step is to compare the maximum likelihood estimates with both schemes. The results are shown in Figure 15. By eye, looking at the limited examples, one interpolation scheme does not appear more favourable over the other. A more thorough investigation took place, studying the $5 \ldots 100$ sets of realisations with both interpolation schemes. 121

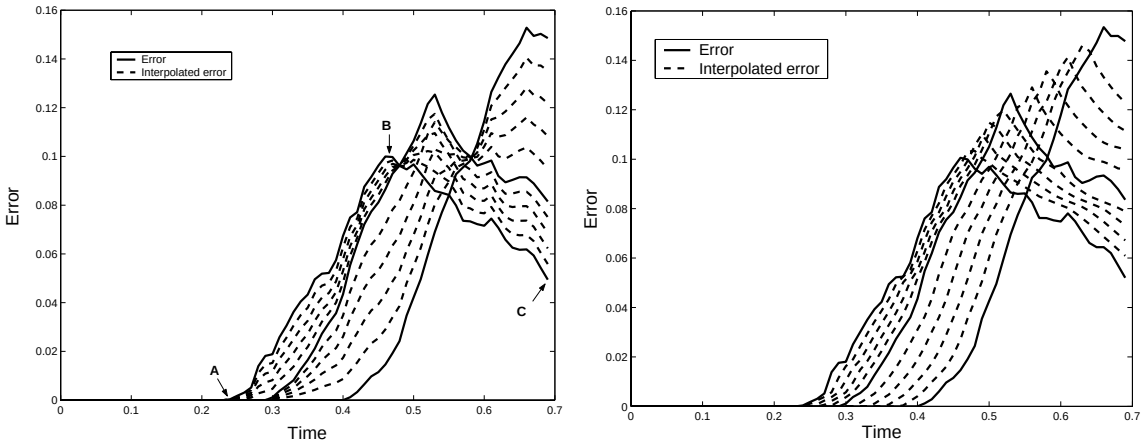

Figure 14. MATLAB's interpolation scheme (left) and improved interpolation scheme (right).

Table 2. Comparison of interpolation schemes.

5...100	PME	FEM
MATLAB	19	30
IMPROVED	57	40
EQUIVALENT	45	51

comparisons are made, 11 different sets of realisations for 11 viscosities. The results are summarised in Table 2, showing on how many occasions each method performed best or if they were equivalent. The improved scheme produced a more accurate estimate more often than MATLAB's linear interpolations scheme. They performed equally well on a number of occasions.

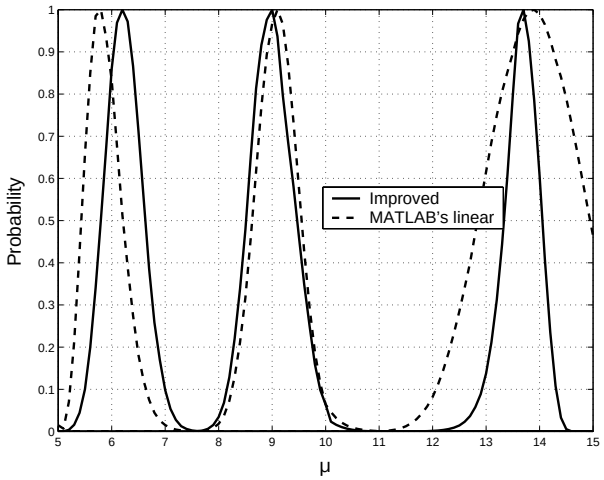

Figure 15. Comparison of interpolation schemes.

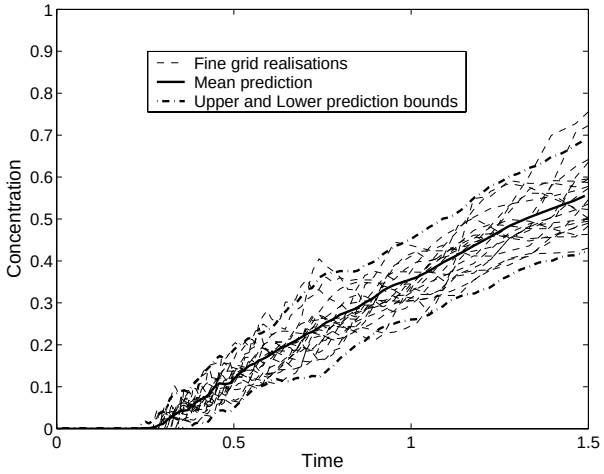

Figure 16. Error model prediction.

7. PREDICTIONS

The error model has shown to significantly reduce the bias introduced from using a coarse model, giving an accurate estimation of the parameter with limited effort. The purpose of determining properties for the system in this way is to be able to use the values to make accurate predictions for oil recovery. The next step is to check how well the error model is able to make a prediction.

The concentration data output from the simulator was defined for each cell in terms of $\xi = X/T$. The data is converted to units of time by realising the length of the reservoir is 1 when compared it to the scalable Todd & Longstaff model, thus $T = 1/\xi$, section 3.2. Hence, we can look forward in time. The 20 fine grid realisations for $\mu = 13$ are extended to $T = 1.5$ and are shown as black dotted curves on Figure 16. To see how the error model solution compares with these fine grid simulation results, the Todd & Longstaff result was calculated for $\mu = 13$ and the error model applied. The error model data consists of mean error and covariance for a range of viscosity values. To apply the error model to the coarse grid solution, the mean error from the error model for $\mu = 13$ is added to the coarse grid solution. This removes the bias incurred from the Todd and Longstaff solution. This is shown by the black solid curve in the figure. Next, $\pm 2\sigma$ for $\mu = 13$ from the error model was added to the newly calculated mean solution. These are bands are the spread of results predicted from using the error model with the coarse grid solution. These bands do encompass most of the fine grid results, making an accurate prediction.

8. CONCLUSIONS AND DISCUSSION

Using a coarse model in simulation for parameter estimation is necessary for a practical computation time. Using a least squares misfit in combination with a coarse grid model introduces simulation error, giving biased and overconfident parameter estimates. This provides a motivation for error modelling in reservoir prediction.

A 2D viscous fingering example was introduced, providing a means for explaining how an error model should be constructed and tested for parameter estimation. The parameter to be estimated is the viscosity of the oil. The error model was constructed in the following way.

- 20 fine grid realisations were generated at 3 base points, $\mu = 5, 10$ and 15.
- A coarse grid approximation known as the Todd & Longstaff model was calculated for the same three viscosity values.
- The simulation error for each viscosity in the example is found by subtracting the coarse grid solution from the fine grid solution.
- The mean and variance of the error for each base point is calculated and MATLAB's linear interpolation function is used to predict the mean error and variance at intermediate viscosity values.
- The error data is added to the coarse grid solution producing a fast simulation model with the bias removed by including mean error and overconfidence avoided by using a fair representation of the spread of error.

A number of variables were studied to minimise the amount of work put into the model with maximised results.

- It was found for the example given that 3 base points were sufficient. There was no significant gain in accuracy for the work put in with using 4 base points. Fewer than 3 base points increased bias effects.
- The number of realisations used for fine grid simulations was varied from $5 \ldots 100$. 20 realisations were more than adequate, with no further improvement gained from using 100 realisations.
- The interpolation scheme was improved, allowing peaks and troughs of the error data to be accurately resolved. This had a positive effect on parameter estimation, predicting values closer to the actual value than with the initial scheme.

Finally the error model was used to make a prediction. The coarse grid result was shown to make an accurate prediction when the error model data was added. The mean error significantly reduced the bias effects while the covariance gave a realistic spread from the mean prediction.

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